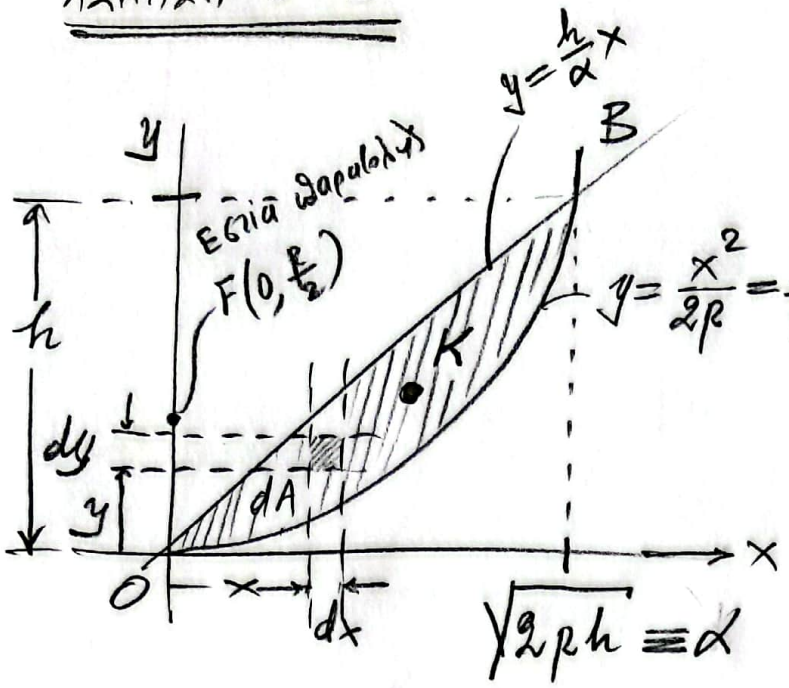
ΑΣΚΗΣΗ 5-2



$y=\frac{x^2}{2p}=\frac{x^2}{b}$

Κατηγορία επιφάνειας μεταξύ ευθείας OB και παραβολής $y=\frac{x^2}{2p}\equiv\frac{x^2}{b}$

Εξίσωση ευθείας OB: $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$$y-0=\frac{h-0}{\alpha-0}(x-0)\Rightarrow \boxed{y=\frac{h}{\alpha}x}$$

1ος τρόπος

$$S_y=\int_A x\,dA=\int_{x=0}^{x=\alpha} x\left(\int_{y=\frac{x^2}{b}}^{y=\frac{h}{\alpha}x} dy\right)dx=\int_0^{\alpha} x\left(\frac{h}{\alpha}x-\frac{x^2}{b}\right)dx=$$

$$=\frac{h}{\alpha}\left[\frac{x^3}{3}\right]_0^{\alpha}-\frac{1}{b}\left[\frac{x^4}{4}\right]_0^{\alpha}=\frac{h\alpha^2}{3}-\frac{\alpha^4}{4b}=\frac{h\,2ph}{3}-\frac{4p^2h^2}{4\cdot 2p}=$$

$$=ph^2\left(\frac{2}{3}-\frac{1}{2}\right)=\boxed{\frac{ph^2}{6}}$$

$$S_x=\int_A y\,dA=\int_{x=0}^{x=\alpha}\left(\int_{y=\frac{x^2}{b}}^{y=\frac{h}{\alpha}x} y\,dy\right)dx=\int_{x=0}^{x=\alpha}\left[\frac{y^2}{2}\right]_{\frac{x^2}{b}}^{\frac{h}{\alpha}x}dx=$$

$$=\frac{1}{2}\int_0^{\alpha}\left(\frac{h^2}{\alpha^2}x^2-\frac{x^4}{b^2}\right)dx=\frac{1}{2}\left\{\frac{h^2}{\alpha^2}\left[\frac{x^3}{3}\right]_0^{\alpha}-\frac{1}{b^2}\left[\frac{x^5}{5}\right]_0^{\alpha}\right\}=$$

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$$= \frac{1}{2}\left(\frac{h^2\alpha}{3} - \frac{\alpha^5}{5b^2}\right) = \frac{1}{2}\left[\frac{h^2\sqrt{2ph}}{3} - \frac{\sqrt{2ph}\,(2ph)^2}{5(2p)^2}\right] =$$

$$= \frac{h^2\sqrt{2ph}}{2}\left(\frac{1}{3} - \frac{1}{5}\right) = \boxed{\frac{h^2\sqrt{2ph}}{15} = S_x'}$$

$$\boxed{A = \int_A dA = \int_{x=0}^{x=\alpha}\left(\int_{y=\frac{x^2}{b}}^{y=\frac{h}{\alpha}x} dy\right)dx = \int_0^{\alpha}\left(\frac{h}{\alpha}x - \frac{x^2}{b}\right)dx =}$$

$$= \frac{h}{\alpha}\left[\frac{x^2}{2}\right]_0^{\alpha} - \frac{1}{b}\left[\frac{x^3}{3}\right]_0^{\alpha} = \frac{h\alpha}{2} - \frac{\alpha^3}{3b} =$$

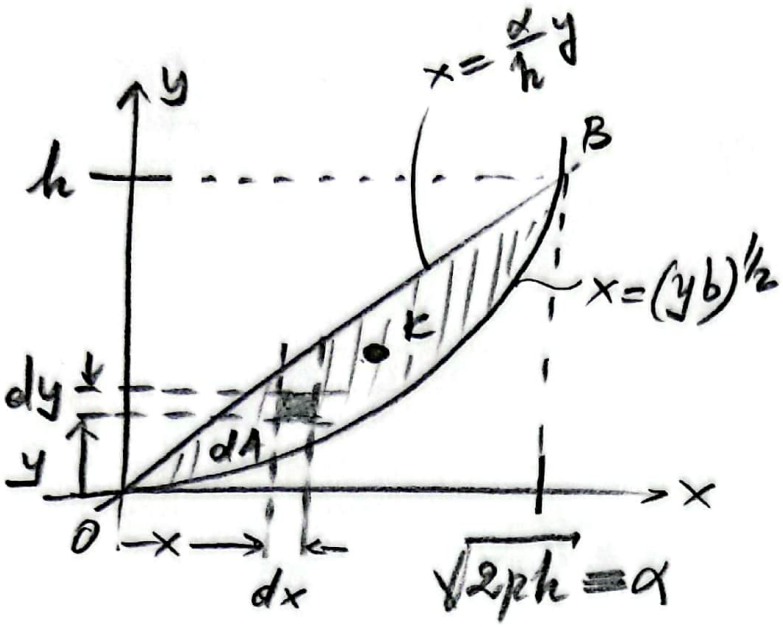
$$= \frac{h}{2}\sqrt{2ph} - \sqrt{2ph}\,\frac{2ph}{3\cdot 2p} = \boxed{\frac{h\sqrt{2ph}}{6}}$$

$$\boxed{x_K = \frac{\int_A x\,dA}{\int_A dA} = \frac{S_y}{A} = \frac{ph^2}{6}\cdot\frac{6}{h\sqrt{2ph}} = \frac{ph\sqrt{2ph}}{2ph} = \frac{\sqrt{2ph}}{2}}$$

$$\boxed{y_K = \frac{\int_A y\,dA}{\int_A dA} = \frac{S_x}{A} = \frac{h^2\sqrt{2ph}}{15}\cdot\frac{6}{h\sqrt{2ph}} = \frac{2h}{5}}$$

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2ος τρόπος | Με αλλαγή σειράς ολοκλήρωσης



$$y = \frac{h}{\alpha}x \Rightarrow x = \frac{\alpha}{h}y$$

$$y = \frac{x^2}{b} \Rightarrow x = (yb)^{1/2}$$

$$(b \equiv 2p)$$

$$S_y = \int_A x\,dA = \int_{y=0}^{y=h}\left(\int_{x=\frac{\alpha}{h}y}^{x=(yb)^{1/2}} x\,dx\right)dy = \int_{y=0}^{y=h}\left[\frac{x^2}{2}\right]_{\frac{\alpha}{h}y}^{(yb)^{1/2}} dy =$$

$$= \frac{1}{2}\int_0^h \left(yb - \frac{\alpha^2}{h^2}y^2\right)dy = \frac{1}{2}\left\{b\left[\frac{y^2}{2}\right]_0^h - \frac{\alpha^2}{h^2}\left[\frac{y^3}{3}\right]_0^h\right\} =$$

$$= \frac{1}{2}\left(\frac{bh^2}{2} - \frac{\alpha^2 h}{3}\right) = \frac{h}{2}\left(\frac{2ph}{2} - \frac{2ph}{3}\right) = \boxed{\frac{ph^2}{6}} \checkmark$$

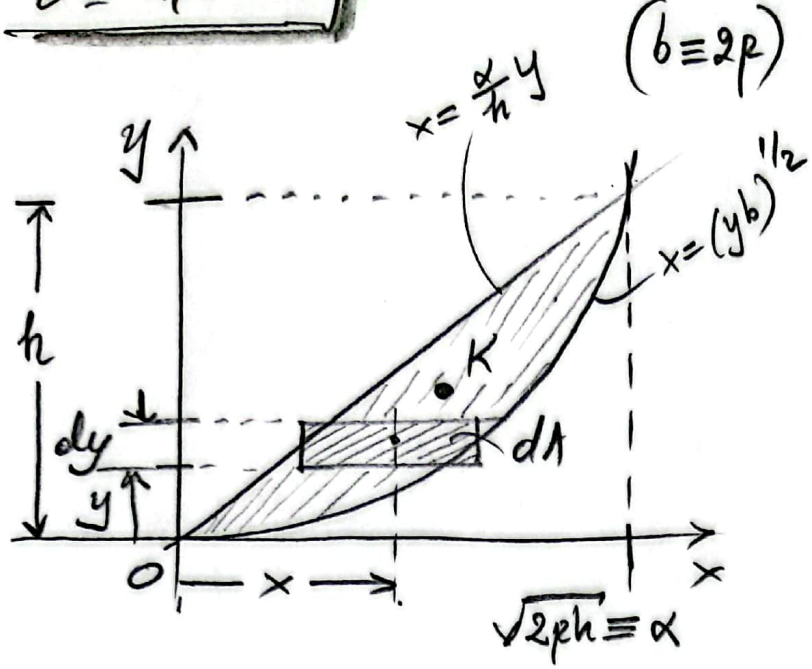
$$S_x = \int_A y\,dA = \int_{y=0}^{y=h} y\left(\int_{x=\frac{\alpha}{h}y}^{x=(yb)^{1/2}} dx\right)dy = \int_0^h \left(\sqrt{b}\,y^{3/2} - \frac{\alpha}{h}y^2\right)dy =$$

$$= \sqrt{b}\left[\frac{y^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right]_0^h - \frac{\alpha}{h}\left[\frac{y^3}{3}\right]_0^h = \sqrt{b}\,\frac{2}{5}h^2\sqrt{h} - \frac{\alpha h^2}{3} =$$

$$= h^2\sqrt{2ph}\left(\frac{2}{5} - \frac{1}{3}\right) = \boxed{\frac{h^2\sqrt{2ph}}{15}} \checkmark \ ,,,$$

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$$dA = \left(\sqrt{b}\, y^{1/2} - \frac{\alpha}{h} y\right) dy =$$

$$= \left(\sqrt{2p}\, y^{1/2} - \frac{\sqrt{2ph}}{h} y\right) dy$$

$$= \sqrt{2p}\left(y^{1/2} - \frac{\sqrt{h}}{h} y\right) dy$$

$$x = \frac{1}{2}\left(\sqrt{b}\, y^{1/2} + \frac{\alpha}{h} y\right) = \frac{1}{2}\left(\sqrt{2p}\, y^{1/2} + \frac{\sqrt{2ph}}{h} y\right) =$$

$$= \frac{\sqrt{2p}}{2}\left(y^{1/2} + \frac{\sqrt{h}}{h} y\right)$$

$$\boxed{A = \int_A dA = \int_{y=0}^{y=h} \sqrt{2p}\left(y^{1/2} - \frac{\sqrt{h}}{h} y\right) dy =}$$

$$= \sqrt{2p}\left\{\left[\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^h - \frac{\sqrt{h}}{h}\left[\frac{y^2}{2}\right]_0^h\right\} = \sqrt{2p}\left(\frac{2}{3} h\sqrt{h} - \frac{\sqrt{h}\, h}{2}\right) =$$

$$= \boxed{\frac{h\sqrt{2ph}}{6}} \checkmark$$

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$$S_y = \int_A x\,dA = \int_{y=0}^{y=h} \frac{\sqrt{2p}}{2}\left(y^{1/2} + \frac{\sqrt{h}}{h}y\right)\sqrt{2p}\left(y^{1/2} - \frac{\sqrt{h}}{h}y\right)dy =$$

$$= p\int_0^h \left(y - \frac{y^2}{h}\right)dy = p\left\{\left[\frac{y^2}{2}\right]_0^h - \frac{1}{h}\left[\frac{y^3}{3}\right]_0^h\right\} =$$

$$= p\left(\frac{h^2}{2} - \frac{h^2}{3}\right) = \boxed{\frac{ph^2}{6}} \checkmark$$

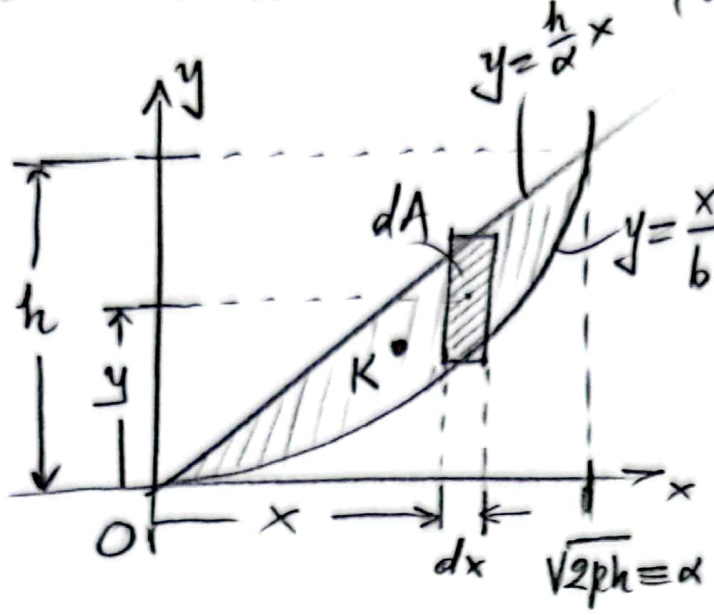
$$S_x = \int_A y\,dA = \int_{y=0}^{y=h} y\sqrt{2p}\left(y^{1/2} - \frac{\sqrt{h}}{h}y\right)dy =$$

$$= \sqrt{2p}\int_0^h \left(y^{3/2} - \frac{\sqrt{h}}{h}y^2\right)dy = \sqrt{2p}\left\{\left[\frac{y^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right]_0^h - \frac{\sqrt{h}}{h}\left[\frac{y^3}{3}\right]_0^h\right\} =$$

$$= \sqrt{2p}\left(\frac{2}{5}h^2\sqrt{h} - \frac{\sqrt{h}\cdot h^2}{3}\right) = \boxed{\frac{h^2\sqrt{2ph}}{15}} \checkmark \ ,,,$$

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4ος τρόπος



$(b\equiv 2p)$

$$dA=\left(\frac{h}{\alpha}x-\frac{x^2}{b}\right)dx$$

$$y=\frac{1}{2}\left(\frac{h}{\alpha}x+\frac{x^2}{b}\right)$$

$$A=\int_A dA=\int_{x=0}^{x=\alpha}\left(\frac{h}{\alpha}x-\frac{x^2}{b}\right)dx=\frac{h}{\alpha}\left[\frac{x^2}{2}\right]_0^{\alpha}-\frac{1}{b}\left[\frac{x^3}{3}\right]_0^{\alpha}=$$

$$=\frac{h\alpha}{2}-\frac{\alpha^3}{3b}=\alpha\left(\frac{h}{2}-\frac{2ph}{3\cdot 2p}\right)=\frac{h\sqrt{2ph}}{6}\ \checkmark$$

$$S_y=\int_A x\,dA=\int_{x=0}^{x=\alpha}x\left(\frac{h}{\alpha}x-\frac{x^2}{b}\right)dx=\frac{h}{\alpha}\left[\frac{x^3}{3}\right]_0^{\alpha}-\frac{1}{b}\left[\frac{x^4}{4}\right]_0^{\alpha}=$$

$$=\frac{h\alpha^2}{3}-\frac{\alpha^4}{4b}=\frac{h\,2ph}{3}-\frac{(2ph)(2ph)}{2\cdot 4\cdot 2p}=\frac{ph^2}{6}\ \checkmark$$

$$S_x=\int_A y\,dA=\int_{x=0}^{x=\alpha}\frac{1}{2}\left(\frac{h}{\alpha}x+\frac{x^2}{b}\right)\left(\frac{h}{\alpha}x-\frac{x^2}{b}\right)dx=$$

$$=\frac{1}{2}\int_0^{\alpha}\left(\frac{h^2}{\alpha^2}x^2-\frac{x^4}{b^2}\right)dx=\frac{1}{2}\left\{\frac{h^2}{\alpha^2}\left[\frac{x^3}{3}\right]_0^{\alpha}-\frac{1}{b^2}\left[\frac{x^5}{5}\right]_0^{\alpha}\right\}=$$

$$=\frac{1}{2}\left(\frac{h^2\alpha}{3}-\frac{\alpha^5}{5b^2}\right)=\frac{\alpha}{2}\left[\frac{h^2}{3}-\frac{(2ph)^2}{5(2p)^2}\right]=\frac{h^2\sqrt{2ph}}{15}\ \checkmark\ \ldots$$

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