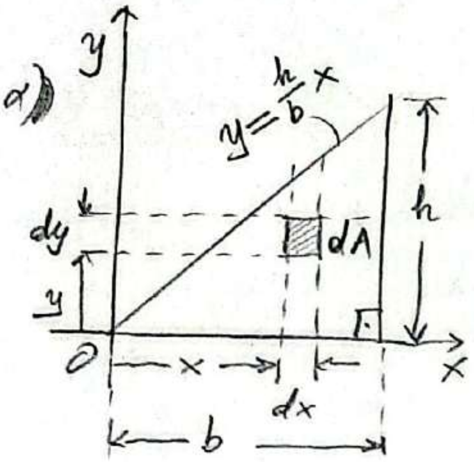
(4-36)

30η ΑΣΚΗΣΗ

Να υπολογισθούν οι επιφανειακές ροπές 2ας τάξης και η φυγόκεντρος ροπή του τριγώνου ως προς:
α) Oxy
β) Kxy

α)



1ος τρόπος

$$I_{xx}=\int_A y^2\,dA=\int_{x=0}^{x=b}\left(\int_{y=0}^{y=\frac{h}{b}x} y^2\,dy\right)dx=$$

$$=\int_0^b\left(\frac{y^3}{3}\right)_{y=0}^{y=\frac{h}{b}x}dx=\frac{1}{3}\int_0^b\frac{h^3}{b^3}x^3\,dx=$$

$$=\frac{h^3}{3b^3}\left.\frac{x^4}{4}\right|_0^b=\frac{h^3}{12b^3}b^4=\frac{bh^3}{12}$$

$$I_{yy}=\int_A x^2\,dA=\int_0^b x^2\left(\int_{y=0}^{y=\frac{h}{b}x}dy\right)dx=\int_0^b x^2\frac{h}{b}x\,dx=\frac{h}{b}\left.\frac{x^4}{4}\right|_0^b=\frac{hb^4}{4b}=\frac{hb^3}{4}$$

$$I_{xy}=-\int_A xy\,dA=-\int_0^b x\left(\int_{y=0}^{y=\frac{h}{b}x}y\,dy\right)dx=-\int_0^b x\left(\frac{y^2}{2}\right)_0^{\frac{h}{b}x}dx=-\frac{1}{2}\int_0^b\frac{h^2}{b^2}x^3\,dx=$$

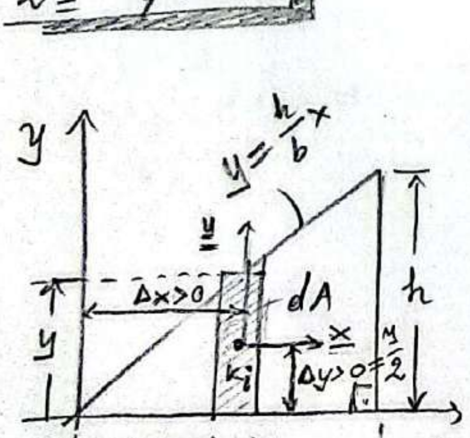
$$=-\frac{h^2}{2b^2}\left.\frac{x^4}{4}\right|_0^b=-\frac{h^2b^4}{8b^2}=-\frac{h^2b^2}{8}$$

2ος τρόπος

$$dA=\underbrace{y}\,dx=\underbrace{\frac{h}{b}}x\,dx$$

$$I_{xx}=\int_A\left(\frac{y}{2}\right)^2dA=\int_0^b\frac{1}{4}\frac{h^2x^2}{b^2}\,\frac{hx}{b}\,dx=$$

$$=\frac{h^3}{4b^3}\int_0^b x^3\,dx=\frac{h^3}{4b^3}\left.\frac{x^4}{4}\right|_0^b=\frac{h^3b^4}{16b^3}=\underline{\frac{h^3b}{16}}\ ?$$



Θ. Παραλλ. Αξόνων:

$$dI_{xx}=d\bar{I}_{xx}+\left(\frac{y}{2}\right)^2dA=\frac{dx\,y^3}{12}+\frac{h^2x^2}{4b^2}\frac{hx}{b}dx=$$

$$=\left(\frac{h^3x^3}{12b^3}+\frac{h^3x^3}{4b^3}\right)dx=\frac{h^3x^3}{3b^3}dx$$

$$I_{xx}=\int_0^b dI_{xx}=\frac{h^3}{3b^3}\int_0^b x^3\,dx=\frac{h^3}{3b^3}\left.\frac{x^4}{4}\right|_0^b=\frac{h^3b^4}{12b^3}=\frac{h^3b}{12}\ \checkmark$$

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$$I_{yy} = \int_A x^2 dA = \int_0^b x^2 \frac{h}{b} x\, dx = \frac{h}{b} \frac{x^4}{4}\Big|_0^b = \frac{hb^3}{4} \checkmark$$

η' Θ. Παραλλ. Αξόνων:

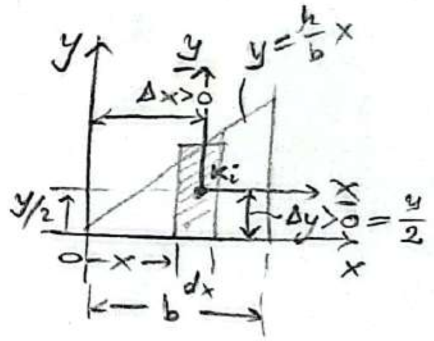
$$dI_{yy} = d\underline{I_{yy}} + x^2 dA = \frac{y\, dx^3}{12} + x^2 \frac{h}{b} x\, dx \cong 0 + \frac{h}{b} x^3 dx$$

$$I_{yy} = \int_0^b dI_{yy} = \int_0^b \frac{h}{b} x^3 dx = \frac{h}{b} \frac{x^4}{4}\Big|_0^b = \frac{hb^3}{4} \checkmark$$

$$I_{xy} = -\int_A xy\, dA = -\int_0^b x \left(\int_{y=0}^{y=\frac{h}{b}x} y\, dy \right) dx = -\int_0^b x \left(\frac{y^2}{2} \right)_0^{\frac{h}{b}x} dx =$$

$$= -\frac{1}{2} \int_0^b x \frac{h^2}{b^2} x^2 dx = -\frac{h^2}{2b^2} \frac{x^4}{4}\Big|_0^b = -\frac{h^2 b^2}{8} \checkmark$$

η' Θ. Παραλλήλων Αξόνων:



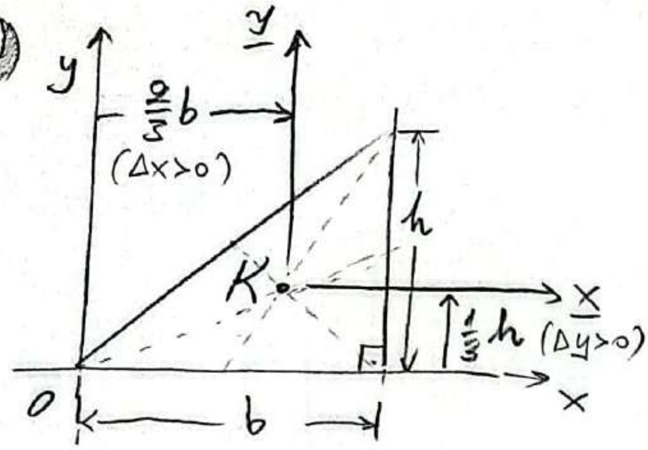
$$dI_{xy} = d\underline{I_{xy}} - \Delta x \Delta y\, dA = 0 -$$

$$= -(+x)(+\frac{y}{2})\, dA = -x \frac{hx}{2b} \frac{hx}{b} dx$$

$$= -\frac{h^2 x^3}{2b^2}$$

$$I_{xy} = \int_0^b dI_{xy} = -\frac{h^2}{2b^2} \frac{x^4}{4}\Big|_0^b = -\frac{h^2 b^2}{8} \checkmark$$

β)



Στο Oxy

$$I_{xx} = \frac{bh^3}{12}, \quad I_{yy} = \frac{hb^3}{4}$$

$$I_{xy} = -\frac{b^2h^2}{8}$$

Στο $K\underline{x}\underline{y}$ με το Θ. Παραλλήλων Αξόνων :

$$\boxed{I_{\underline{xx}} = I_{xx} - \underbrace{\left(\frac{h}{3}\right)}_{\Delta y}^2 A = \frac{bh^3}{12} - \frac{h^2}{9}\,\frac{bh}{2} = \frac{bh^3}{36}}$$

$$\boxed{I_{\underline{yy}} = I_{yy} - \underbrace{\left(\frac{2}{3}b\right)}_{\Delta x}^2 A = \frac{hb^3}{4} - \frac{\overset{2}{\cancel{4}}}{9}b^2\,\frac{bh}{\cancel{2}} = \frac{hb^3}{36}}$$

$$\boxed{I_{\underline{xy}} = I_{xy} + \underbrace{\left(\frac{h}{3}\right)}_{\Delta y}\underbrace{\left(\frac{2b}{3}\right)}_{\Delta x} A = -\frac{b^2h^2}{8} + \frac{\cancel{2}hb}{9}\,\frac{bh}{\cancel{2}} = -\frac{b^2h^2}{72}}$$