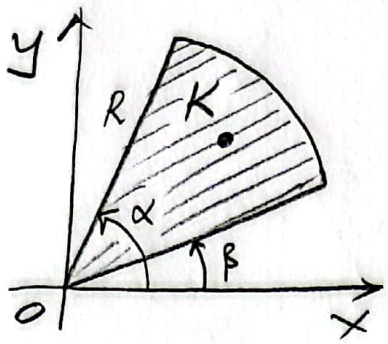
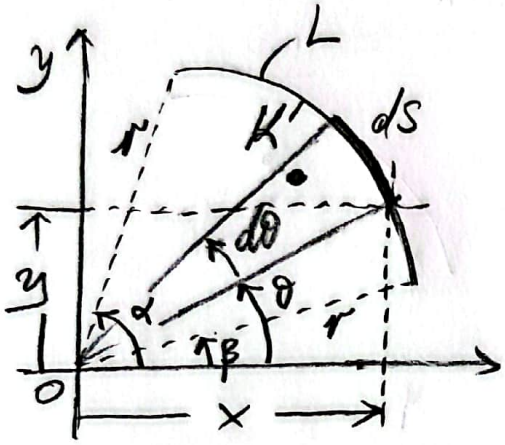
ΑΣΚΗΣΗ 5-3



Κεντροειδές κυκλικού τομέα

1ος τρόπος Πρώτα υπολογισμός κεντροειδούς K', τόξου κύκλου, μήκους L.



Θεωρία

$$x_K = \frac{\int_L x\,ds}{\int_L ds}, \quad y_K = \frac{\int_L y\,ds}{\int_L ds}$$

$$ds = r\,d\theta, \quad x = r\cos\theta, \quad y = r\sin\theta$$

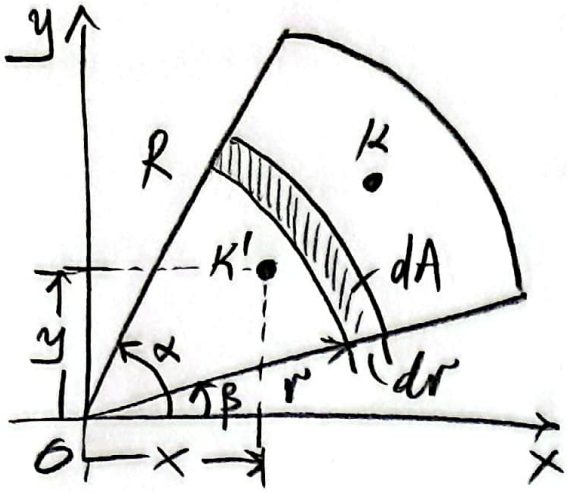
$$L = \int_L ds = \int_\beta^\alpha r\,d\theta = r(\alpha-\beta) \qquad (\alpha>0,\ \alpha<0)$$

$$S_y = \int_L x\,ds = \int_\beta^\alpha r^2\cos\theta\,d\theta = r^2\left[\sin\theta\right]_\beta^\alpha = r^2(\sin\alpha-\sin\beta)$$

$$S_x = \int_L y\,ds = \int_\beta^\alpha r^2\sin\theta\,d\theta = -r^2\left[\cos\theta\right]_\beta^\alpha = r^2(\cos\beta-\cos\alpha)$$

$$\boxed{x_{K'} = \frac{S_y}{L} = r\frac{\sin\alpha-\sin\beta}{\alpha-\beta}} \qquad \boxed{y_{K'} = \frac{S_x}{L} = r\frac{\cos\beta-\cos\alpha}{\alpha-\beta}}$$

Σαρώθηκε με το CamScanner



$$dA = r(\alpha-\beta)\,dr$$

$$\boxed{A = \int_A dA = (\alpha-\beta)\int_0^R r\,dr = \frac{R^2(\alpha-\beta)}{2}}$$

$$\boxed{S_y = \int_A x\,dA \overset{x\equiv x_K'}{=\!=} \int_0^R r\,\frac{\sin\alpha-\sin\beta}{\alpha-\beta}\,r(\alpha-\beta)\,dr =}$$

$$= (\sin\alpha-\sin\beta)\left[\frac{r^3}{3}\right]_0^R = \boxed{\frac{R^3(\sin\alpha-\sin\beta)}{3}}$$

$$\boxed{S_x = \int_A y\,dA \overset{y\equiv y_K'}{=\!=} \int_0^R r\,\frac{\cos\beta-\cos\alpha}{\alpha-\beta}\,r(\alpha-\beta)\,dr =}$$

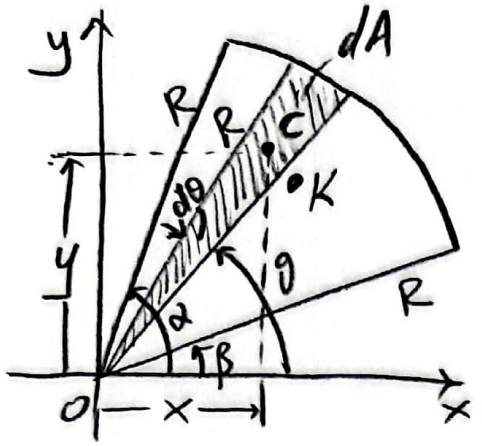
$$= (\cos\beta-\cos\alpha)\left[\frac{r^3}{3}\right]_0^R = \boxed{\frac{R^3(\cos\beta-\cos\alpha)}{3}}$$

$$\boxed{x_K = \frac{S_y}{A} = \frac{R^3(\sin\alpha-\sin\beta)}{3}\,\frac{2}{R^2(\alpha-\beta)} = \frac{2R(\sin\alpha-\sin\beta)}{3(\alpha-\beta)}}$$

$$\boxed{y_K = \frac{S_x}{A} = \frac{R^3(\cos\beta-\cos\alpha)}{3}\,\frac{2}{R^2(\alpha-\beta)} = \frac{2R(\cos\beta-\cos\alpha)}{3(\alpha-\beta)}}$$

Σαρώθηκε με το CamScanner

2ος τρόπος



Το dA σχεδόν (ισοσκελές) τρίγωνο

$$dA = \frac{1}{2} \cdot R\,d\theta \cdot R = \frac{R^2 d\theta}{2}$$

Το κεντροειδές C του dA στα $\frac{2}{3}R$ από το O $\Rightarrow$

$$x = \frac{2}{3}R\cos\theta, \quad y = \frac{2}{3}R\sin\theta$$

$$\boxed{A = \int_A dA = \int_\beta^\alpha \frac{R^2 d\theta}{2} = \frac{R^2}{2}(\alpha - \beta)} \checkmark$$

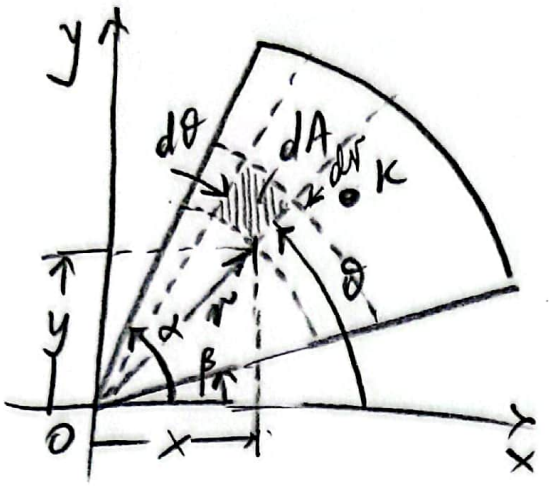
$$\boxed{S_y = \int_A x\,dA = \int_\beta^\alpha \frac{2}{3}R\cos\theta\,\frac{R^2 d\theta}{2} = \frac{R^3}{3}\left[\sin\theta\right]_\beta^\alpha = \frac{R^3(\sin\alpha - \sin\beta)}{3}} \checkmark$$

$$\boxed{S_x = \int_A y\,dA = \int_\beta^\alpha \frac{2}{3}R\sin\theta\,\frac{R^2 d\theta}{2} = \frac{R^3}{3}\left[-\cos\theta\right]_\beta^\alpha = \frac{R^3(\cos\beta - \cos\alpha)}{3}} \checkmark$$

...

Σαρώθηκε με το CamScanner

3ος τρόπος



$$dA = r\,d\theta\,dr$$

$$x = r\cos\theta, \quad y = r\sin\theta$$

$$\boxed{A = \int_A dA = \int_0^R r\,dr \int_\beta^\alpha d\theta = \frac{R^2(\alpha-\beta)}{2}} \checkmark$$

$$\boxed{S_y = \int_A x\,dA = \int_0^R \int_\beta^\alpha r\cos\theta\, r\,d\theta\,dr = \int_0^R r^2\,dr \int_\beta^\alpha \cos\theta\,d\theta =}$$

$$= \boxed{\frac{R^3}{3}(\sin\alpha - \sin\beta)} \checkmark$$

$$\boxed{S_x = \int_A y\,dA = \int_0^R \int_\beta^\alpha r\sin\theta\, r\,d\theta\,dr = \int_0^R r^2\,dr \int_\beta^\alpha \sin\theta\,d\theta =}$$

$$= \boxed{\frac{R^3}{3}(\cos\beta - \cos\alpha)} \checkmark$$

Σαρώθηκε με το CamScanner