Functional Analysis I
(Problem sheet 7)

Exercise 1. *Let X be a vector space and $\|\cdot\|, |||\cdot|||$ be two norms on X for which there exists $c > 0$such that $\|x\| \leq c|||x|||$, for all $x \in X$. If we assume that X is a Banach space with both norms show that $\|\cdot\|$ and $|||\cdot|||$ are equivalent.*

Exercise 2. *For any $x = (x_n) \in c_0$ we define*

$$\|x\| = \sum_{n=1}^{\infty} \frac{1}{2^n} |x_n| .$$

Show thar $\|\cdot\|$ is a normand that $(c_0, \|\cdot\|)$ is not a Banach space.

Exercise 3. *Let X, Y be Banach spaces and $T : X \to Y$ be a bounded linear operator onto. Show that there exists $M > 0$ such that for all $y \in Y$ there exists $x \in X$ with $y = Tx$ and $\|x\| \leq M\|y\|$.*

Exercise 4. *Let X, Y be normed spaces and $T : X \to Y$ be a bounded linear operator onto which is also an open mapping. Show that if X is a Banach space, then so is Y.*