PROBLEM SET 4

PROBLEM 1: Let X, Y be topological spaces and $f: X \to Y$ has closed graph. Show that if $K \subseteq Y$ is compact then $f^{-1}(K) \subseteq X$ is closed.

PROBLEM 2: Let H be a Hilbert space and $A, B: H \to H$ linear operators such that

$$(A(x), u) = (x, B(u)) \quad \forall x, u \in H.$$

Show that $A, B \in \mathcal{L}(H)$.

PROBLEM 3: Let X, Y, Z be Banach spaces and $\beta: X \times Y \to Z$ a bilinear map which is separately continuo[us]. Show that $\beta(\cdot,\cdot)$ is jointly continuous.

PROBLEM 4: Let X, Y be topological spaces, $D \subseteq X$ dense $f, g: X \to Y$ continuous maps such that

$$f(u) = g(u) \quad \forall u \in D.$$

Show that $f \equiv g$.

PROBLEM 5: Let X, Y be Banach spaces and $A \in \mathcal{L}(X, Y)$. Show that A^* is 1-1 iff $A(X) \subseteq Y$ is dense