The chemomechanical constitutive equation for the poroelastic medium has the following form

$$\underset{\sim}{\sigma}' = \lambda \, \mathrm{tr}\, \underset{\sim}{\varepsilon}' \underset{\sim}{I} + 2\mu \underset{\sim}{\varepsilon}' \quad (10)$$

where the effective stress $\underset{\sim}{\sigma}'$ tensor is defined by the relation

$$\underset{\sim}{\sigma}' = \underset{\sim}{\sigma} + p_w \underset{\sim}{I} \quad (11)$$

and the chemoelastic strain $\underset{\sim}{\varepsilon}'$ is defined by the relation

$$\underset{\sim}{\varepsilon}' = \underset{\sim}{\varepsilon} + \alpha_c C \underset{\sim}{I} \quad (12)$$

where α_c is the coefficient of chemical contraction and ε is the total strain of the medium, λ and μ are the Lamé constants of the solid skeleton.

The balance of momentum equations (3) and (4), lead to the following equivalent equations

$$\mathrm{div}\, \underset{\sim}{\sigma} = 0 \quad (13)$$

$$n^w (\vec{v}_w - \vec{v}_s) = -K_H \vec{\nabla} p_w \quad (14)$$

where K_H is the hydraulic permeability of the fluid diffusing within the pores of the solid.

~~Boundary and initial conditions for the uniaxial deformation of the strip, along the $x = x_1$ direction, where we have the smallest dimension L_1, in the orthogonal strip of dimensions $L_1 \times L_2 \times L_3$.~~

Remember that $L_1 << L_2 << L_3$.

1) Initial condition for the displacement u_s, for $x \in [-\ell, \ell]$ where $L_1 = 2L$

$$(IC)_u : u_s(x, t = 0) = 0 \quad (14)$$

2) Boundary condition for u_s

$$(BC)_u : u_s(x = 0, t) = 0 \quad (15)$$

3) Boundary condition for the stress σ'_{xx}

$$\sigma'_{xx}(x = \pm\ell, t) = \overbrace{\left(H \frac{\partial u_s}{\partial x} + 3K\alpha_c C\right)}^{\sigma'_{xx}}(x = \pm\ell, t) = 0 \quad (16)$$

4) Boundary condition for the pore pressure p_w

$$(BC)_{p_w}: \; p_w(x=\pm l, t)=0. \quad (15)$$

The diffusion problem of the chemical, uncouples from the deformation problem of the medium.

$$(FE)_c: \; \frac{\partial C}{\partial t} - D\frac{\partial^2 C}{\partial x^2} = 0 \quad (16)$$

$$(IC)_c: \; C(x, t=0) = 0 \quad (17)$$

$$(BC)_c: \; C(\pm l, t) = C_0 H(t) \quad (18)$$

with $H(t)$ a step function

Returning to the deformation problem, we can combine the FE (mass and momentum balance equations) and end up with 2 equations in terms of 2 unknowns, the solid skeleton displacement u_s and the pore pressure p_w. There two equations are:

$$k_H \frac{\partial p_w}{\partial x} = \frac{\partial u_s}{\partial t} \quad (19)$$

$$\frac{\partial^2 u_s}{\partial x^2} - \frac{1}{H k_H}\frac{\partial u_s}{\partial t} + 3\alpha_c \frac{K}{H}\frac{\partial C}{\partial x} = 0 \quad (20)$$

and can be solved for u_s and p_w.

The tensile force $F_3(t)$, along the x_3 longitudinal direction of the strip, is given via the relation

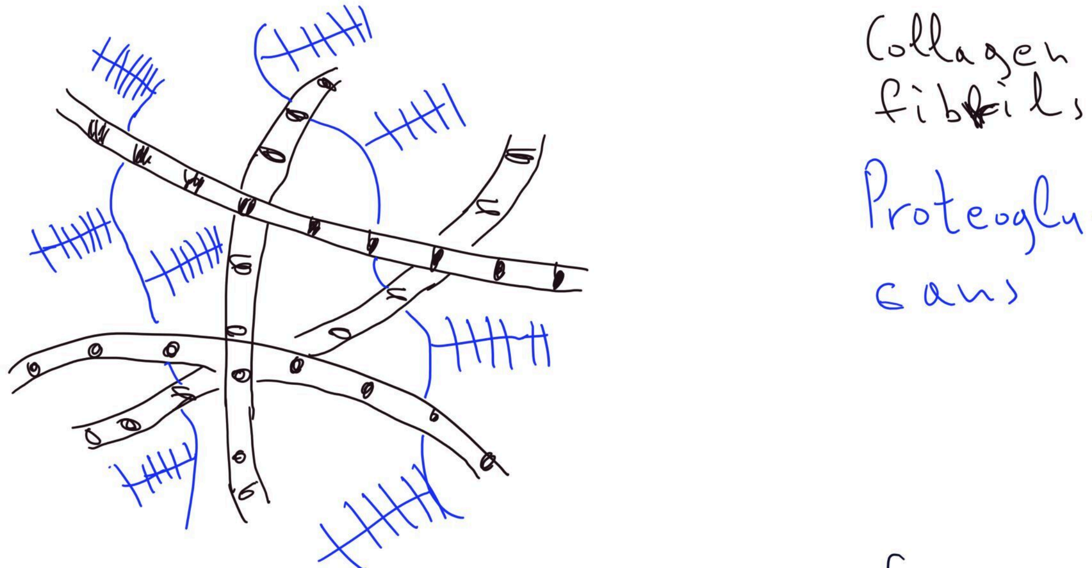
$$F(t) = L_2 \int_{-\ell}^{\ell} \sigma_{33}^s(x,t)\,dx \qquad (21)$$

where

$$\sigma_{33}^s = \lambda \frac{\partial u_s}{\partial x} + 3K\alpha_c C - n^s p_w \qquad (22)$$

elastic + chemical + fluid contributions to the stress

Illustration of the cartilage structure



Proteoglycan repulsion ~~causes~~ force the collagen fibrils to stretch.

Crucial role in the models for cartilage, play the electrochemical potentials:

$$g_k^{ec} = \underbrace{\hat{v}_k p_w}_{\text{mechanical part}} + \underbrace{RT \ln x_k}_{\text{chemical part}} + \underbrace{z_k F \phi}_{\text{electrical part}}$$

$\hat{v}_k$: molar volume of species k

p_w: fluid (water) pressure in the pores

R: universal gas constant

T: absolute temperature

x_k: the concentration of the species k in the mixture

z_k: is the valence of species k

F: Faraday's constant

ϕ: electrical field