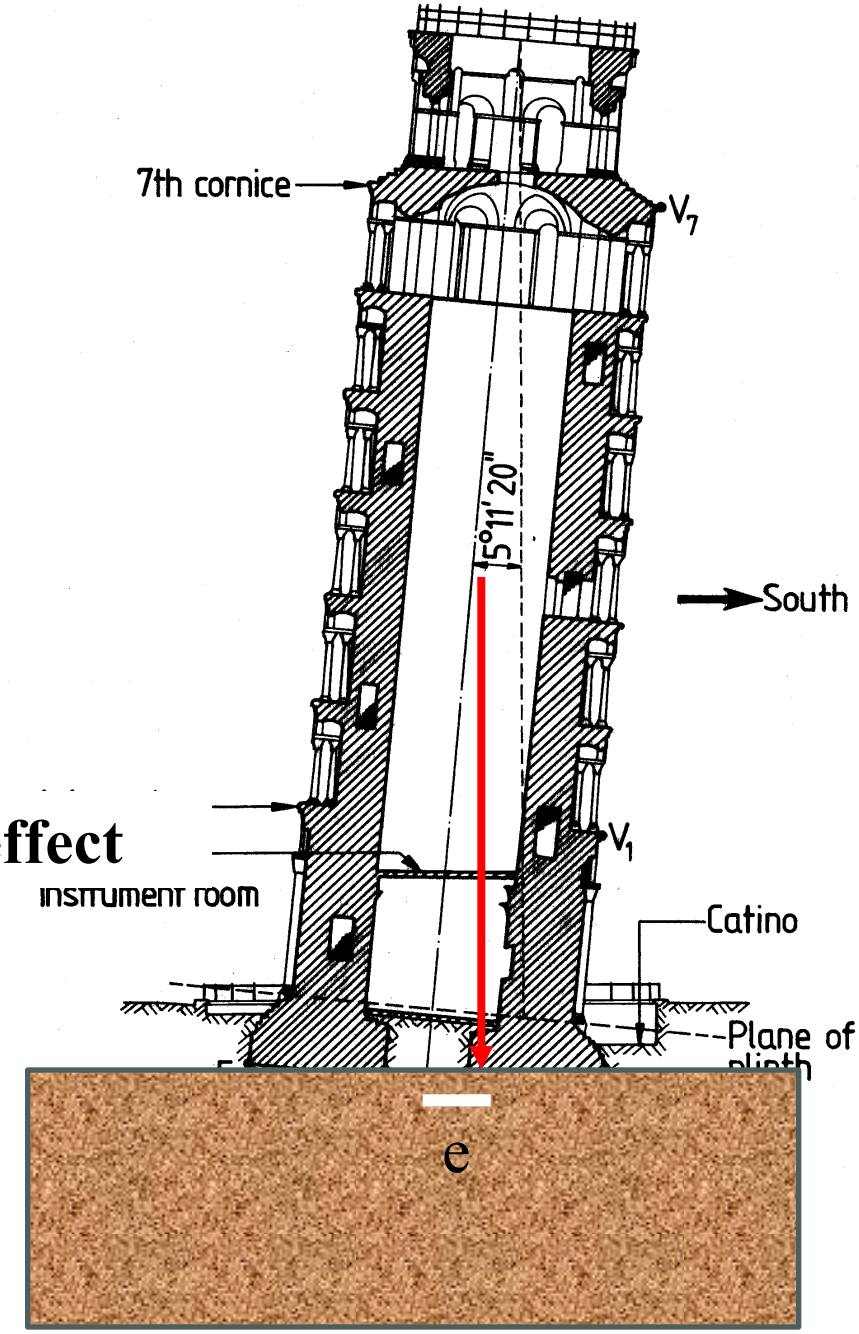
1. Consider a tall, relatively rigid structure resting on a **deformable** soil foundation.
2. Inclination α creates eccentricity e between the vertical load and the original foundation centreline.
3. The structure's weight W then generates an overturning moment: $M_{overturning}=We=Wh\sin\theta$

This is the destabilising geometric effect



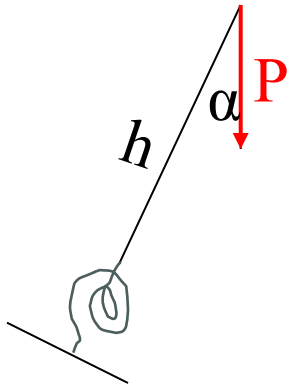
The role of foundation rotational stiffness

The soil provides resistance against foundation rotation.

For small rotations, we can approximate its moment–rotation response as: $M_{resisting}=K_{\theta}\theta$

where K_{θ} is the rotational stiffness of the soil–foundation system

CRITICAL CONDITION: $K_{\theta}=<Wh$ ($M_{resisting}<M_{overturning}$)



The essential point is that leaning instability can develop without a conventional bearing-capacity failure

- **Geometric destabilisation**: Increasing inclination increases load eccentricity and overturning moment
- **Geotechnical response**: Soil deformation, **consolidation** and nonlinear foundation stiffness determine whether the foundation can resist the additional moment

The tower's substantial weight, high centre of gravity, shallow foundation and compressible underlying deposits created conditions in which relatively small changes in inclination could generate significant additional overturning moments

Imagine perturbing the tower by a small additional rotation

The incremental restoring moment is: $dMr = \frac{dMr}{d\theta} d\theta$

The incremental destabilising moment is: $dMg = Whcos\theta d\theta$

<u>At critical state</u>: $\frac{dMr}{d\theta} = Whcos\theta$

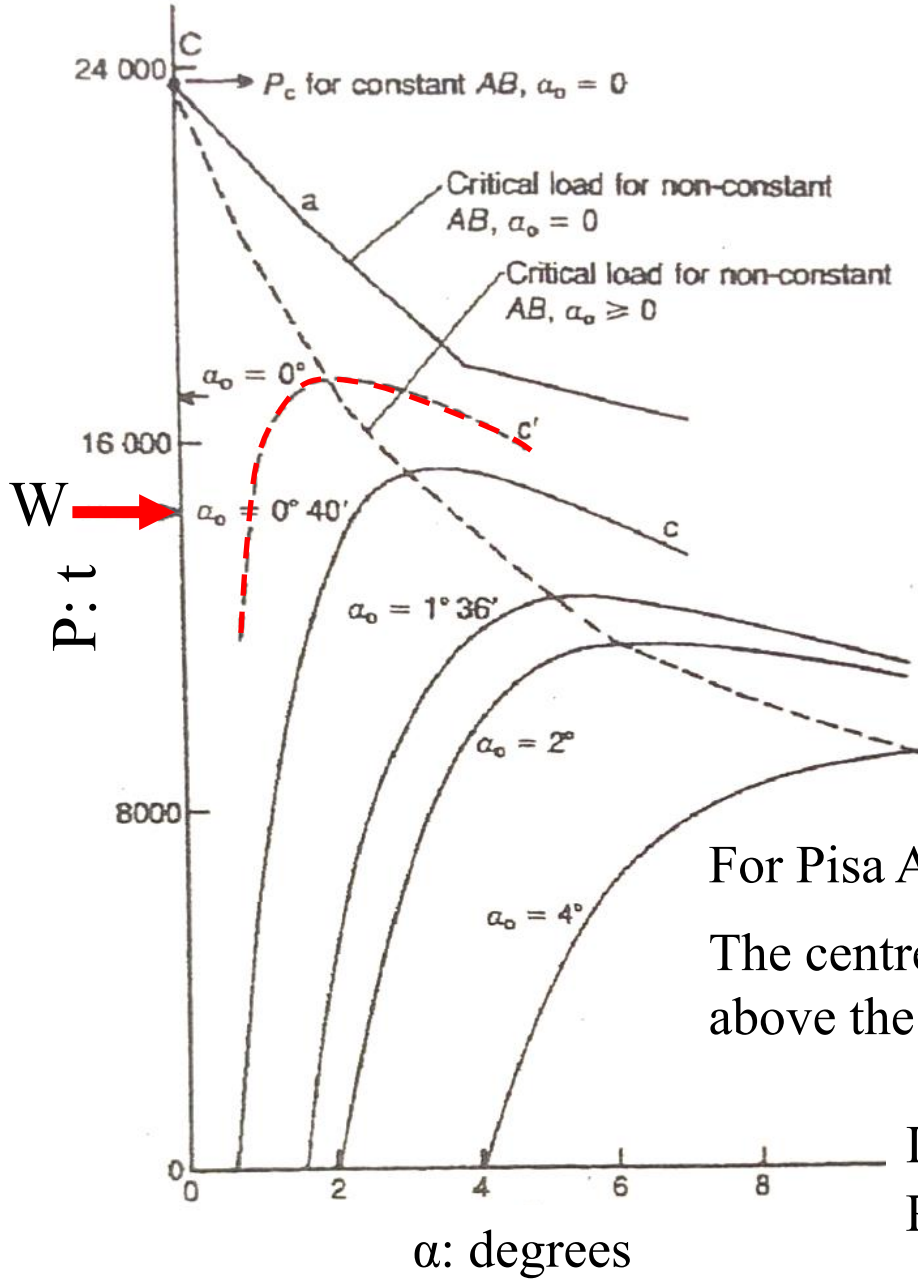
The critical quantity is the tangent rotational stiffness of the soil–foundation system (non-linear)

Lancellotta represents the soil–foundation restoring moment with an exponential relationship: $Mr = A[1 - e^{-B(\theta - \theta o)}]$

where:

- A is the asymptotic restoring moment capacity attained for large θ
- B controls how rapidly rotational stiffness decreases with rotation, with units of inverse radians
- θo represents the initial inclination or imperfection, including the effect of accumulated creep rotation
- θ-θo is the rotation that mobilises the spring resistance

Differentiating gives the *tangent rotational stiffness*: $\frac{dMr}{d\theta} = K_{\theta,t}$



$$K_{\theta,t} = ABe^{-B(\theta-\theta o)}$$ θ=θo??

Notice that the stiffness progressively decreases as rotation increases (plot $K_{\theta,t}/AB$ ↓ and M_r/A ↑ and reproduce curve for θo=αo=2)

The soil may still be developing additional resisting moment, but its incremental stiffness can become insufficient to counteract the destabilising effect of gravity.

For each possible rotation θ, the equilibrium load is: $P_\theta = [M_r = A[1 - e^{-B(\theta-\theta o)}]/h\sin\theta$

$\theta_o=0$, $P_{critical}=AB/h$ (a curve)

For Pisa AB=550000tm/rad & A=60000tm (c curve)

The centre of gravity is approximately h=23 m above the foundation level → Pc=24000 t

If tendons applied F=300t at h_1=11m

$P_\theta h\sin\theta - Fh_1 = A[1 - e^{-B(\theta-\theta o)}]$ (c' curve)

Leaning instability is a coupled geometric–geotechnical instability

Mechanism	Governing phenomenon
Bearing-capacity failure	Inadequate soil resistance to foundation loading
Differential settlement	Unequal vertical displacements beneath the foundation
Leaning instability	Loss of rotational stability due to interaction of self-weight eccentricity and foundation rotational stiffness
Structural buckling	Loss of structural stability under compressive loading

- In leaning instability the critical condition depends on the nonlinear moment–rotation response of the foundation, rather than on ultimate bearing capacity