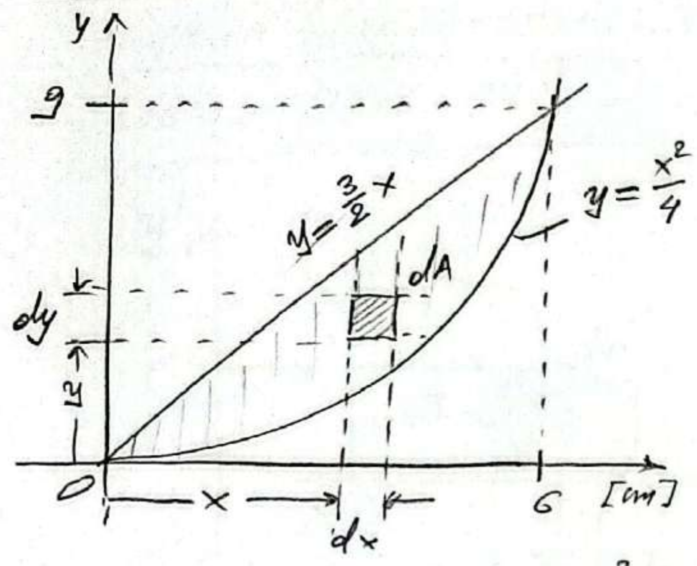
23η ΑΣΚΗΣΗ (δες και 24η)

Να υπολογισθούν οι επιφανειακές ροπές, και η φυγόκεντρος ροπή 2ας τάξεως της επιφανείας του σχήματος



$$I_{xx}=\int_A y^2 dA=\int_{x=0}^{x=6}\left(\int_{y=\frac{x^2}{4}}^{y=\frac{3}{2}x} y^2 dy\right)dx=\int_0^6\left(\frac{y^3}{3}\right)_{\frac{x^2}{4}}^{\frac{3}{2}x}dx=$$

$$=\frac{1}{3}\int_0^6\left(\frac{27}{8}x^3-\frac{x^6}{64}\right)dx=\frac{1}{24}\left(27\frac{x^4}{4}\Big|_0^6-\frac{1}{8}\frac{x^7}{7}\Big|_0^6\right)=$$

$$=\frac{1}{96}\left(27\times6^4-\frac{1}{14}6^7\right)=\frac{1}{96}\left(27\times1296-\frac{279936}{14}\right)=156.21\ cm^4$$

$$I_{yy}=\int_A x^2 dA=\int_{x=0}^{x=6}x^2\left(\int_{y=\frac{x^2}{4}}^{y=\frac{3}{2}x}dy\right)dx=\int_0^6 x^2\left(\frac{3}{2}x-\frac{x^2}{4}\right)dx=$$

$$=\frac{1}{2}\int_0^6\left(3x^3-\frac{1}{2}x^4\right)dx=\frac{1}{2}\left(3\frac{x^4}{4}\Big|_0^6-\frac{1}{2}\frac{x^5}{5}\Big|_0^6\right)=$$

$$=\frac{1}{4}\left(\frac{3}{2}\times6^4-\frac{6^5}{5}\right)=\frac{1}{4}\left(\frac{3}{2}\times1296-\frac{7776}{5}\right)=97.2\ cm^4$$

$$I_{xy}=-\int_A xy\,dA=-\int_{x=0}^{x=6}x\left(\int_{y=\frac{x^2}{4}}^{y=\frac{3}{2}x}y\,dy\right)dx=$$

$$=-\int_0^6 x\left(\frac{y^2}{2}\right)_{\frac{x^2}{4}}^{\frac{3}{2}x}dx=-\frac{1}{2}\int_0^6 x\left(\frac{9}{4}x^2-\frac{x^4}{16}\right)dx=$$

$$=-\frac{1}{8}\int_0^6\left(9x^3-\frac{x^5}{4}\right)dx=-\frac{1}{8}\left(9\frac{x^4}{4}\Big|_0^6-\frac{1}{4}\frac{x^6}{6}\Big|_0^6\right)=-\frac{1}{32}\left(9\times6^4-\frac{\cancel{6^6}\,6^5}{\cancel{6}}\right)=-121.5\ cm^4$$