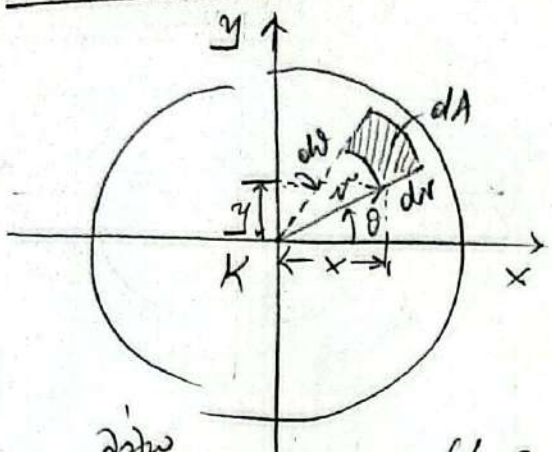
28η ΑΣΚΗΣΗ

Να υπολογισθούν, οι επιφανειακές ροπές, η φυγόκεντρος και η πολική ροπή 2ας τάξεως της κυκλικής διατομής.



$$dA = r\,d\theta\,dr\,,\quad x = r\cos\theta\,,\quad y = r\,\eta\mu\theta$$

$$I_{xx} \overset{\text{λόγω}}{\underset{\text{συμμ.}}{=\!=\!=}} I_{yy} = \iint_A y^2\,dA = \int_0^R\int_0^{2\pi} r^2 \sin^2\theta\, r\,dr\,d\theta =$$

$$= \int_0^R r^3\,dr \int_0^{2\pi} \frac{1-\cos 2\theta}{2}\,d\theta = \left(\frac{r^4}{4}\right)_0^R \cdot \frac{1}{2}\left(\int_0^{2\pi} d\theta - \int_0^{2\pi}\cos 2\theta\,d\theta\right) =$$

$$= \frac{R^4}{8}\left(2\pi - \cancel{\frac{\sin 2\theta}{2}\Big|_0^{2\pi}}^{\,0}\right) = \frac{\pi R^4}{4}\ \text{cm}^4$$

$$I_K = I_{xx} + I_{yy} = \frac{\pi R^4}{2}\ \text{cm}^4$$

$$I_{xy} = -\iint_A xy\,dA = -\int_0^R\int_0^{2\pi} r\cos\theta \cdot r\,\eta\mu\theta \cdot r\,d\theta\,dr = -\int_0^R r^3\,dr \int_0^{2\pi} \frac{\sin 2\theta}{2}\,d\theta =$$

$$= -\frac{R^4}{4}\cdot\frac{1}{2}\left(-\frac{\cos 2\theta}{2}\right)_0^{2\pi} = +\frac{R^4}{16}\left(\cancel{\cos 4\pi}^{\,1} - \cancel{\cos 0}^{\,1}\right) = 0\ \text{cm}^4$$

Αν τεταρτοκύκλιο:

$$I_{xx} = I_{yy} = \frac{R^4}{8}\left(\int_0^{\pi/2} d\theta - \int_0^{\pi/2} \cos 2\theta\,d\theta\right) = \frac{R^4}{8}\left(\frac{\pi}{2} - \cancel{\frac{\eta\mu 2\theta}{2}\Big|_0^{\frac{\pi}{2}}}^{\,0}\right) = \frac{\pi R^4}{16}\ \text{cm}^4$$

$$I_{xy} = -\frac{R^4}{16}\left(-\cos 2\theta\right)_0^{\pi/2} = \frac{R^4}{16}\left(\cos\pi - \cos 0\right) = \frac{R^4}{16}(-1-1) = -\frac{R^4}{8}\ \text{cm}^4$$