

## Functional Analysis I

### 1st week

Linear spaces, subspaces, basis and dimension of a linear space.

### 2nd week

Normed spaces, open and closed ball of a normed space. Open and closed sets of a normed space. Continuity of the norm and of addition/scalar multiplication. Convergent and Cauchy sequences. The definition of a Banach space. Examples of Banach spaces:  $\mathbb{R}^n$ ,  $C([a, b])$ ,  $l^p$  for  $1 \leq p < \infty$ .

### 3rd/4th week

Finite dimensional normed spaces. Equivalent norms. We proved that all norms on a finite dimensional space are equivalent and then that every finite dimensional space is a Banach space. We then proved that if for a subspace  $Y$  of a normed space  $X$  we have that  $Y^\circ \neq \emptyset$ , then  $Y = X$ . We concluded with the very important Theorem that the dimension of a Banach space is either finite or uncountable. Bounded linear operators.

### 5th week

Main properties of bounded linear operators. Examples: the Volterra integral operator and the right shift. The norm of a bounded linear operator.

### 6th week

We gave the definition of codimension of a subspace and we proved that the codimension of the kernel of a linear functional is 1. The separability of finite dimensional spaces, of  $c_{00}$  and of  $l^p$ , for  $1 \leq p < \infty$ . The non-separability of  $l^\infty$ .

### 7th week

We stated the Hahn-Banach Theorem and proved some of its corollaries.

### 8th week

Using the corollaries of the Hahn-Banach theorem we proved that if  $X^*$  is separable, then  $X$  is separable and that  $X$  is finite dimensional if and only if its closed unit ball is compact. We begun the proof of the Hahn-Banach theorem.

### 9th week

We finished the proof of the Hahn-Banach theorem. We showed the  $X$  may be embedded to its second dual. We also showed that if  $p > 1$ , then  $(l^p)^* = l^q$  for all  $\frac{1}{p} + \frac{1}{q} = 1$  and hence that if  $p > 1$ , then  $l^p$  is reflexive.