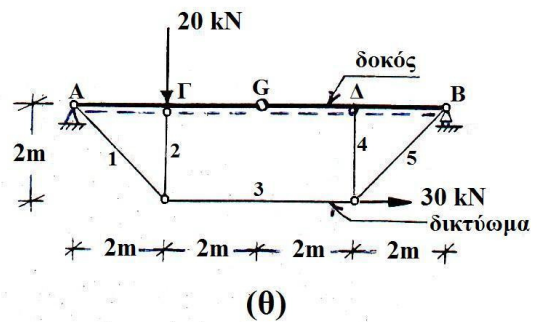
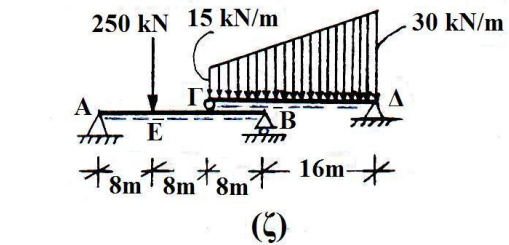
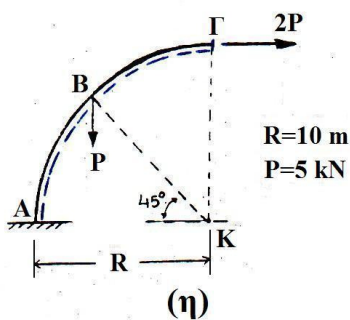
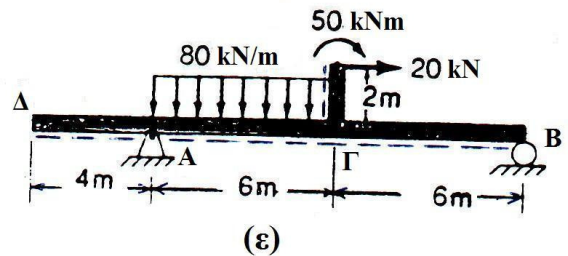
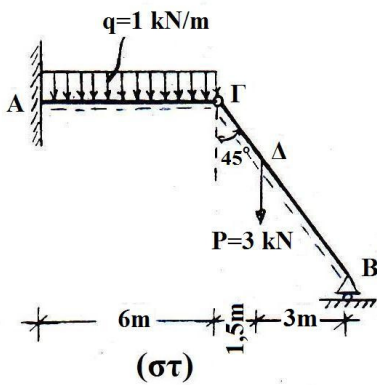
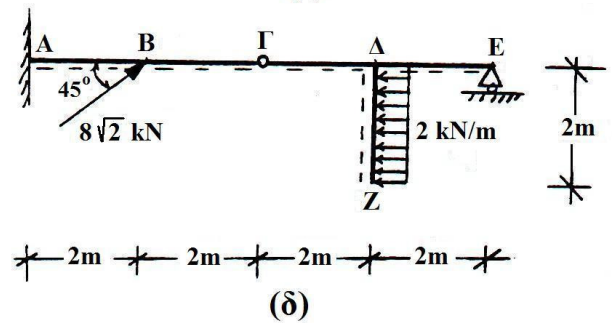
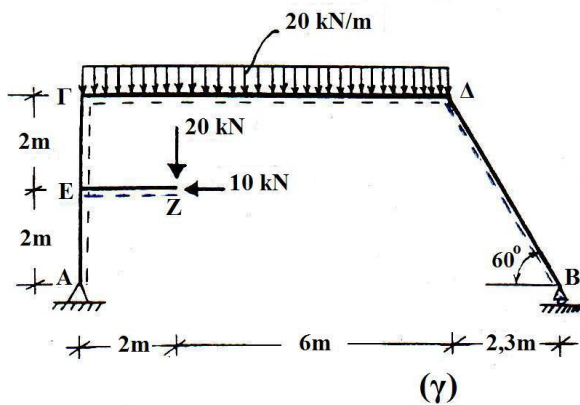
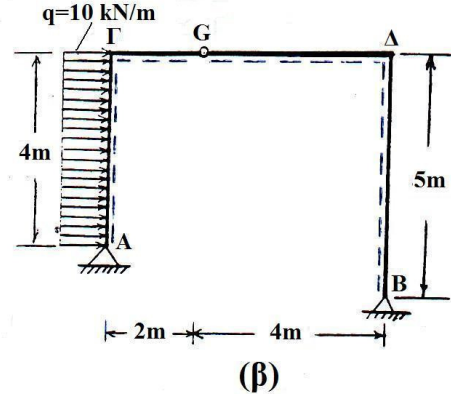
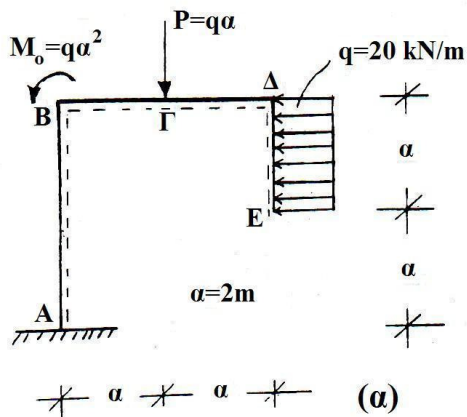


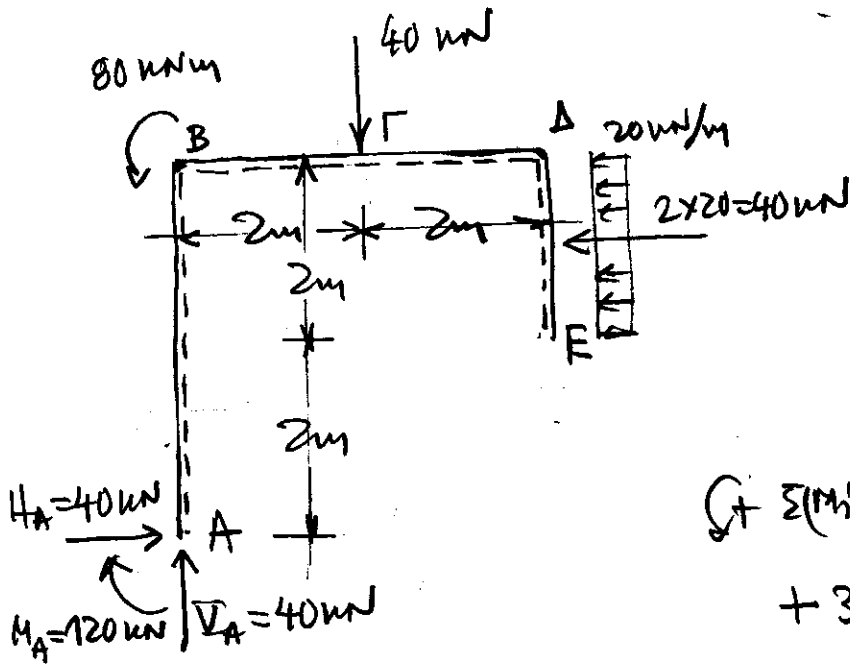
1. Στους παρακάτω φορείς να γίνουν τα διαγράμματα αξονικών δυνάμεων (Δ.Α.Δ.), τεμνουσών δυνάμεων (Δ.Τ.Δ.) και ροπών κάμψεως (Δ.Ρ.Κ.).



Άσκηση 1 (α)

"Πλαίσιο πρόβλημα"

Υπόψη Ανάλυ. - Εξ. Ενω. loop



$$\sum x_i = 0 \Rightarrow H_A - 40 = 0$$

$$\boxed{H_A = 40 \text{ kN}}$$

$$\sum y_i = 0 \Rightarrow V_A - 40 = 0$$

$$\boxed{V_A = 40 \text{ kN}}$$

$$\sum (M_i)_A = 0 \Rightarrow -M_A + 80 - 2 \cdot 40 +$$

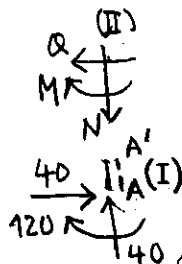
$$+ 3 \cdot 40 = 0 \Rightarrow \boxed{M_A = 120 \text{ kNm}}$$

Διαιρέματα:

Βεβαιώνει πως ^{N, Q, M} σε χαρακτηριστικά σημεία της αραμής (β ζόνος)

Σημείο Α:

1^η εντομή -
- κατω (I)



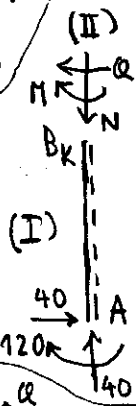
$$N_{A'} = -40 \text{ kN}$$

$$Q_{A'} = -40 \text{ kN}$$

$$M_{A'} = 120 \text{ kNm}$$

Σημείο Β κίον:

1^η εντομή -
- κατω (I)



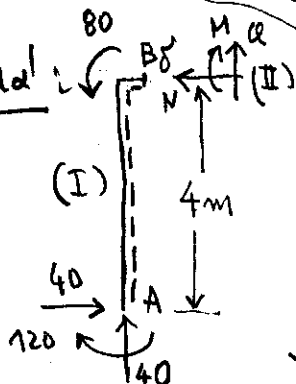
$$N_{Bk} = -40 \text{ kN}$$

$$Q_{Bk} = -40 \text{ kN}$$

$$M_{Bk} = 120 - 4 \cdot 40 = -40 \text{ kNm}$$

Σημείο Β δέσμη:

1^η εντομή -
- κατω (I)



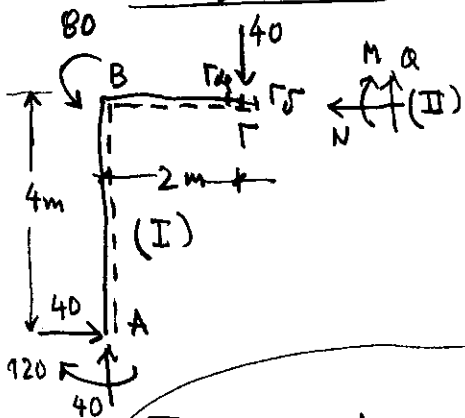
$$N_{B\delta} = -40 \text{ kN}$$

$$Q_{B\delta} = 40 \text{ kN}$$

$$M_{B\delta} = 120 - 4 \cdot 40 - 80 = -120 \text{ kNm}$$

.../...

Στοιχείο Γ



1^η ενδοχυσή -
- τέταρτη (I)

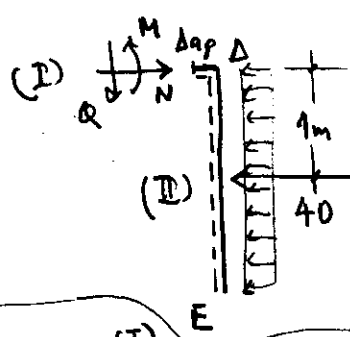
$$N_{\Gamma} = -40 \text{ kN}$$

$$Q_{\Gamma_{\alpha\beta}} = 40 \text{ kN}$$

$$Q_{\Gamma\beta} = 40 - 40 = 0$$

$$M_{\Gamma} = 120 + 2 \cdot 40 - 4 \cdot 40 - 80 = -40 \text{ kNm}$$

Στοιχείο Δ τέταρτη



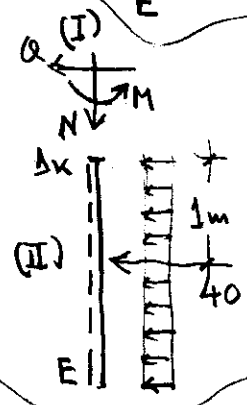
$$N_{\Delta\alpha\beta} = -40 \text{ kN}$$

$$Q_{\Delta\alpha\beta} = 0$$

$$M_{\Delta\alpha\beta} = -1 \cdot 40 = -40 \text{ kNm}$$

2^η ενδοχυσή -
- τρίτη (II)

Στοιχείο Δ τρίτη



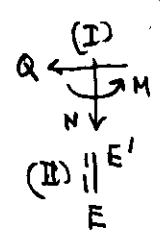
$$N_{\Delta\kappa} = 0$$

$$Q_{\Delta\kappa} = 40 \text{ kN}$$

$$M_{\Delta\kappa} = -1 \cdot 40 = -40 \text{ kNm}$$

2^η ενδοχυσή -
- τρίτη (II)

Στοιχείο E



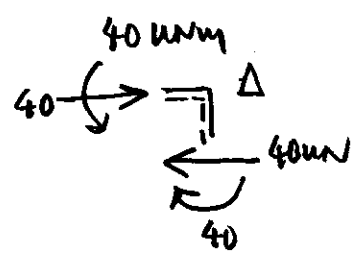
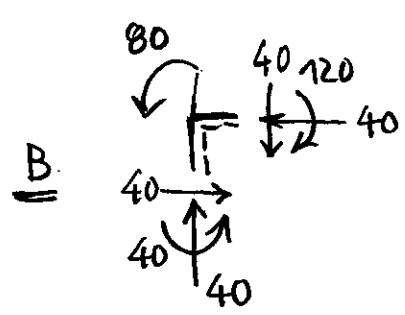
$$N_{E'} = 0$$

$$Q_{E'} = 0$$

$$M_{E'} = 0$$

2^η ενδοχυσή -
- τρίτη (II)

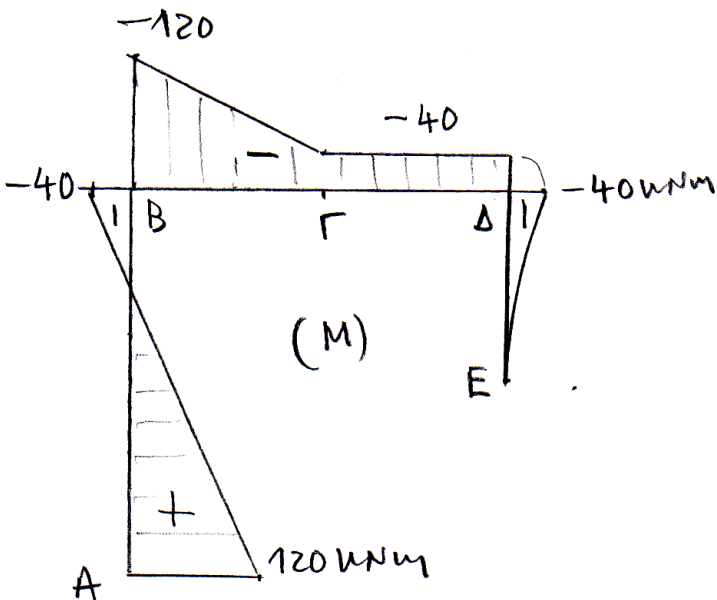
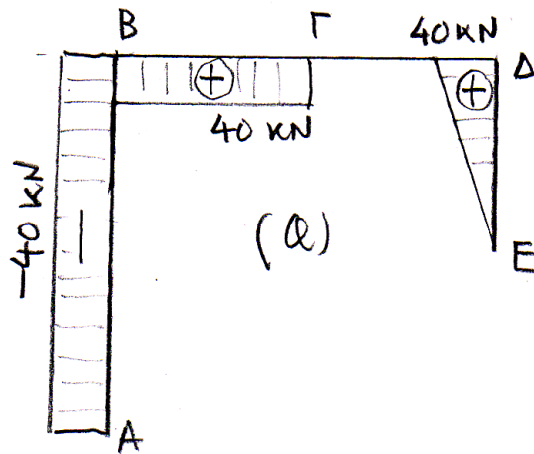
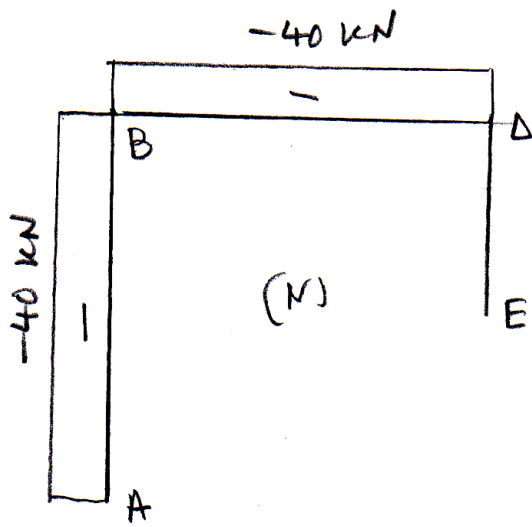
ισορροπία κόμβων



AB, BΓ, ΓΔ: $q=0 \Rightarrow Q = \text{const.} \Rightarrow M: \text{γραμμ. \u03b5\u03c4\u03b9\u03c3}$

\u0394E: $q = \text{const.} \Rightarrow Q: \text{γραμμ. \u03b5\u03c4\u03b9\u03c3} \Rightarrow M: \text{\u039d\u03c1\u03b1\u03c6\u03b1\u03b2\u03b7}$

\u0391 \u0391\u03c1\u03b7\u03b8\u03b5\u03c4\u03b1\u03b9 \u03c1\u03b9\u03c3 \u03c3\u03c4\u03b1\u03b4\u03b9\u03b1\u03b9\u03b5\u03c3 \u03ba\u03b1\u03b9 \u03b5\u03c1\u03c9\u03c4\u03b7\u03c1\u03b5 \u03c1\u03b5 \u03c4\u03b7\u03bd \u03ba\u03c4\u03b1\u03b2\u03b1\u03bd\u03b7 \u03b3\u03c1\u03b1\u03c6\u03bc\u03b9\u03bd.



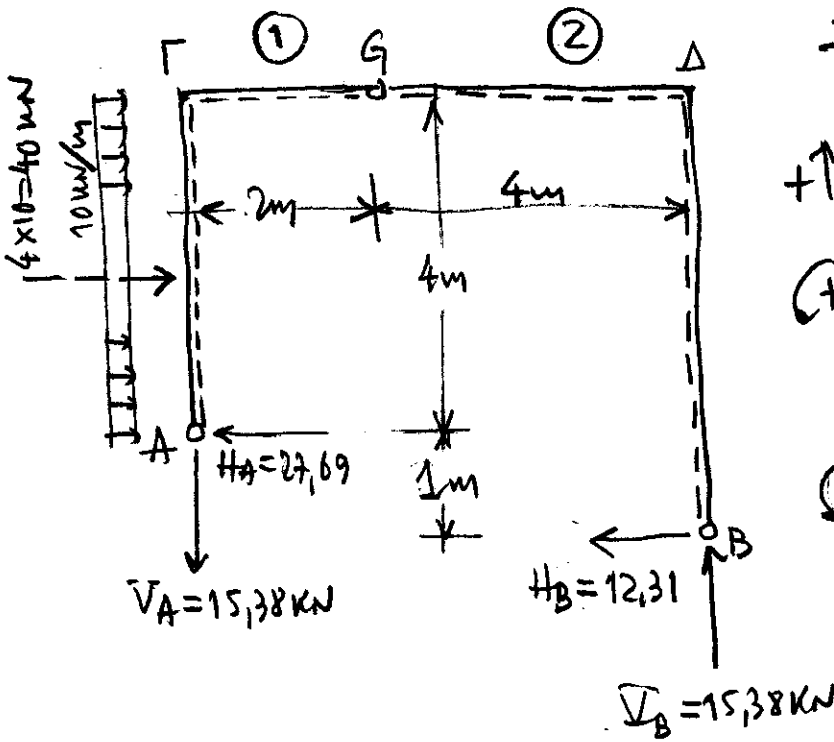
$Q_E = 0 \Rightarrow$

$M_E: \text{\u0391\u03c1\u03b7\u03b8\u03b5\u03c4\u03b1\u03b9} =$
 $= \text{\u0391\u03c1\u03b7\u03b8\u03b5\u03c4\u03b1\u03b9 \u03b5\u03c1\u03c9\u03c4\u03b7\u03c1\u03b5}$

Άσκηση 1 (B)

"Τριάρθρωτο ζόζο"

Υπόψη: Άντλ. - Εξ. Στατ. loop.



$$\rightarrow \sum X_i = 0 \text{ m' } 40 - H_A - H_B = 0 \quad (1)$$

$$\uparrow \sum Y_i = 0 \text{ m' } -V_A + V_B = 0 \quad (2)$$

$$\begin{aligned} \circlearrowleft \sum (M_i)_A = 0 \text{ m' } \\ -2 \cdot 40 + 6 V_B - 1 \cdot H_B = 0 \quad (3) \end{aligned}$$

$$\circlearrowleft \sum (M_i)_G = 0 \text{ m' } 4 V_B - 5 H_B = 0 \quad (4)$$

(3) & (4) \Rightarrow

$H_B = 12,31 \text{ kN}$
$V_B = 15,38 \text{ kN}$

(1) & (2) \Rightarrow

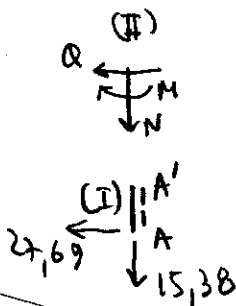
$H_A = 27,69 \text{ kN}$
$V_A = 15,38 \text{ kN}$

Διευρύνματα:

Βεβαιώνει υπόψη & φέρνει στο μάτι στοιχεία της ανάλυσης. (6 ζώνες)

Στοιχείο Α

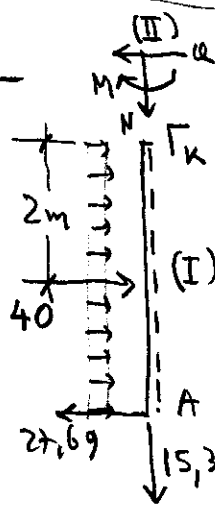
1^η εντάξη -
- καίω (I)



$$\begin{aligned} N_{A'} &= 15,38 \text{ kN} \\ Q_{A'} &= 27,69 \text{ kN} \\ M_{A'} &= 0 \end{aligned}$$

Στοιχείο Γ καίω

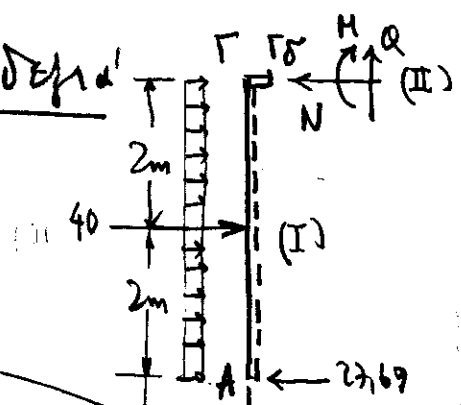
1^η εντάξη -
- καίω (I)



$$\begin{aligned} N_{\Gamma K} &= 15,38 \text{ kN} \\ Q_{\Gamma K} &= 27,69 - 40 = -12,31 \text{ kN} \\ M_{\Gamma} &= 4 \cdot 27,69 - 2 \cdot 40 = 30,76 \text{ kNm} \end{aligned}$$

Στοιχείο Γ Δεξιά

1^η επιλογή -
- καίσιμ (I)



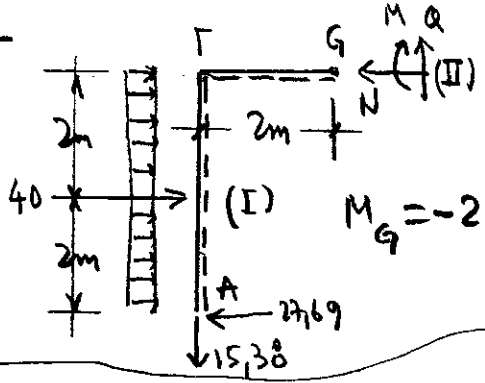
$$N_{\Gamma\delta} = 27,69 - 40 = -12,31 \text{ kN}$$

$$Q_{\Gamma\delta} = -15,38 \text{ kN}$$

$$M_{\Gamma} = 4 \cdot 27,69 - 2 \cdot 40 = 30,76 \text{ kNm}$$

Στοιχείο Γ

1^η επιλογή -
- κεντροειδής (I)



$$N_{\Gamma} = 27,69 - 40 = -12,31 \text{ kN}$$

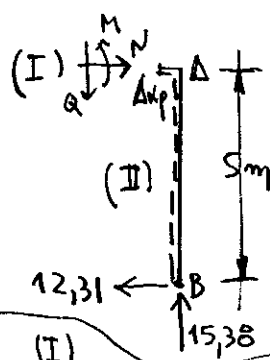
$$Q_{\Gamma} = -15,38 \text{ kN}$$

$$M_{\Gamma} = -2 \cdot 15,38 + 4 \cdot 27,69 - 2 \cdot 40 = 0$$

Γ : Δρθρων

Στοιχείο Δ Δεξιά

2^η επιλογή -
- καίσιμ (II)



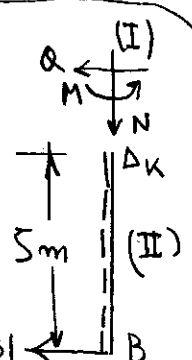
$$N_{\Delta\delta\phi} = -12,31 \text{ kN}$$

$$Q_{\Delta\delta\phi} = -15,38 \text{ kN}$$

$$M_{\Delta} = -5 \cdot 12,31 = -61,55 \text{ kNm}$$

Στοιχείο Δ καίσιμ

2^η επιλογή -
- καίσιμ (II)



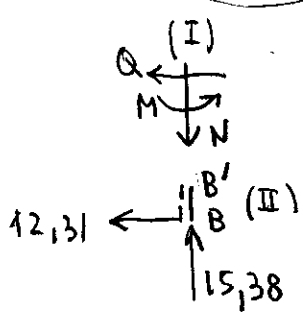
$$N_{\Delta\kappa} = -15,38 \text{ kN}$$

$$Q_{\Delta\kappa} = 12,31 \text{ kN}$$

$$M_{\Delta} = -5 \cdot 12,31 = -61,55 \text{ kNm}$$

Στοιχείο Β

2^η επιλογή -
- καίσιμ (II)

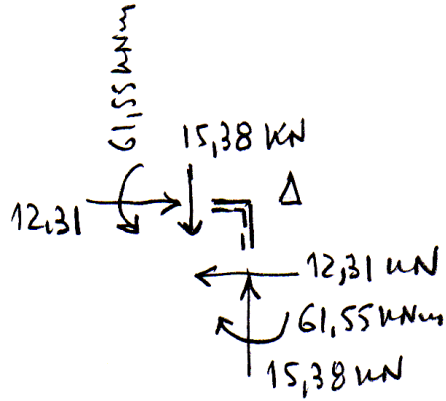
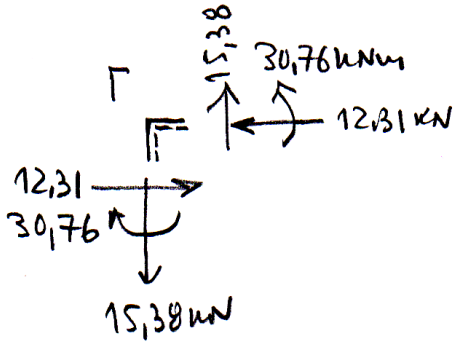


$$N_{B'} = -15,38 \text{ kN}$$

$$Q_{B'} = 12,31 \text{ kN}$$

$$M_{B'} = 0$$

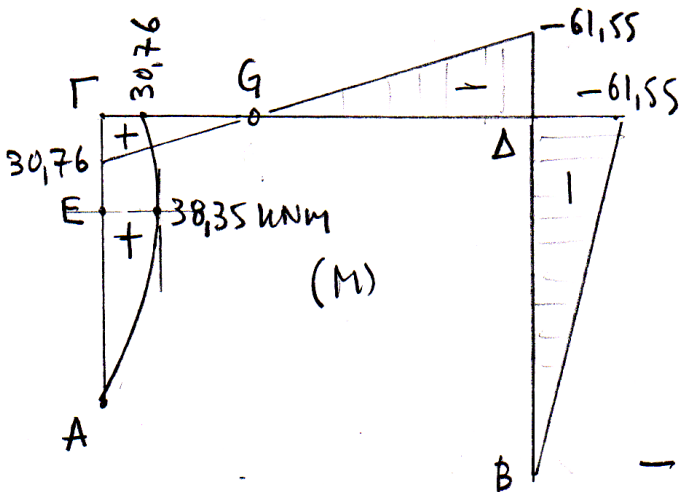
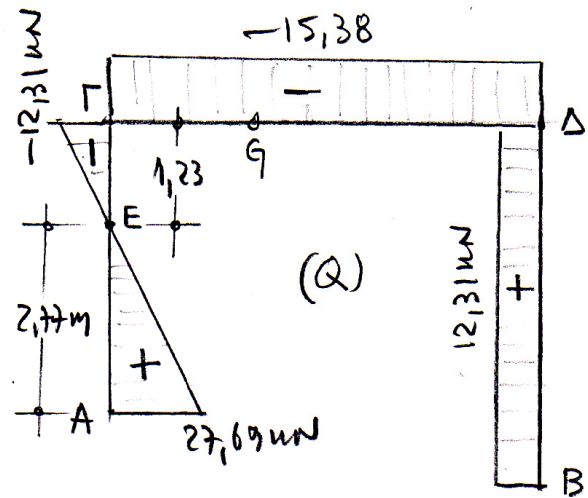
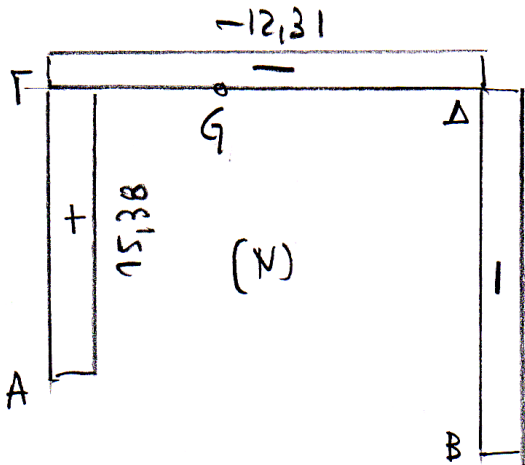
ισορροπία κόμβων



ΑΓ: $q = \text{const} \Rightarrow Q: \text{γραμμ. μεταβ.} \Rightarrow M: \text{παράβολο}$

ΓΔ, ΔΒ: $q = 0 \Rightarrow Q = \text{const} \Rightarrow M: \text{γραμμ. μεταβ.}$

• υποθέτουμε ως σταθερές και εντάξει τις με κατάλληλη γροφυκή



$Q_E = 0 \Rightarrow M_E: \text{απόστατο - κταυρόσημο ή ελαττωτικό}$

$$M_E = M_A + \int_A^E Q dx =$$

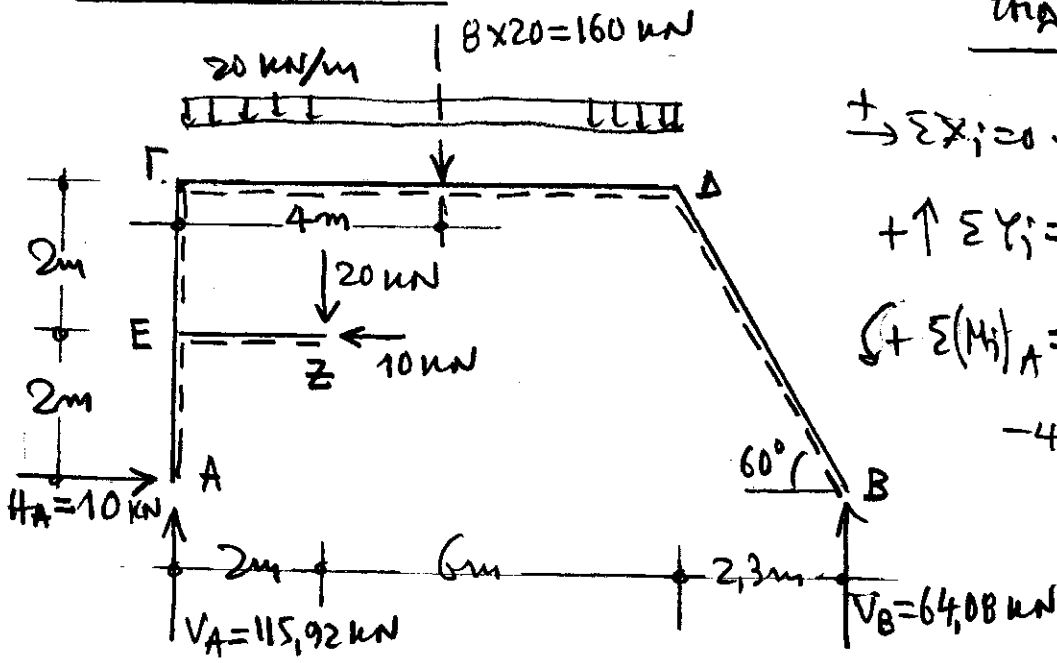
$$= 0 + \frac{1}{2} \cdot 2,77 \cdot 27,69 = 38,35 \text{ kNm}$$

.../...

"Απείρομο μέγιστο"

Άσκηση 1 (8)

Υπόδειξη: Εξ. στα. λογ. - Εξ. στα. λογ.



$$\begin{aligned} \rightarrow \sum X_i = 0 & \quad H_A - 10 = 0 \\ \uparrow \sum Y_i = 0 & \quad V_A + V_B - 20 - 160 = 0 \\ \sum (M_i)_A = 0 & \quad -2 \cdot 20 + 2 \cdot 10 - 4 \cdot 160 + 10,3 \cdot V_B = 0 \end{aligned}$$

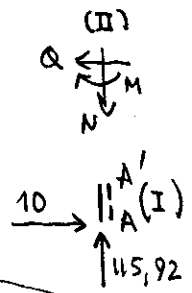
$H_A = 10 \text{ kN}$
$V_A = 115,92 \text{ kN}$
$V_B = 64,08 \text{ kN}$

Διευθετώματα:

Βεβαιώνει επίς $\sqrt{N, Q, M}$ σε χρωματισμένα σημεία με μέγιστο (6 ζεύγη)

Σημείο Α

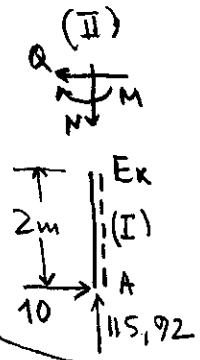
1^η επιλογή -
- καίω (I)



$$\begin{aligned} N_A' &= -115,92 \text{ kN} \\ Q_A' &= -10 \text{ kN} \\ M_A' &= 0 \end{aligned}$$

Σημείο Ε καίω

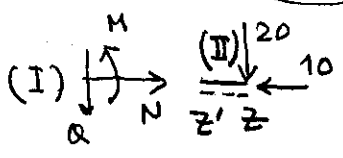
1^η επιλογή -
- καίω (I)



$$\begin{aligned} N_{EK} &= -115,92 \text{ kN} \\ Q_{EK} &= -10 \text{ kN} \\ M_{EK} &= -2 \cdot 10 = -20 \text{ kNm} \end{aligned}$$

Σημείο Ζ

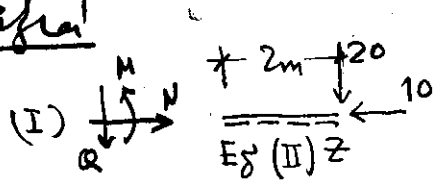
2^η επιλογή -
- διαίω (II)



$$\begin{aligned} N_{Z'} &= -10 \text{ kN} \\ Q_{Z'} &= 20 \text{ kN} \\ M_{Z'} &= 0 \end{aligned}$$

Συμμετοχή E δέξια

2^η εντομή -
- δέξια (II)



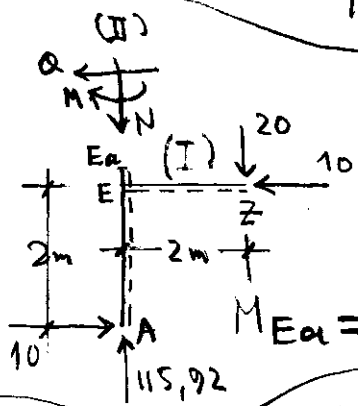
$$N_{E\delta} = -10 \text{ kN}$$

$$Q_{E\delta} = 20 \text{ kN}$$

$$M_{E\delta} = -2 \cdot 20 = -40 \text{ kNm}$$

Συμμετοχή E ανω

1^η εντομή -
- ανω (I)



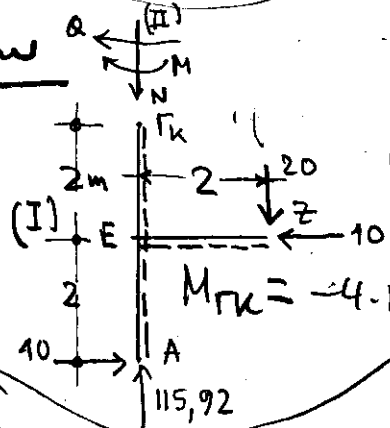
$$N_{E\alpha} = -115,92 + 20 = -95,92 \text{ kN}$$

$$Q_{E\alpha} = -10 + 10 = 0$$

$$M_{E\alpha} = -2 \cdot 10 + 2 \cdot 20 = 20 \text{ kNm}$$

Συμμετοχή Γ κατω

1^η εντομή -
- κατω (I)



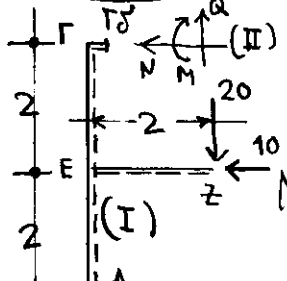
$$N_{\Gamma\kappa} = -115,92 + 20 = -95,92 \text{ kN}$$

$$Q_{\Gamma\kappa} = -10 + 10 = 0$$

$$M_{\Gamma\kappa} = -4 \cdot 10 + 2 \cdot 10 + 2 \cdot 20 = 20 \text{ kNm}$$

Συμμετοχή Γ δεξιά

1^η εντομή -
- κατω (I)



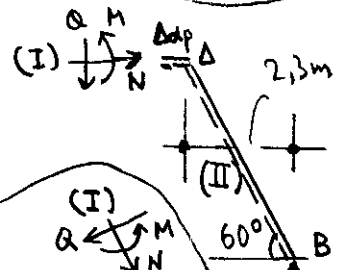
$$N_{\Gamma\delta} = 0$$

$$Q_{\Gamma\delta} = -64,08 + 160 = 95,92 \text{ kN}$$

$$M_{\Gamma\delta} = 10,3 \cdot 64,08 - 4 \cdot 160 = 20 \text{ kNm}$$

Συμμετοχή Δ αριστερά

2^η εντομή -
- κατω (II)



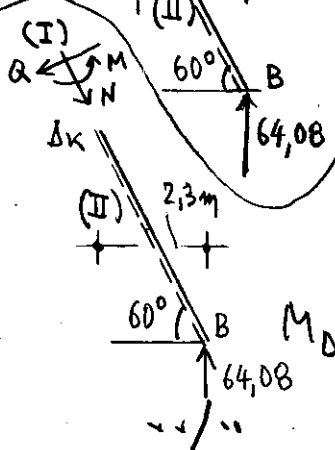
$$N_{\Delta\alpha} = 0$$

$$Q_{\Delta\alpha} = -64,08 \text{ kN}$$

$$M_{\Delta\alpha} = 2,3 \cdot 64,08 = 147,38 \text{ kNm}$$

Συμμετοχή Δ κατω

2^η εντομή -
- κατω (II)



$$N_{\Delta\kappa} = -64,08 \cdot \cos 30^\circ = -55,49 \text{ kN}$$

$$Q_{\Delta\kappa} = -64,08 \cdot \sin 30^\circ = -32,04 \text{ kN}$$

$$M_{\Delta\kappa} = 2,3 \cdot 64,08 = 147,38 \text{ kNm}$$

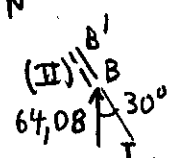
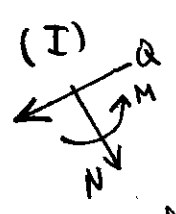
Σημείο Β

2^η επιλογή -
- νότιο (II)

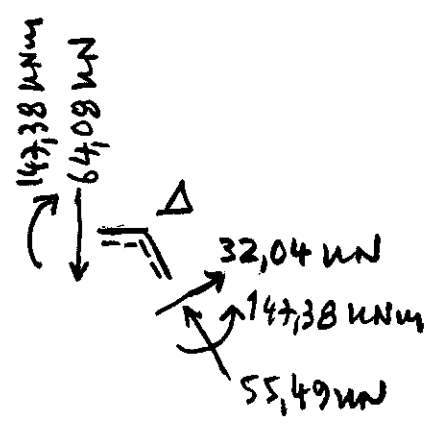
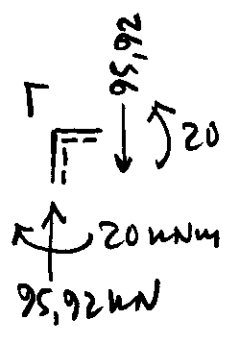
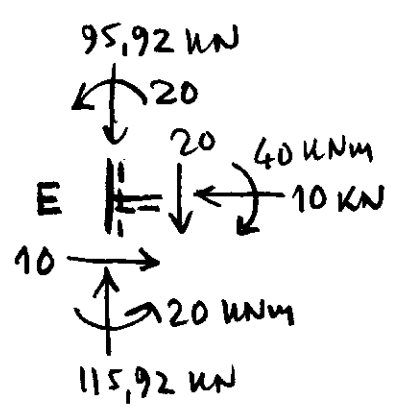
$$N_{B'} = -64,08 \cdot \cos 30^\circ = -55,49 \text{ kN}$$

$$Q_{B'} = -64,08 \cdot \sin 30^\circ = -32,04 \text{ kN}$$

$$M_{B'} = 0$$



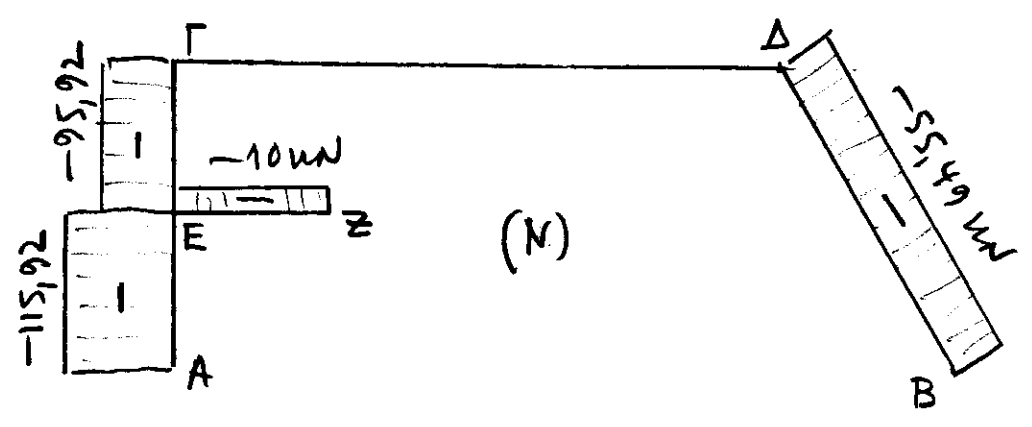
Ισορροπιές νότιο

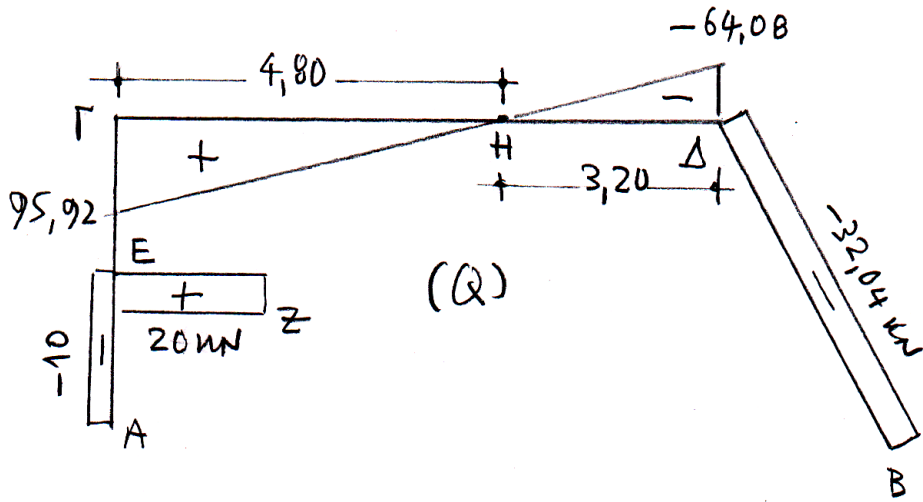


ΑΕ, ΕΒ, ΕΓ, ΔΒ: $q=0 \Rightarrow Q = \text{const} \Rightarrow M = \text{γραμμ. μεταβ.}$

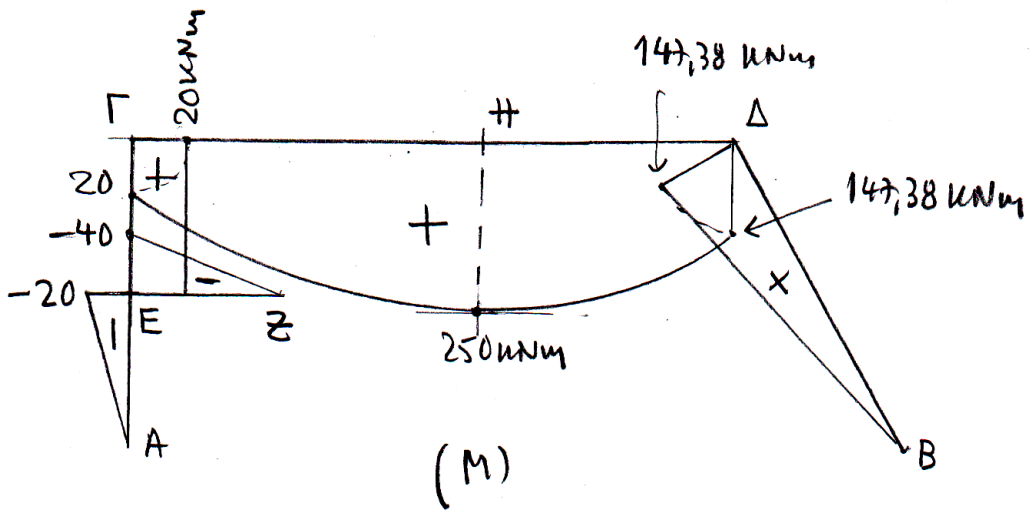
ΓΔ: $q = \text{const} \Rightarrow Q = \text{γραμμ. μεταβ.} \Rightarrow M = \text{παραβολή}$

• υποθέτουμε ως εστιαμίες και ενώνουμε τις με κατάλληλη γραμμή.





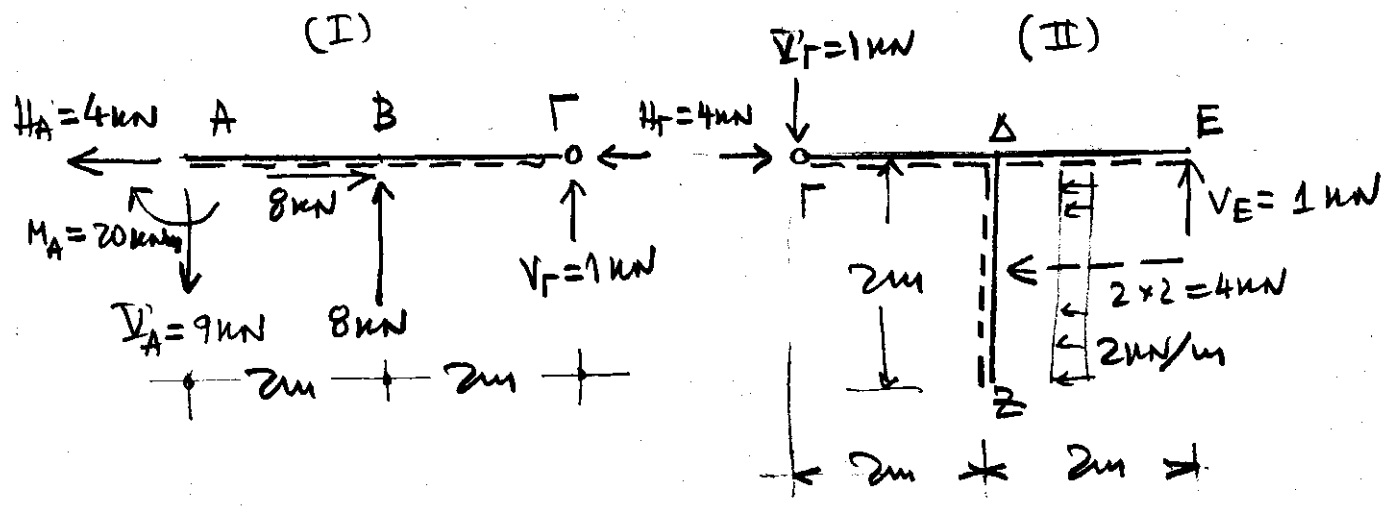
$Q_H = 0 \Rightarrow M_H = \text{използвато}$



$$M_H = M_r + \frac{1}{2} q l^2 [a]_{\Gamma}^H = 20 + \frac{1}{2} 4,80 \cdot 95,92 = 250 \text{ kNm}$$

Άσκηση 1(δ)

$v' = 1, v = (3 + 1 = 4) = (3 + 1 = 4)$
κλιμακωμένη index
 $v + u = 3 \cdot 2$



(II) υποθετιθέμε 2° - δίνονται 1°

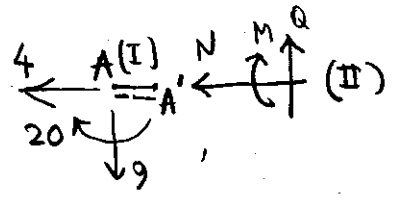
Εξ. στατ. loop. $\rightarrow \begin{cases} V_E = 1 \text{ kN} \\ H_\Gamma = 4 \text{ kN} \\ V_\Gamma = 1 \text{ kN} \end{cases}$

(I) υποθετιθέμε 1° - δίνονται 2°

Εξ. στατ. loop. $\Rightarrow \begin{cases} H_A = 4 \text{ kN} \\ V_A = 9 \text{ kN} \\ M_A = 20 \text{ kNm} \end{cases}$

Δι' αρεθμματα βρισκόμε υπε'σ' σε χ' κλιμακωμένα'
 ομπωτα ει' αυξηση (b ζε'νος)

Σταθ' A 1° ενταξη - σταθ' (I) $N_A = 4 \text{ kN}$



$Q_A = -9 \text{ kN}$
 $M_A = 20 \text{ kNm}$

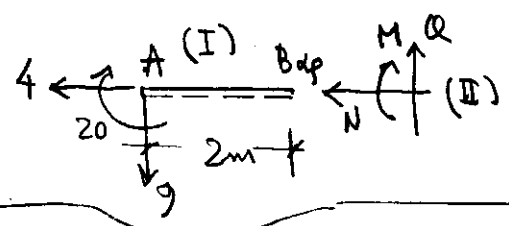
Σημείο Β κρούση!

1^η επιλογή -
- κρούση (I)

$N_{B\psi} = 4 \text{ kN}$

$Q_{B\psi} = -9$

$M_{B\psi} = 20 - 2 \cdot 9 = 2 \text{ kNm}$



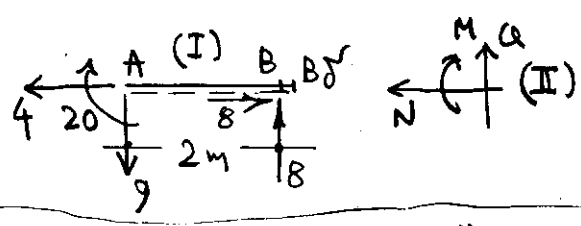
Σημείο Β δόξια

1^η επιλογή -
- δόξια (I)

$N_{B\delta} = 4 - 8 = -4 \text{ kN}$

$Q_{B\delta} = -9 + 8 = -1 \text{ kN}$

$M_{B\delta} = 20 - 2 \cdot 9 = 2 \text{ kNm}$



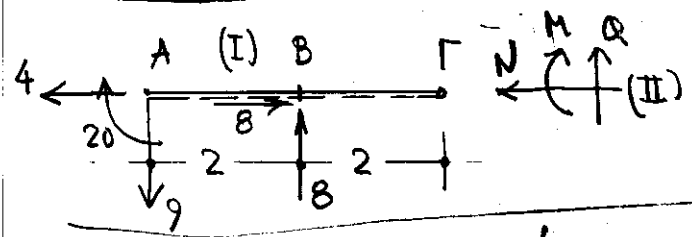
Σημείο Γ

1^η επιλογή -
- κρούση (I)

$N_{\Gamma} = 4 - 8 = -4 \text{ kN}$

$Q_{\Gamma} = -9 + 8 = -1 \text{ kN}$

$M_{\Gamma} = 20 - 4 \cdot 9 + 2 \cdot 8 = 0$ λείπει!



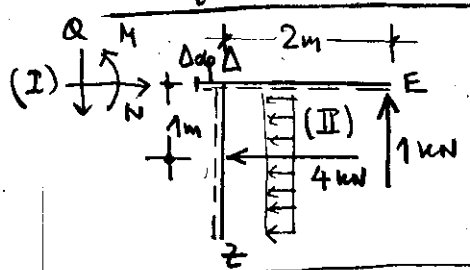
Σημείο Δ κρούση!

2^η επιλογή -
- δόξια (II)

$N_{\Delta\psi} = -4 \text{ kN}$

$Q_{\Delta\psi} = -1 \text{ kN}$

$M_{\Delta\psi} = 2 \cdot 1 - 1 \cdot 4 = -2 \text{ kNm}$



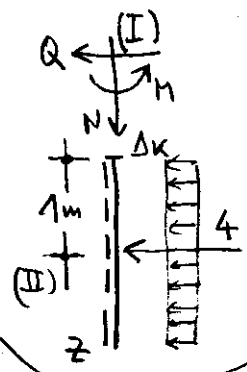
Σημείο Δ κρούση

2^η επιλογή -
- κρούση (II)

$N_{\Delta\kappa} = 0$

$Q_{\Delta\kappa} = 4 \text{ kN}$

$M_{\Delta\kappa} = -1 \cdot 4 = -4 \text{ kNm}$



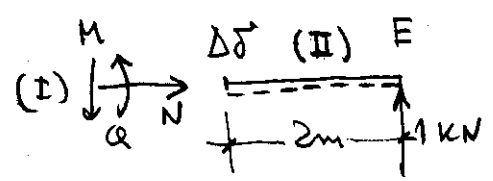
Σημείο Δ δόξια!

2^η επιλογή -
- δόξια (II)

$N_{\Delta\delta} = 0$

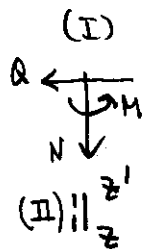
$Q_{\Delta\delta} = -1$

$M_{\Delta\delta} = 2 \cdot 1 = 2 \text{ kNm}$



Σημείο Z

2^η εντάξη -
- υάρω (II)



$$N_z = 0$$

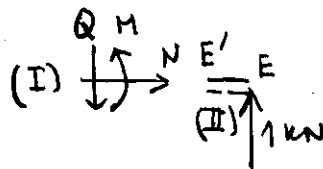
$$Q_z = 0$$

$$M_z = 0$$

Σημείο E

2^η εντάξη -
- δέξιο (II)

46

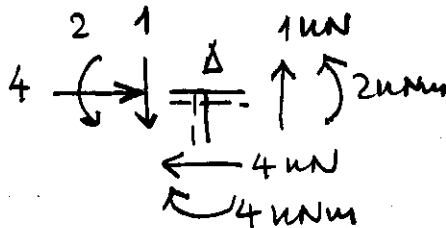


$$N_E = 0$$

$$Q_E = -1 \text{ kN}$$

$$M_E = 0$$

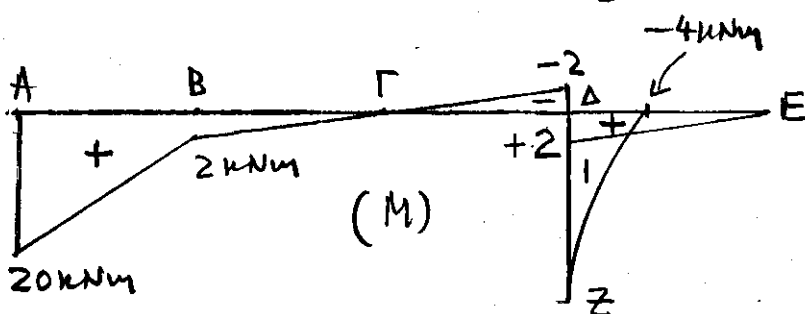
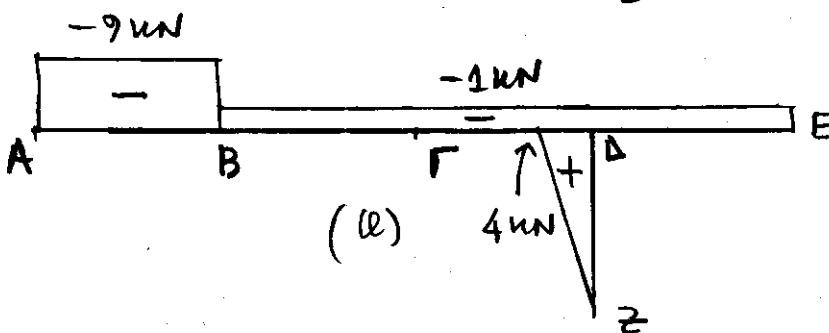
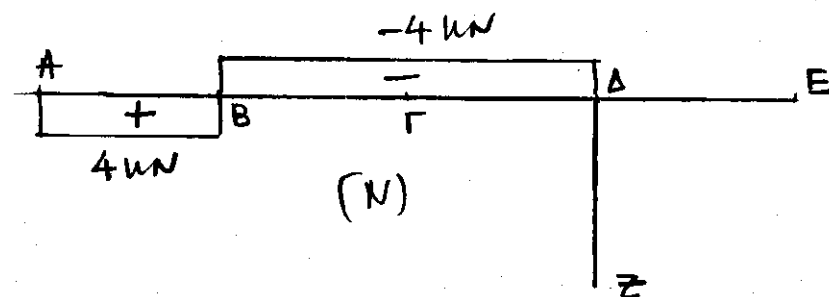
• Ισορροπία κόμβων Δ



ΑΒ, ΒΓ, ΓΔ, ΔΕ: $q=0 \Rightarrow Q = \text{const} \rightarrow M$: γραμμ. μεταβ.

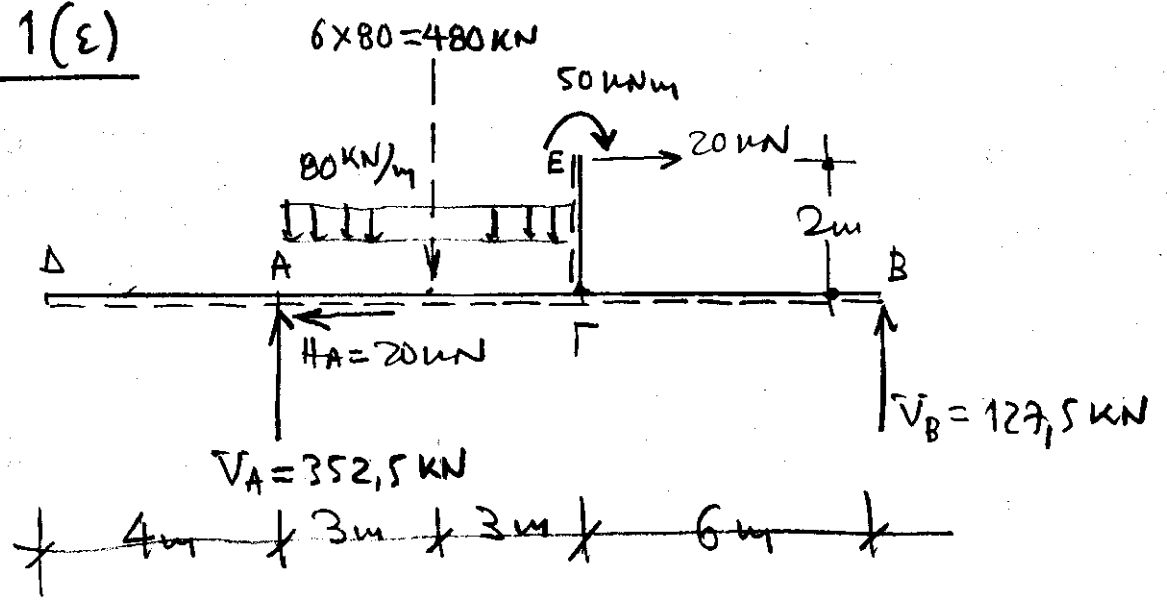
ΔΖ: $q = \text{const} \rightarrow Q$: γραμμ. μεταβ. $\rightarrow M$: παραβολή

• Προσδιορίστε ως 2 σταθμικές και εντάξε με την κατάλληλη γραμμή.



$Q_z = 0 \Rightarrow$
 M_z : κυρτότατο:
καταόριστη
εφαρμογή

Άσκηση 1 (ε)



Εξ. εντά κοφ.

$\rightarrow \sum X_i = 0 \quad \text{ή} \quad -H_A + 20 = 0 \Rightarrow \boxed{H_A = 20 \text{ kN}}$

$\uparrow \sum Y_i = 0 \quad \text{ή} \quad V_A - 480 + V_B = 0 \quad (1)$

$(+ \sum (M_i)_A = 0 \quad \text{ή} \quad -3 \cdot 480 - 50 - 2 \cdot 20 + 12 \cdot V_B = 0 \Rightarrow \boxed{V_B = 127,50 \text{ kN}}$

$(1) \Rightarrow \boxed{V_A = 352,50 \text{ kN}}$

Διεύρυνση Βεβαιώνει πως ^(N, Q, M) σε χαρακτηριστικά σημεία ηε εντάγη (6 ζώνες)

Σημείο Δ 1^η εντάγη - αριστερό (I)

(I) $\Delta \Rightarrow \Delta'$

$N_\Delta = 0$
 $Q_\Delta = 0$
 $M_\Delta = 0$

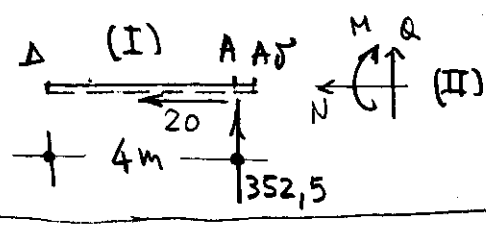
Σημείο Ααριστερά 1^η εντάγη - αριστερό (I)

(I) $\Delta \Rightarrow \Delta'$

$Q_{A\alpha\pi} = 0$
 $N_{A\alpha\pi} = 0$
 $M_{A\alpha\pi} = 0$

Σημείο Α δεξιά

1^η επιλογή -
- αριστερό (I)



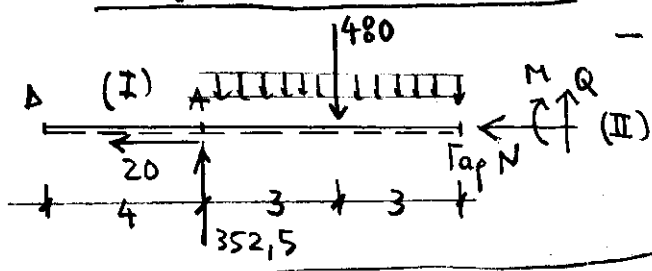
$$N_{A\delta} = 20 \text{ kN}$$

$$Q_{A\delta} = 352,5 \text{ kN}$$

$$M_{A\delta} = 0$$

Σημείο Γ αριστερά

1^η επιλογή -
- αριστερό (I)



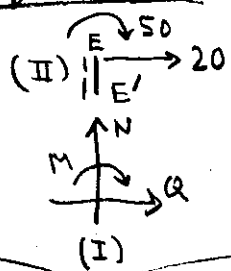
$$N_{\Gamma\alpha\beta} = 20 \text{ kN}$$

$$Q_{\Gamma\alpha\beta} = 352,5 - 480 = -127,5 \text{ kN}$$

$$M_{\Gamma\alpha\beta} = 6 \cdot 352,5 - 3 \cdot 480 = 675 \text{ kNm}$$

Σημείο Ε

2^η επιλογή -
- κάτω (II)



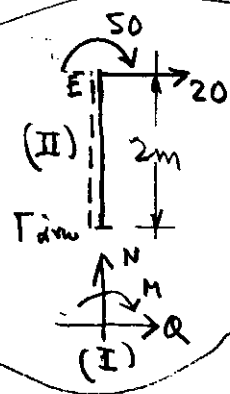
$$N_E = 0$$

$$Q_E = 20 \text{ kN}$$

$$M_E = 50 \text{ kNm}$$

Σημείο Γ κάτω

2^η επιλογή -
- κάτω (II)



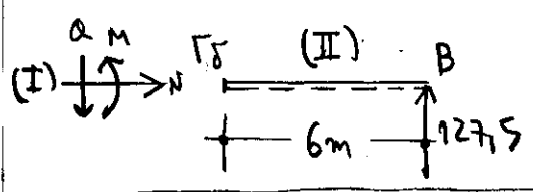
$$N_{\Gamma\alpha} = 0$$

$$Q_{\Gamma\alpha} = 20 \text{ kN}$$

$$M_{\Gamma\alpha} = 50 + 2 \cdot 20 = 90 \text{ kNm}$$

Σημείο Γ δεξιά

2^η επιλογή -
- δεξιά (II)



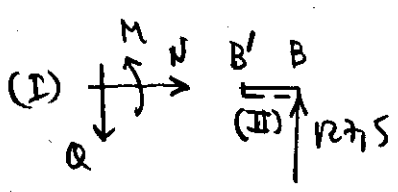
$$N_{\Gamma\delta} = 0$$

$$Q_{\Gamma\delta} = -127,5 \text{ kN}$$

$$M_{\Gamma\delta} = 6 \cdot 127,5 = 765 \text{ kNm}$$

Σημείο Β

2^η επιλογή -
- δεξιά (II)

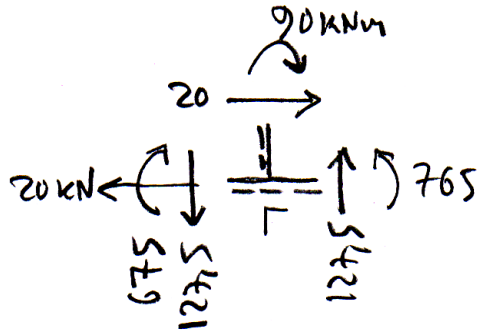


$$N_B = 0$$

$$Q_B = -127,5 \text{ kN}$$

$$M_B = 0$$

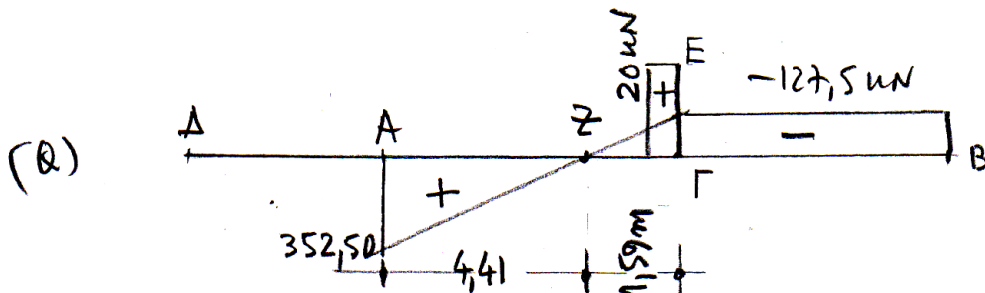
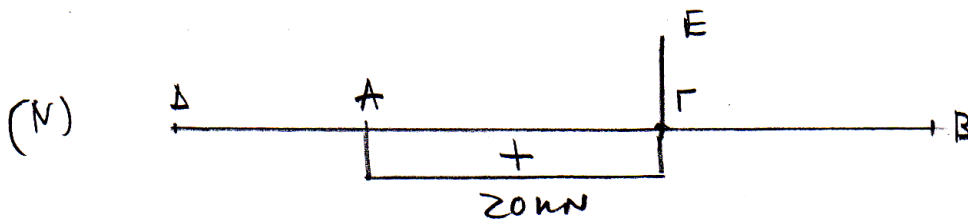
Ισορροπία κόμβου Γ



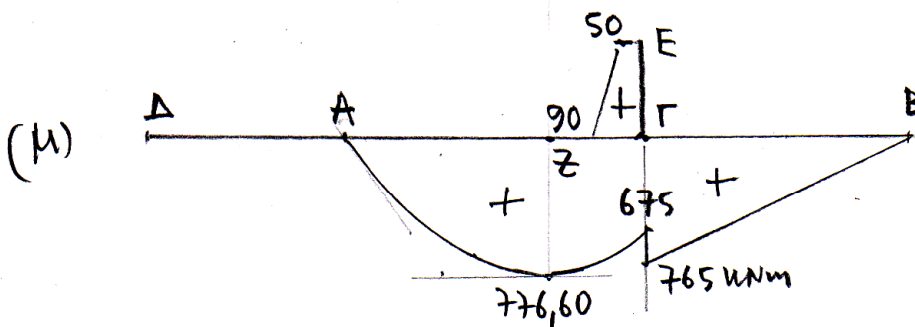
$\Delta A, \Gamma E, \Gamma B: q=0 \Rightarrow Q = const \Rightarrow M: \text{γραμμ. κλίση}$

$A\Gamma: q = const \Rightarrow Q = \text{γραμμ. μίση}$ $\Rightarrow M: \text{παραβολή}$

- αποδοτέωμε ως τετράγωνο και εύραχε δε την κατακόνη γραμμή



$Q_z = 0 \Rightarrow$
 $M_z: \text{απότομο}$

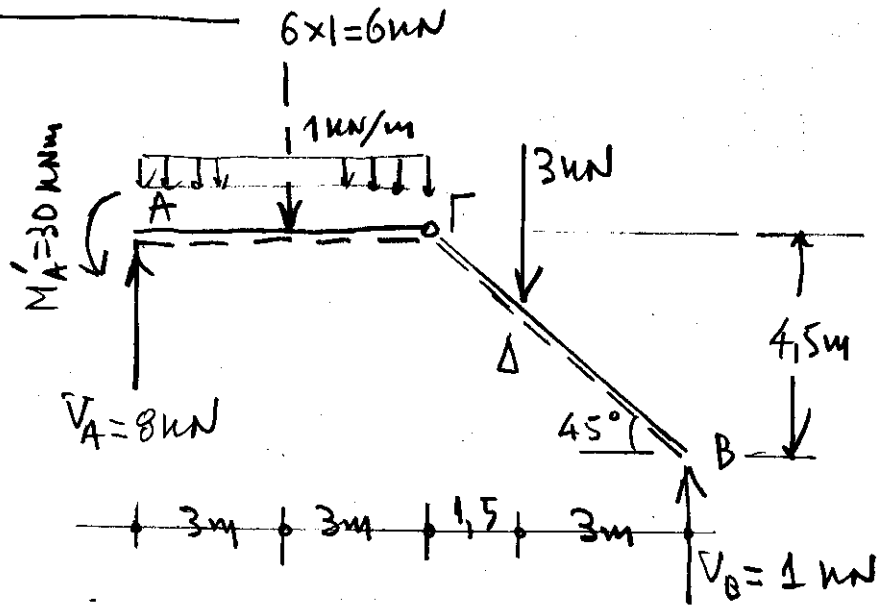


$$M_z = M_A + \int_A^z [Q] dz =$$

$$= 0 + \frac{1}{2} \cdot 4,41 \cdot 352,5 =$$

$$= 776,60 \text{ kNm}$$

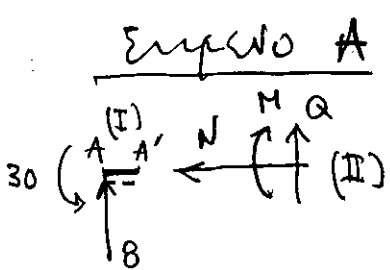
Άσκηση 1 (52)



Αντικείμενο 2ος Αξον 3(5), ΟΜΑΔΑ Δ'

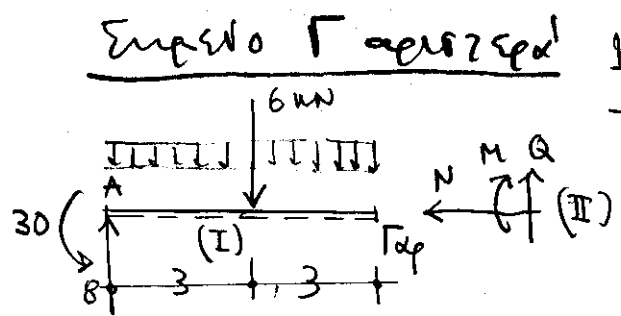
$H_A = 0$
 $V_A = 8 \text{ kN}$
 $M'_A = 30 \text{ kNm}$
 $V_B = 1 \text{ kN}$

Διαγράμματα: Βρίσκουμε πρώτα σε χαρτί μισόβρωμα τα μεγέθη με τα μισά (6 ζεύγη)



1^η ενταξη -
- εφιστέρο (I)

$N_A = 0$
 $Q_A = 8 \text{ kN}$
 $M_A = -30 \text{ kNm}$



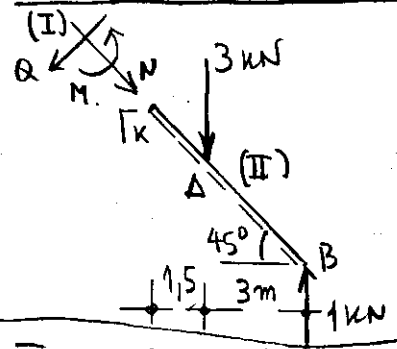
1^η ενταξη -
- εφιστέρο (I)

$N_{\Gamma_{αρ}} = 0$
 $Q_{\Gamma_{αρ}} = 8 - 6 = 2 \text{ kN}$

$M_{\Gamma_{αρ}} = -30 + 6 \cdot 8 - 3 \cdot 6 = 0$ εφθρων!!

..//..

Σημείο Γ κλίση



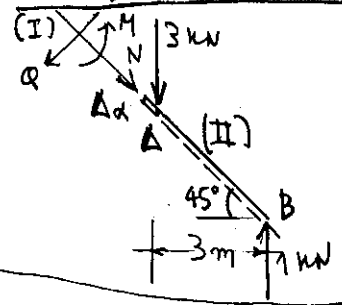
2^η επιλογή -
- κλίση (II)

$$N_{\Gamma\kappa} = -1 \frac{\sqrt{2}}{2} + 3 \frac{\sqrt{2}}{2} = 1,414 \text{ kN}$$

$$Q_{\Gamma\kappa} = -1 \frac{\sqrt{2}}{2} + 3 \frac{\sqrt{2}}{2} = 1,414 \text{ kN}$$

$$M_{\Gamma\kappa} = 4,5 \cdot 1 - 1,5 \cdot 3 = 0 \text{ αρθρουν!!}$$

Σημείο Δ άνω



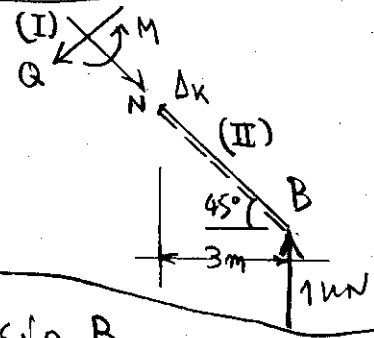
2^η επιλογή -
- κλίση (II)

$$N_{\Delta\alpha} = -1 \frac{\sqrt{2}}{2} + 3 \frac{\sqrt{2}}{2} = 1,414 \text{ kN}$$

$$Q_{\Delta\alpha} = -1 \frac{\sqrt{2}}{2} + 3 \frac{\sqrt{2}}{2} = 1,414 \text{ kN}$$

$$M_{\Delta\alpha} = 3 \cdot 1 = 3 \text{ kNm}$$

Σημείο Δ κλίση



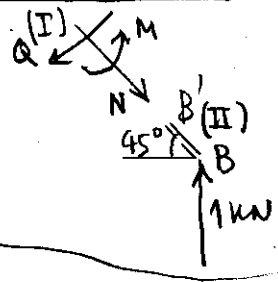
2^η επιλογή -
- κλίση (II)

$$N_{\Delta\kappa} = -1 \frac{\sqrt{2}}{2} = -0,707 \text{ kN}$$

$$Q_{\Delta\kappa} = -1 \frac{\sqrt{2}}{2} = -0,707 \text{ kN}$$

$$M_{\Delta\kappa} = 3 \cdot 1 = 3 \text{ kNm}$$

Σημείο Β



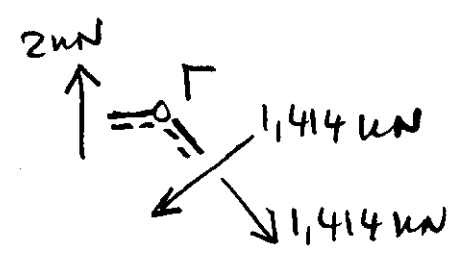
2^η επιλογή -
- κλίση (II)

$$N_B = -1 \frac{\sqrt{2}}{2} = -0,707 \text{ kN}$$

$$Q_B = -1 \frac{\sqrt{2}}{2} = -0,707 \text{ kN}$$

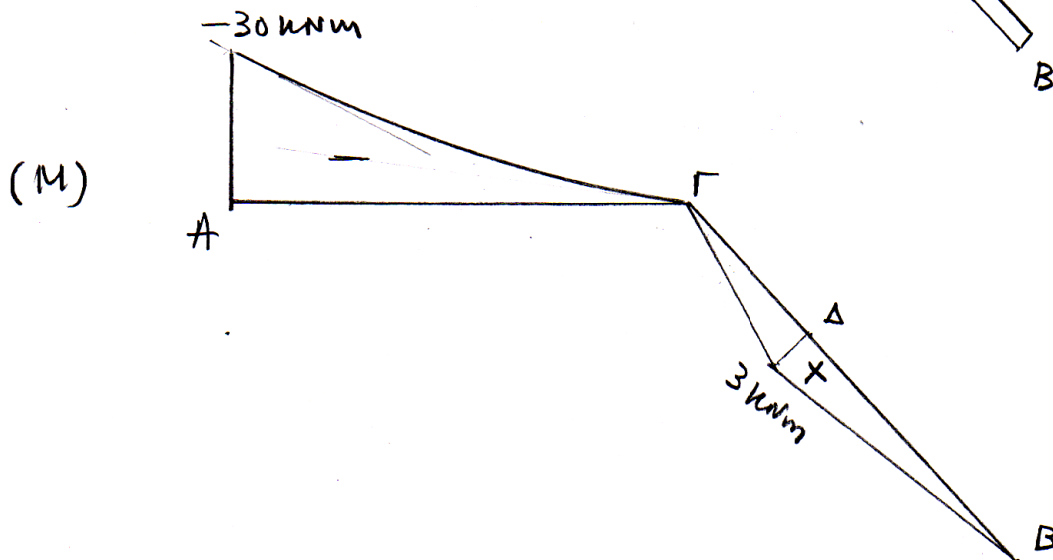
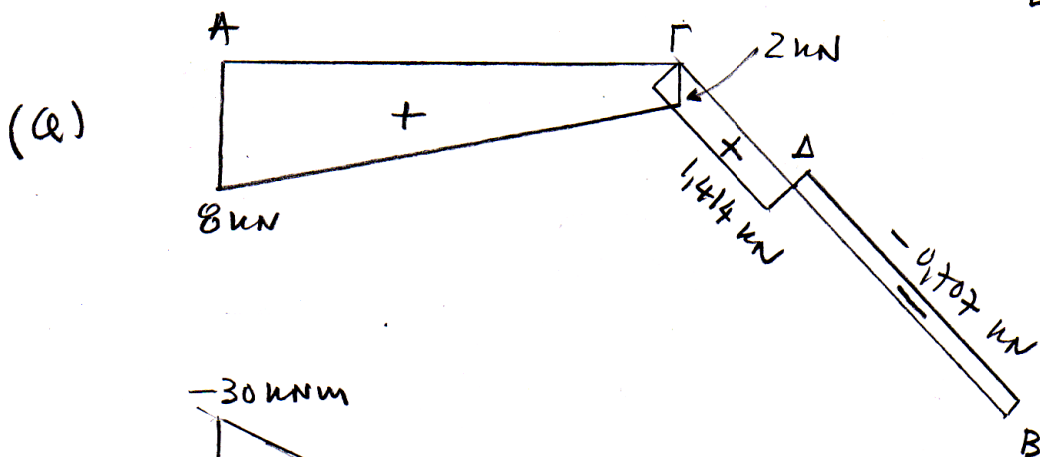
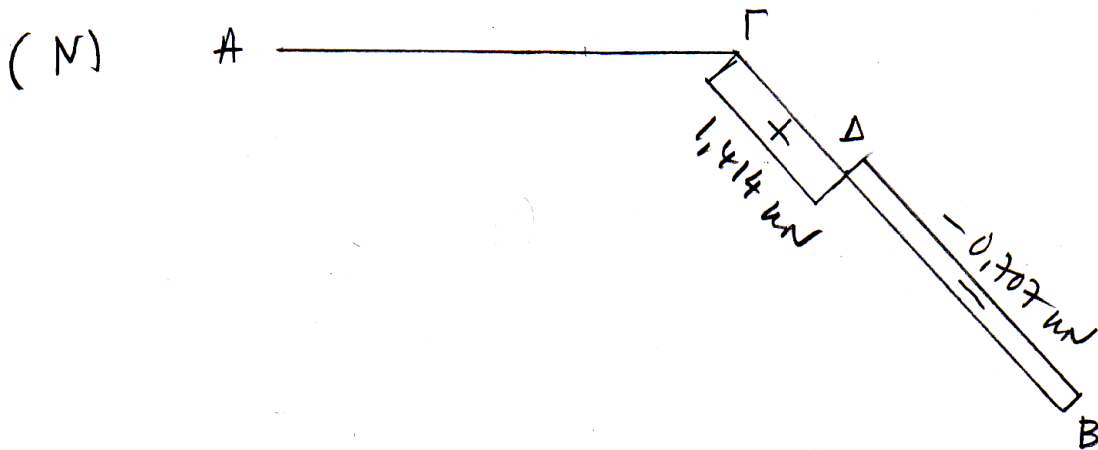
$$M_B = 0$$

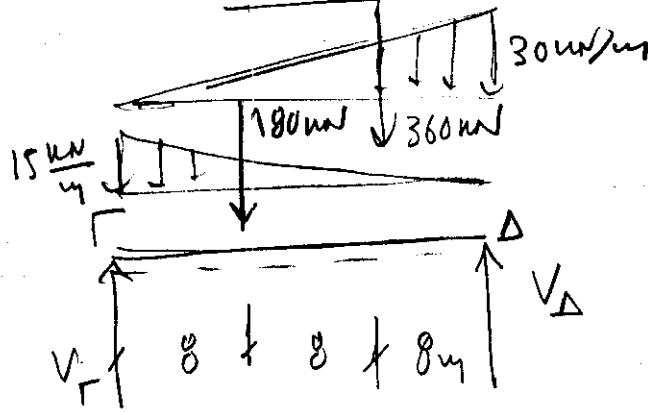
• Ισορροπία αρθρουν Γ



ΑΓ : $q = \text{const} \Rightarrow Q = \text{γραμμ. μετ.} \Rightarrow M: \text{παράβολο!}$
 ΓΒ : $q = 0 \Rightarrow Q = \text{const.} \Rightarrow M: \text{γραμμ. μετ.}$

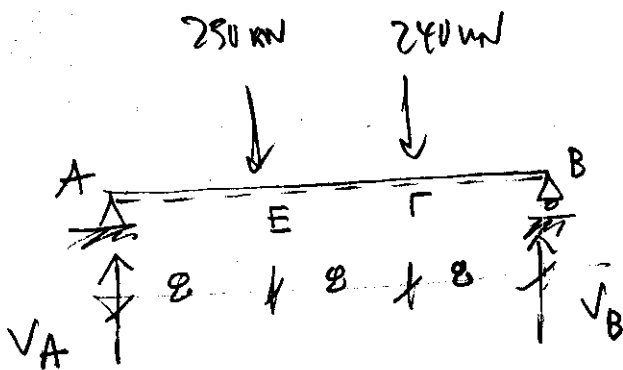
• υποθέτουμε ως σταθμούς και ενώνουμε με μια κατάλληλη γραμμή.





$$\sum Y_i = 0 \Rightarrow V_r - 180 - 360 + V_D = 0 \Rightarrow V_r + V_D = 540 \text{ kN} \Rightarrow \boxed{V_r = 240 \text{ kN}}$$

$$\sum (M_i)_r = 0 \Rightarrow -8 \cdot 180 - 16 \cdot 360 + 24 \cdot V_D = 0 \Rightarrow \boxed{V_D = 300 \text{ kN}}$$

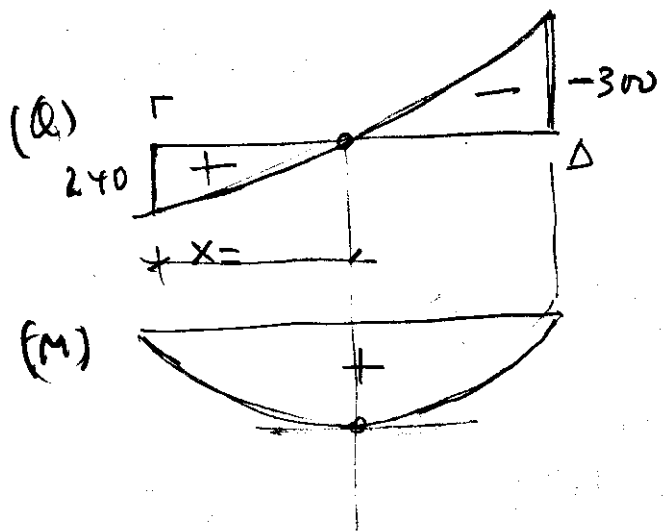
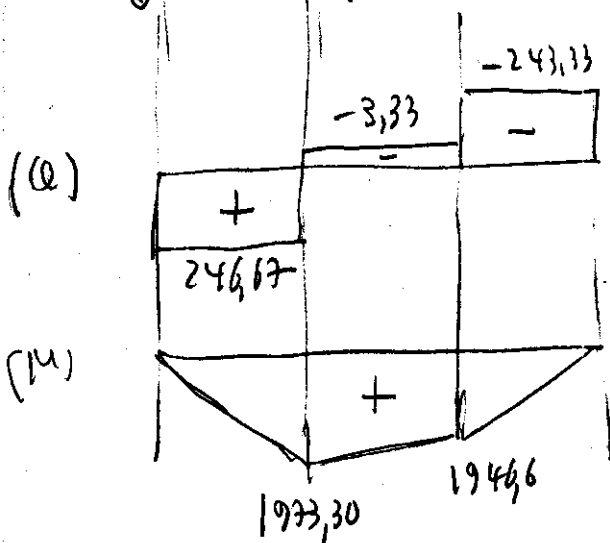


$$\sum Y_i = 0 \Rightarrow V_A - 250 - 240 + V_B = 0 \Rightarrow$$

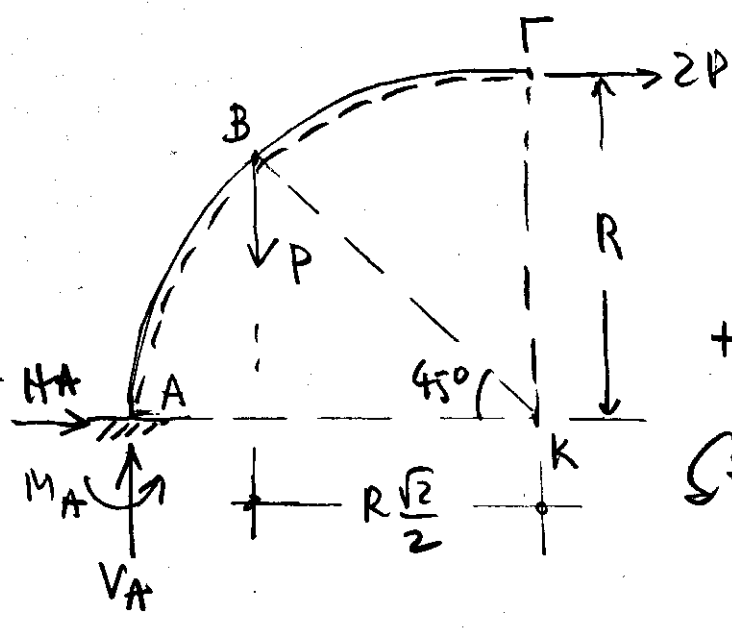
$$V_A + V_B = 490 \text{ kN} \Rightarrow$$

$$\boxed{V_A = 246,67 \text{ kN}}$$

$$\sum (M_i)_A = 0 \Rightarrow -8 \cdot 250 - 16 \cdot 240 + 24 \cdot V_B = 0 \Rightarrow \boxed{V_B = 243,33 \text{ kN}}$$



Άσκηση 1(η)



Υπόψ. Αρχή -
- Εξισ. Στατ. Ισορ.

$$\rightarrow \sum X_i = 0 \text{ ή } H_A + 2P = 0 \Rightarrow \boxed{H_A = -2P}$$

$$+\uparrow \sum Y_i = 0 \text{ ή } V_A - P = 0 \Rightarrow \boxed{V_A = P}$$

$$\curvearrow + \sum (M_i)_A = 0 \text{ ή } M_A - R(1 - \frac{\sqrt{2}}{2})P - R \cdot 2P = 0 \Rightarrow$$

$$\boxed{M_A = RP(3 - \frac{\sqrt{2}}{2}) = 2,293 RP}$$

" " όπως προσα κελευστων γιατι θα λογαριαστω και δεξιά "

Επισημ. απαιτούμενων συσχετισμων N(φ), Q(φ), M(φ)

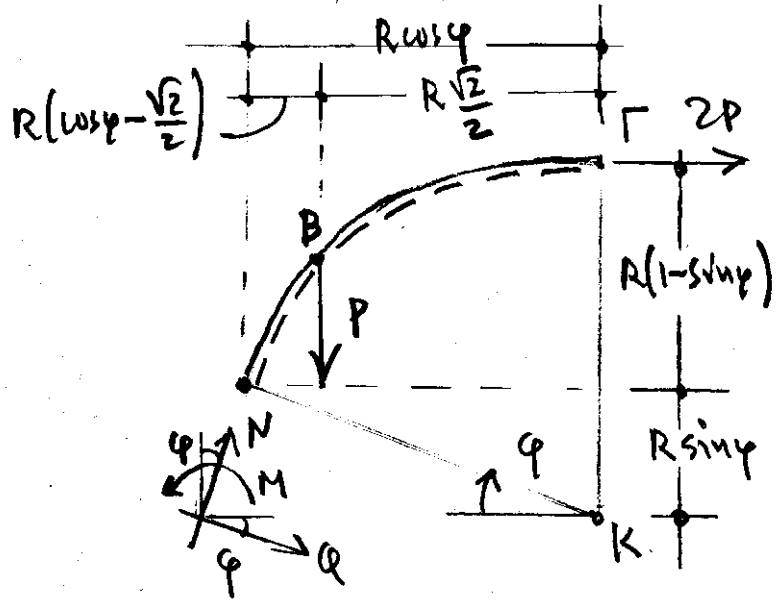
Διευθετήσατε τις συνιστώσες (b radius) των

δυνάμεων των σημείων και του γωνίας φ. Από δεξιά.

AB $0 \leq \varphi \leq 45^\circ$

$$N(\varphi) = -P \cos \varphi + 2P \sin \varphi = P(2 \sin \varphi - \cos \varphi) \quad (1)$$

$$Q(\varphi) = P \sin \varphi + 2P \cos \varphi = P(\sin \varphi + 2 \cos \varphi) \quad (2)$$



.../...

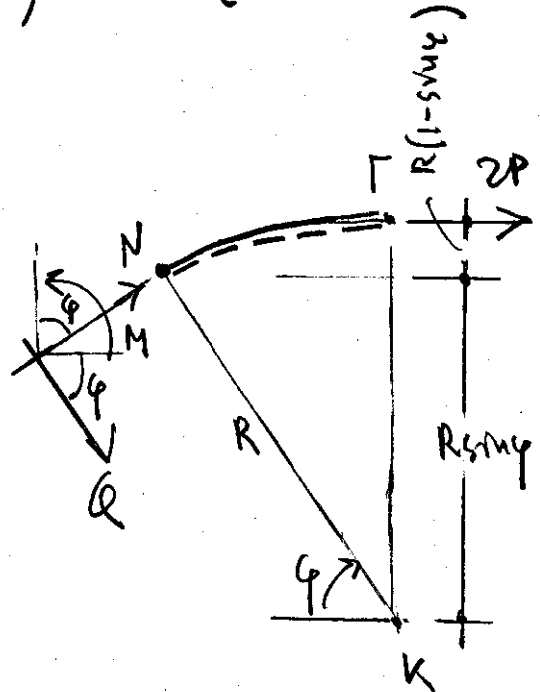
$$\begin{aligned}
 M(\varphi) &= -R \left(\cos\varphi - \frac{\sqrt{2}}{2} \right) P - R(1 - \sin\varphi) 2P = \\
 &= -RP \left(2 - \frac{\sqrt{2}}{2} + \cos\varphi - 2\sin\varphi \right) = \\
 &= -RP (1,293 + \cos\varphi - 2\sin\varphi) \quad (3)
 \end{aligned}$$

BT $45^\circ \leq \varphi \leq 90^\circ$

$$N(\varphi) = 2P \sin\varphi \quad (4)$$

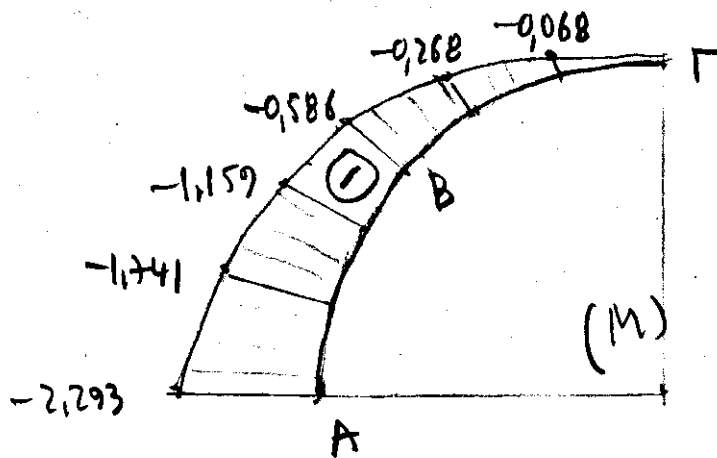
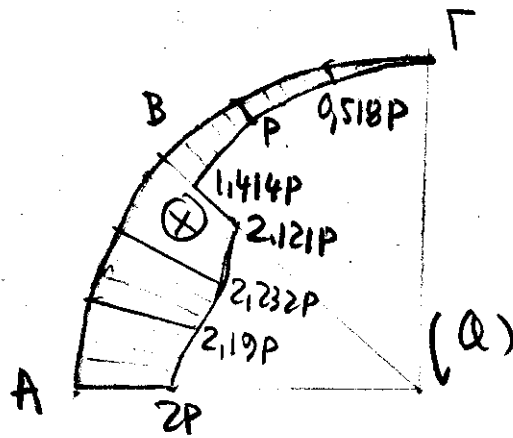
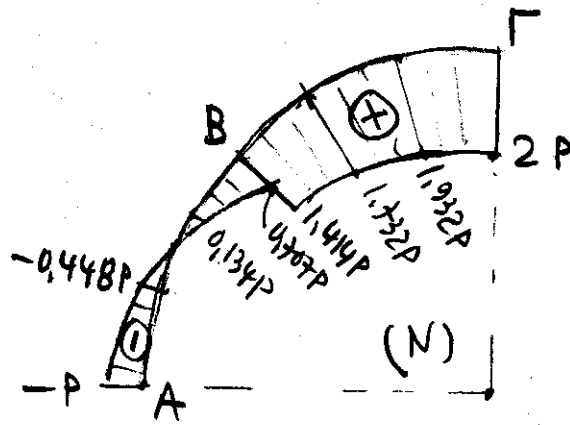
$$Q(\varphi) = 2P \cos\varphi \quad (5)$$

$$\begin{aligned}
 M(\varphi) &= -R(1 - \sin\varphi) 2P = \\
 &= -RP (2 - 2\sin\varphi) \quad (6)
 \end{aligned}$$

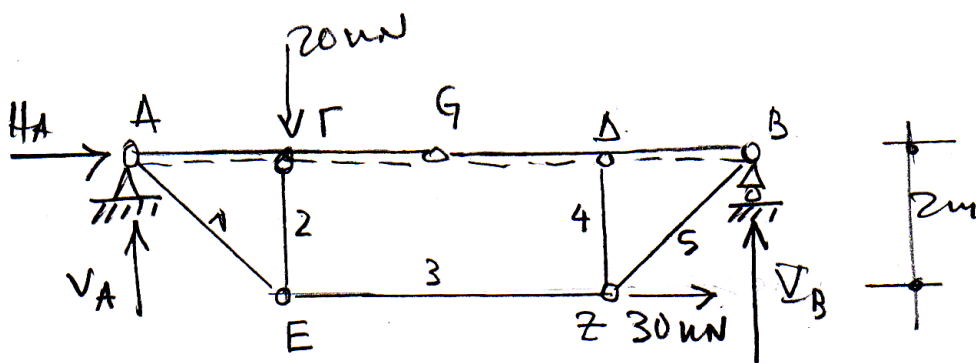


- Sifat-sifat rupa dan bentuk (1) ÷ (6) berdasarkan sifat-sifat dan sifat-sifat:

φ	N	Q	M
0	-P	2,0 P	-2,293 RP
15°	-0,448 P	2,19 P	-1,741 RP
30°	0,134 P	2,232 P	-1,159 RP
45°	0,707 P	2,121 P	-0,586 RP
	1,414 P	1,414 P	
60°	1,732 P	P	-0,268 RP
75°	1,932 P	0,518 P	-0,068 RP
90°	2,0 P	0	0



Assignment 1 (9)



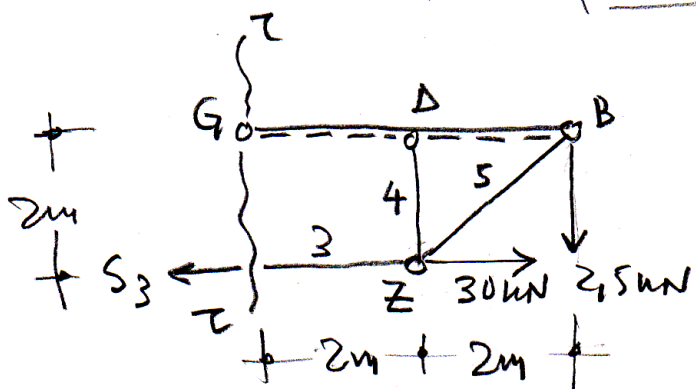
Trussstruktur - E3 - stat. Lösung.

$$\rightarrow \sum X_i = 0 \text{ u' } H_A + 30 = 0 \Rightarrow \boxed{H_A = -30 \text{ kN}}$$

$$\uparrow \sum Y_i = 0 \text{ u' } V_A - 20 + V_B = 0 \quad (1)$$

$$(+ \sum (M_i))_A = 0 \text{ u' } -2 \cdot 20 + 2 \cdot 30 + 8 \cdot V_B = 0 \Rightarrow \boxed{V_B = -2,5 \text{ kN}}$$

$$(1) \Rightarrow \boxed{V_A = 22,5 \text{ kN}}$$



2m u' Ritter z-z

$$(+ \sum (M_i))_G = 0 \text{ u' } -2 \cdot S_3 + 2 \cdot 30 - 4 \cdot 2,5 = 0 \Rightarrow \boxed{S_3 = 25 \text{ kN}}$$

loop. Lösung Z

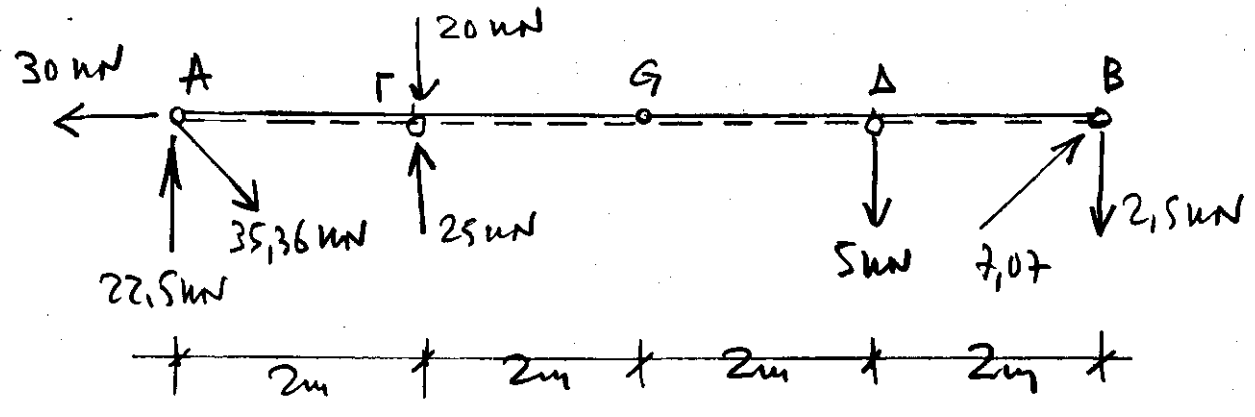
$$\left. \begin{aligned} \rightarrow \sum X_i = 0 \text{ u' } -25 + 30 + S_5 \frac{\sqrt{2}}{2} = 0 \\ \uparrow \sum Y_i = 0 \text{ u' } S_4 + S_5 \frac{\sqrt{2}}{2} = 0 \end{aligned} \right\} \Rightarrow \begin{cases} \boxed{S_5 = -7,07 \text{ kN}} \\ \boxed{S_4 = 5 \text{ kN}} \end{cases}$$

loop. Lösung E

$$\left. \begin{aligned} \rightarrow \sum X_i = 0 \text{ u' } -S_1 \frac{\sqrt{2}}{2} + 25 = 0 \\ \uparrow \sum Y_i = 0 \text{ u' } S_1 \frac{\sqrt{2}}{2} + S_2 = 0 \end{aligned} \right\} \Rightarrow \begin{cases} \boxed{S_1 = 35,36 \text{ kN}} \\ \boxed{S_2 = -25 \text{ kN}} \end{cases}$$

~\dots

ΔΕΕ δομολ



Διευρύνματα: βρίσκουμε τις N, Q, M χαρακτηριστικά
συντεταγμένα με αναγωγή (6 ζεύγη)

Συντεταγμένα Α $1^{\text{ο}}$ επιλογή - $N_A = 30 - \frac{\sqrt{2}}{2} 35,36 = 5 \text{ kN}$
 - τεταγμένο (I) $Q_A = 22,5 - \frac{\sqrt{2}}{2} 35,36 = -2,5 \text{ kN}$
 $M_A = 0$

Συντεταγμένα Γ τεταγμένα $1^{\text{ο}}$ επιλογή - $N_{\Gamma\Gamma} = 30 - \frac{\sqrt{2}}{2} 35,36 = 5 \text{ kN}$
 - τεταγμένο (I) $Q_{\Gamma\Gamma} = 22,5 - \frac{\sqrt{2}}{2} 35,36 = -2,5 \text{ kN}$
 $M_{\Gamma\Gamma} = 2 \left(22,5 - \frac{\sqrt{2}}{2} 35,36 \right) = -5 \text{ kNm}$

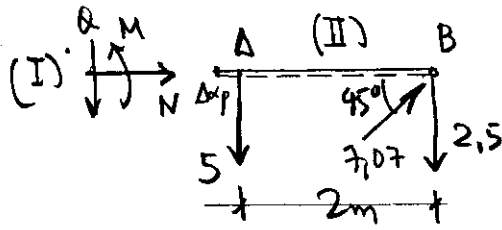
Συντεταγμένα Γ δεξιά $1^{\text{ο}}$ επιλογή - $N_{\Gamma\delta} = 30 - \frac{\sqrt{2}}{2} 35,36 = 5 \text{ kN}$
 - τεταγμένο (I) $Q_{\Gamma\delta} = 22,5 - \frac{\sqrt{2}}{2} 35,36 + 25 - 20 = 2,5 \text{ kN}$
 $M_{\Gamma\delta} = 2 \left(22,5 - \frac{\sqrt{2}}{2} 35,36 \right) = -5 \text{ kNm}$

Συντεταγμένα Γ $2^{\text{ο}}$ επιλογή - $N_G = 7,07 \frac{\sqrt{2}}{2} = 5 \text{ kN}$
 - δεξιά (II) $Q_G = 2,5 - \frac{\sqrt{2}}{2} 7,07 + 5 = 2,5 \text{ kN}$
 $M_G = -4 \cdot 2,5 + 4 \cdot \frac{\sqrt{2}}{2} 7,07 - 2,5 = 0$
 κέρδισμα!!

Σημείο Δ κεντράρι

2^η επιλογή -
- δεξιά (II)

$$N_{\Delta\psi} = \frac{\sqrt{2}}{2} \cdot 7,07 = 5 \text{ kN}$$



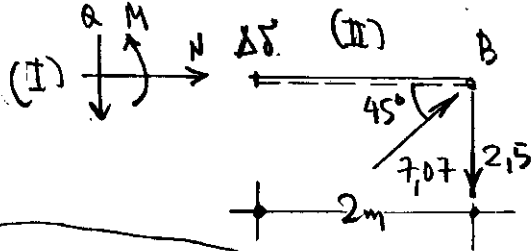
$$Q_{\Delta\psi} = 2,5 - \frac{\sqrt{2}}{2} \cdot 7,07 + 5 = 2,5 \text{ kN}$$

$$M_{\Delta\psi} = -2 \cdot 2,5 + 2 \cdot \frac{\sqrt{2}}{2} \cdot 7,07 = 5 \text{ kNm}$$

Σημείο Δ δεξιά

2^η επιλογή -
- δεξιά (II)

$$N_{\Delta\delta} = \frac{\sqrt{2}}{2} \cdot 7,07 = 5 \text{ kN}$$



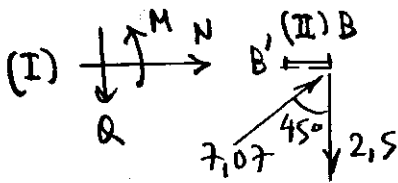
$$Q_{\Delta\delta} = 2,5 - \frac{\sqrt{2}}{2} \cdot 7,07 = -2,5 \text{ kN}$$

$$M_{\Delta\delta} = -2 \cdot 2,5 + 2 \cdot \frac{\sqrt{2}}{2} \cdot 7,07 = 5 \text{ kNm}$$

Σημείο Β

2^η επιλογή -
- δεξιά (II)

$$N_B = \frac{\sqrt{2}}{2} \cdot 7,07 = 5 \text{ kN}$$



$$Q_B = 2,5 - \frac{\sqrt{2}}{2} \cdot 7,07 = -2,5 \text{ kN}$$

$$M_B = 0$$

ΑΓ, ΓΓ, ΓΔ, ΔΒ: $q=0 \Rightarrow Q=\text{const} \rightarrow M$ -γραμμ. ή σταθ.

• Λοιοθέρματα ως επιρροές και ενδυνάμει με την ακτινωτή γραμμή.

