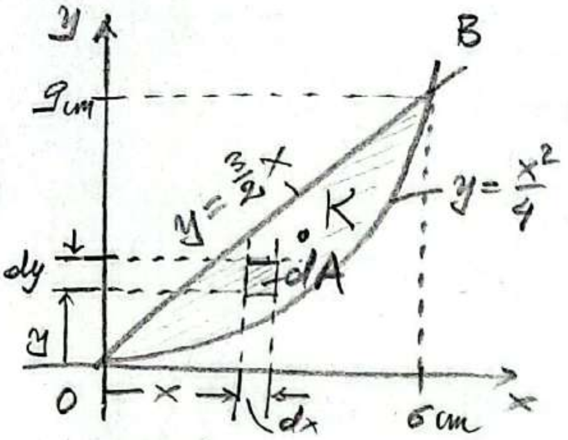
24η ΑΣΚΗΣΗ

Να υπολογισθούν οι συντεταγμένες x_K, y_K του κεντροειδούς K της επιφάνειας



OB (ευθεία): $y = \gamma x + \delta$

$x=0,\ y=0 \Rightarrow \delta = 0$

$x=6,\ y=9 \Rightarrow \gamma = \frac{9}{6} = \frac{3}{2}$

$\Rightarrow y = \frac{3}{2}x$

1ος τρόπος

$$\boxed{S_y} = \int_A x\,dA = \int_{x=0}^{x=6} x\left(\int_{y=\frac{x^2}{4}}^{y=\frac{3}{2}x} dy\right)dx = \int_0^6 x\left(\frac{3}{2}x - \frac{x^2}{4}\right)dx =$$

$$= \frac{3}{2}\,\frac{x^3}{3}\Big|_0^6 - \frac{1}{4}\,\frac{x^4}{4}\Big|_0^6 = \frac{1}{2}216 - \frac{1}{16}1296 = \boxed{27\ cm^3}$$

$$\boxed{S_x} = \int_A y\,dA = \int_{x=0}^{6}\left(\int_{y=\frac{x^2}{4}}^{y=\frac{3}{2}x} y\,dy\right)dx = \int_0^6 \left(\frac{y^2}{2}\right)_{\frac{x^2}{4}}^{\frac{3}{2}x} dx =$$

$$= \frac{1}{2}\int_0^6 \left(\frac{9}{4}x^2 - \frac{x^4}{16}\right)dx = \frac{1}{8}\left(9\frac{x^3}{3}\Big|_0^6 - \frac{1}{4}\,\frac{x^5}{5}\Big|_0^6\right) =$$

$$= \frac{1}{8}\left(3\times 6^3 - \frac{1}{20}6^5\right) = \boxed{32.4\ cm^3}$$

$$\boxed{A} = \int_A dA = \int_{x=0}^{x=6}\left(\int_{y=\frac{x^2}{4}}^{y=\frac{3}{2}x} dy\right)dx = \int_0^6\left(\frac{3}{2}x - \frac{x^2}{4}\right)dx = \frac{1}{2}\left(3\,\frac{x^2}{2}\Big|_0^6 - \frac{1}{2}\,\frac{x^3}{3}\Big|_0^6\right) =$$

$$= \frac{1}{4}\left(3\times 36 - \frac{216}{3}\right) = 27 - 18 = \boxed{9 cm^2}$$

$$\boxed{x_K = \frac{\int_A x\,dA}{\int_A dA} = \frac{S_y}{A} = \frac{27}{9} = 3cm},\quad \boxed{y_K = \frac{\int_A y\,dA}{\int_A dA} = \frac{S_x}{A} = \frac{32.4}{9} = 3.6cm^2}$$

Αλλαγή σειράς ολοκλήρωσης

$$y=\frac{3}{2}x \Rightarrow \underline{x=\frac{2}{3}y}\ , \quad y=\frac{x^2}{4} \Rightarrow \underline{x=2y^{1/2}}$$

$$S_y=\int_A x\,dA=\int_{y=0}^{y=9}\left(\int_{x=\frac{2}{3}y}^{x=2y^{1/2}} x\,dx\right)dy=\int_0^9\left(\frac{x^2}{2}\right)_{\frac{2}{3}y}^{2y^{1/2}}dy=$$

$$=\frac{1}{2}\int_0^9\left(4y-\frac{4}{9}y^2\right)dy=2\left(\frac{y^2}{2}\Big|_0^9-\frac{1}{9}\frac{y^3}{3}\Big|_0^9\right)=2\left(\frac{81}{2}-\cancel{\frac{1}{9}}\,\frac{\cancel{9}\times 81}{3}\right)=$$

$$=\frac{2\times 81}{6}=\boxed{27\ cm^3}\ \checkmark$$

$$S_x=\int_A y\,dA=\int_{y=0}^{y=9}y\left(\int_{x=\frac{2}{3}y}^{x=2y^{1/2}}dx\right)dy=\int_0^9 y\left(2y^{1/2}-\frac{2}{3}y\right)dy=$$

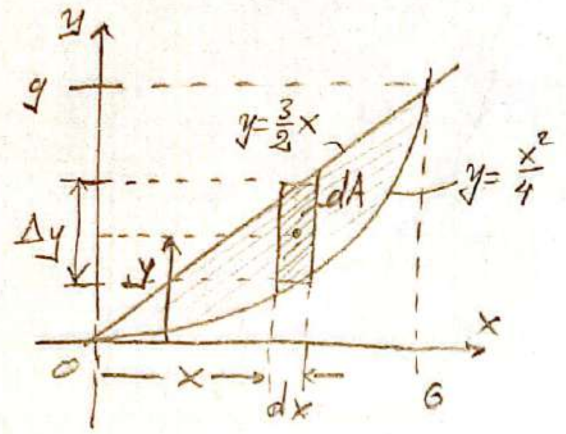
$$=2\int_0^9\left(y^{\frac{3}{2}}-\frac{1}{3}y^2\right)dy=2\left(\frac{y^{\frac{3}{2}+1}}{\frac{3}{2}+1}\Big|_0^9-\frac{1}{3}\frac{y^3}{3}\Big|_0^9\right)=$$

$$=\frac{4}{5}243-\frac{2}{9}729=194.4-162=\boxed{32.4\ cm^3}\ \checkmark$$

$$A=\int_A dA=\int_{y=0}^{y=9}\left(\int_{x=\frac{2}{3}y}^{x=2y^{1/2}}dx\right)dy=\int_0^9\left(2y^{1/2}-\frac{2}{3}y\right)dy=$$

$$=2\left(\frac{y^{1/2+1}}{\frac{1}{2}+1}\Big|_0^9-\frac{1}{3}\frac{y^2}{2}\Big|_0^9\right)dy=\frac{4}{3}27-\frac{3\times 27}{3}=\frac{27}{3}=\boxed{9\ cm^2}\ \checkmark$$

2ος τρόπος



$$dA = \Delta y\, dx = \left(\frac{3}{2}x - \frac{x^2}{4}\right)dx$$

$$y = \frac{1}{2}\left(\frac{x^2}{4} + \frac{3}{2}x\right)$$

$$A = \int_A dA = \int_{x=0}^{x=6}\left(\frac{3}{2}x - \frac{x^2}{4}\right)dx =$$

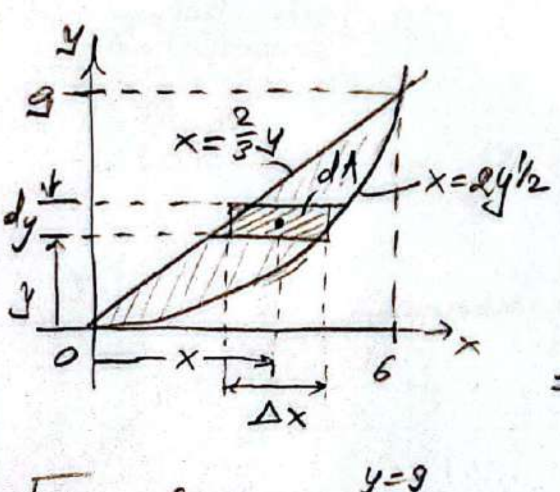
$$= \frac{1}{2}\left(3\,\frac{x^2}{2}\Big|_0^6 - \frac{1}{2}\,\frac{x^3}{3}\Big|_0^6\right) = \frac{1}{4}\left(3\times 36 - \frac{216}{3}\right) = \boxed{9\ cm^2}$$

$$S_y = \int_A x\, dA = \int_{x=0}^{x=6} x\left(\frac{3}{2}x - \frac{x^2}{4}\right)dx = \frac{1}{2}\left(3\,\frac{x^3}{3}\Big|_0^6 - \frac{1}{2}\,\frac{x^4}{4}\Big|_0^6\right) = \frac{216}{2} - \frac{1296}{16} = \boxed{27\ cm^3}$$

$$S_x = \int_A y\, dA = \int_{x=0}^{x=6}\frac{1}{2}\left(\frac{x^2}{4} + \frac{3}{2}x\right)\left(\frac{3}{2}x - \frac{x^2}{4}\right)dx = \frac{1}{8}\int_{x=0}^{x=6}\left[(3x)^2 - \left(\frac{x^2}{2}\right)^2\right]dx =$$

$$= \frac{1}{8}\int_0^6\left(9x^2 - \frac{x^4}{4}\right)dx = \frac{1}{8}\left(9\,\frac{x^3}{3}\Big|_0^6 - \frac{1}{4}\,\frac{x^5}{5}\Big|_0^6\right) = \frac{1}{8}\left(3\times 6^3 - \frac{6^5}{20}\right) = \boxed{32.4\ cm^3}$$

3ος τρόπος



$$dA = \Delta x\, dy = \left(2y^{1/2} - \frac{2}{3}y\right)dy$$

$$x = \frac{1}{2}\left(\frac{2}{3}y + 2y^{1/2}\right)$$

$$A = \int_A dA = \int_{y=0}^{y=9}\left(2y^{1/2} - \frac{2}{3}y\right)dy =$$

$$= 2\cdot\left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\Big|_0^9 - \frac{1}{3}\,\frac{y^2}{2}\Big|_0^9\right) = \frac{4\times 27}{3} - \frac{3\times 27}{3} = \boxed{9\ cm^2}$$

$$S_y = \int_A x\, dA = \int_{y=0}^{y=9}\frac{1}{2}\left(\frac{2}{3}y + 2y^{1/2}\right)\left(2y^{1/2} - \frac{2}{3}y\right)dy = 2\int_0^9\left[\left(y^{1/2}\right)^2 - \left(\frac{y}{3}\right)^2\right]dy =$$

$$= 2\int_0^9\left(y - \frac{y^2}{9}\right)dy = 2\left(\frac{y^2}{2}\Big|_0^9 - \frac{1}{9}\,\frac{y^3}{3}\Big|_0^9\right) = 2\left(\frac{81}{2} - \frac{1}{\cancel{9}}\,\frac{\cancel{9}\times 81}{3}\right) = \boxed{27\ cm^3}$$

$$S_x = \int_A y\, dA = \int_{y=0}^{y=9} y\left(2y^{1/2} - \frac{2}{3}y\right)dy = 2\left(\frac{y^{\frac{5}{2}}}{\frac{5}{2}}\Big|_0^9 - \frac{1}{3}\,\frac{y^3}{3}\Big|_0^9\right) = 194.4 - 162 = \boxed{32.4\ cm^3}$$