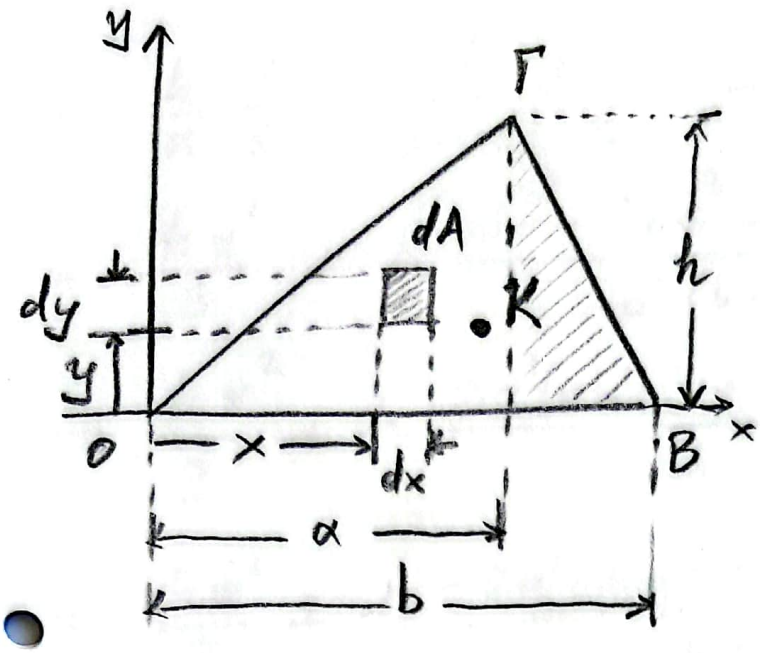
ΑΣΚΗΣΗ 5-1



Κεντροειδές τριγωνικής επιφάνειας

Εξίσωση ευθείας

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\underline{O\Gamma}: \; y - 0 = \frac{h - 0}{\alpha - 0}(x - 0)$$

$$\boxed{y = \frac{h}{\alpha} x}$$

$$\underline{\Gamma B}: \; y - h = \frac{0 - h}{b - \alpha}(x - \alpha) \Rightarrow y = -\frac{h}{b-\alpha} x + \frac{h\alpha}{b-\alpha} + h$$

$$\boxed{y = -\frac{h}{b-\alpha} x + \frac{h}{b-\alpha} b}$$

1ος τρόπος

$$S_y = \int_A x \, dA = \int_{A_1} x \, dA + \int_{A_2} x \, dA =$$

$$= \int_{x=0}^{x=\alpha} x \left(\int_{y=0}^{y=\frac{h}{\alpha}x} dy \right) dx + \int_{x=\alpha}^{x=b} x \left(\int_{y=0}^{y=-\frac{h}{b-\alpha}x + \frac{bh}{b-\alpha}} dy \right) dx =$$

$$= \int_{x=0}^{x=\alpha} \frac{h}{\alpha} x^2 \, dx + \int_{x=\alpha}^{x=b} \left(\frac{bh}{b-\alpha} x - \frac{h}{b-\alpha} x^2 \right) dx =$$

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$$= \frac{h}{\alpha}\left[\frac{x^3}{3}\right]_0^{\alpha} + \frac{bh}{b-\alpha}\left[\frac{x^2}{2}\right]_{\alpha}^{b} - \frac{h}{b-\alpha}\left[\frac{x^3}{3}\right]_{\alpha}^{b} =$$

$$= \frac{h}{\alpha}\frac{\alpha^3}{3} + \frac{bh}{b-\alpha}\frac{b^2-\alpha^2}{2} - \frac{h}{b-\alpha}\frac{b^3-\alpha^3}{3} =$$

$$= \frac{h\alpha^2}{3} + h\frac{3b^3 - 3ba^2 - 2b^3 + 2\alpha^3}{6(b-\alpha)} =$$

$$= \frac{h\alpha^2}{3} + h\frac{2\alpha^2(\alpha-b) + b(b^2-\alpha^2)}{6(b-\alpha)} =$$

$$= \frac{h\alpha^2}{3} + h\frac{-2\alpha^2(b-\alpha) + b(b-\alpha)(b+\alpha)}{6(b-\alpha)} =$$

$$= \frac{h\alpha^2}{3} - \frac{h\alpha^2}{3} + \frac{bh(\alpha+b)}{6} = S_y$$

$$A = \int_A dA = \int_{A_1} dA + \int_{A_2} dA =$$

$$= \int_{x=0}^{x=\alpha}\left(\int_{y=0}^{y=\frac{h}{\alpha}x} dy\right)dx + \int_{x=\alpha}^{x=b}\left(\int_{y=0}^{y=-\frac{h}{b-\alpha}x+\frac{bh}{b-\alpha}} dy\right)dx =$$

$$= \frac{h}{\alpha}\left[\frac{x^2}{2}\right]_0^{\alpha} + \frac{bh}{b-\alpha}[x]_{\alpha}^{b} - \frac{h}{b-\alpha}\left[\frac{x^2}{2}\right]_{\alpha}^{b} =$$

$$= \frac{h\alpha^2}{2\alpha} + \frac{bh}{b-\alpha}(b-\alpha) - \frac{h}{b-\alpha}\frac{b^2-\alpha^2}{2} =$$

$$= \frac{h\alpha}{2} + bh - \frac{h(b+\alpha)}{2} = \frac{bh}{2}$$

(προφανώς ίσο των εδώ x ύψος / 2)

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$$\boxed{x_K = \frac{\int_A x\,dA}{\int_A dA} = \frac{S_y}{A} = \frac{bh(a+b)}{6}\cdot\frac{2}{bh} = \frac{a+b}{3}}$$

$$\boxed{S_x = \int_A y\,dA = \int_{A_1} y\,dA + \int_{A_2} y\,dA =}$$

$$= \int_{x=0}^{x=a}\left(\int_{y=0}^{y=\frac{h}{a}x} y\,dy\right)dx + \int_{x=a}^{x=b}\left(\int_{y=0}^{y=-\frac{h}{b-a}x+\frac{bh}{b-a}} y\,dy\right)dx =$$

$$\bullet\ \int_0^a \left[\frac{y^2}{2}\right]_0^{\frac{h}{a}x} dx + \int_a^b \left[\frac{y^2}{2}\right]_0^{-\frac{h}{b-a}x+\frac{bh}{b-a}} dx =$$

$$= \int_0^a \frac{h^2}{2a^2}x^2\,dx + \int_a^b \frac{h^2x^2+b^2h^2-2bh^2x}{2(b-a)^2}\,dx =$$

$$= \frac{h^2}{2a^2}\left[\frac{x^3}{3}\right]_0^a + \frac{h^2}{2(b-a)^2}\left\{\left[\frac{x^3}{3}\right]_a^b + b^2[x]_a^b - 2b\left[\frac{x^2}{2}\right]_a^b\right\} =$$

$$\bullet\ = \frac{h^2a}{6} + \frac{h^2}{2(b-a)^2}\left(\frac{b^3-a^3}{3} + b^3 - b^2a - b^3 + ba^2\right) =$$

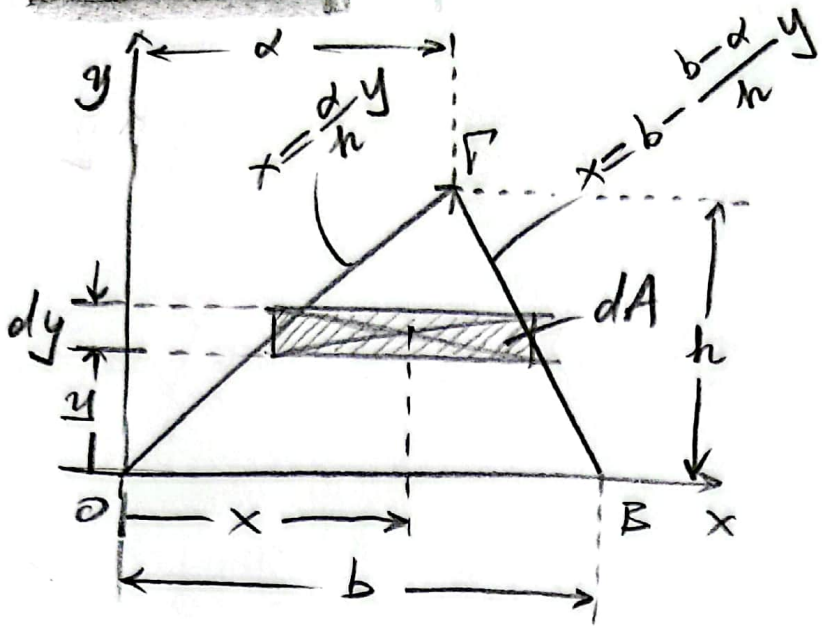
$$= \frac{h^2a}{6} + \frac{h^2}{6(b-a)^2}\left(b^3-a^3-3b^2a+3ba^2\right) =$$

$$= \frac{h^2a}{6} + \frac{h^2}{6(b-a)^2}(b-a)^3 = \boxed{\frac{bh^2}{6}}$$

$$\boxed{y_K = \frac{\int_A y\,dA}{\int_A dA} = \frac{S_x}{A} = \frac{bh^2}{6}\,\frac{2}{bh} = \frac{h}{3}}$$

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2ος τρόπος



$$dA = \left(b - \frac{b-\alpha}{h}y - \frac{\alpha}{h}y\right)dy =$$

$$= \left(b - \frac{b}{h}y\right)dy$$

$$x = \frac{b - \frac{b-\alpha}{h}y + \frac{\alpha}{h}y}{2} =$$

$$= \frac{1}{2}\left(b - \frac{b-2\alpha}{h}y\right)$$

$$\boxed{S_y = \int_A x\,dA = \int_{y=0}^{y=h} \frac{1}{2}\left(b - \frac{b-2\alpha}{h}y\right)\left(b - \frac{b}{h}y\right)dy =}$$

$$= \frac{1}{2}\int_0^h \left(b^2 - \frac{b^2}{h}y - b\frac{b-2\alpha}{h}y + b\frac{b-2\alpha}{h^2}y^2\right)dy =$$

$$= \frac{1}{2}\left\{b^2\left[y\right]_0^h - \frac{b^2}{h}\left[\frac{y^2}{2}\right]_0^h - b\frac{b-2\alpha}{h}\left[\frac{y^2}{2}\right]_0^h + b\frac{b-2\alpha}{h^2}\left[\frac{y^3}{3}\right]_0^h\right\} =$$

$$= \frac{1}{2}\left(b^2h - \frac{b^2h}{2} - b\frac{b-2\alpha}{2}h + b\frac{b-2\alpha}{3}h\right) =$$

$$= \frac{bh}{12}\left(6\alpha + 2b - 4\alpha\right) = \boxed{\frac{bh(\alpha+b)}{6}} \checkmark$$

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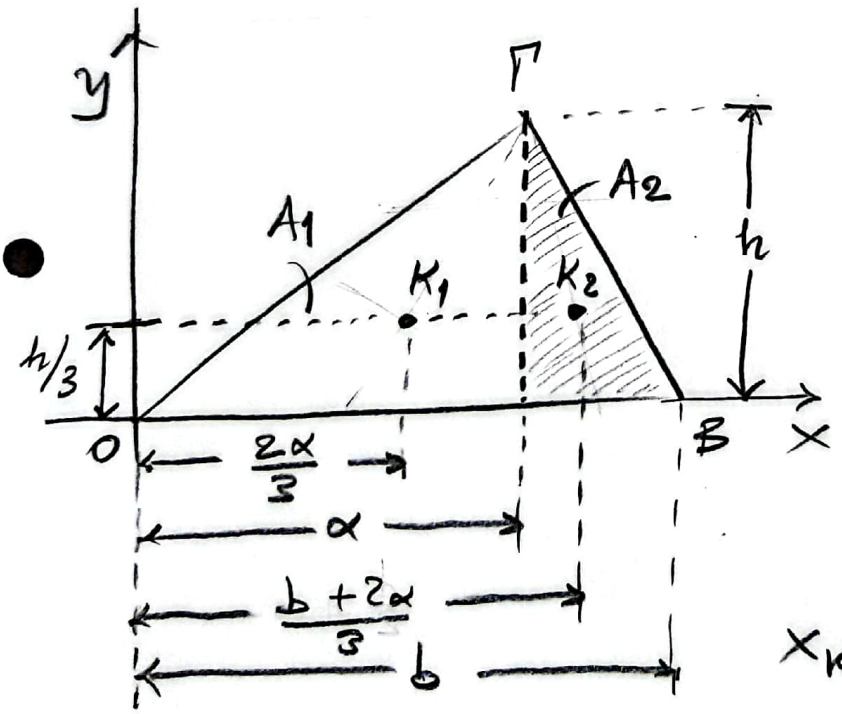
$$A = \int_A dA = \int_0^h \left(b - \frac{b}{h}y\right) dy = b\left[y\right]_0^h - \frac{b}{h}\left[\frac{y^2}{2}\right]_0^h =$$

$$= bh - \frac{bh}{2} = \boxed{\frac{bh}{2}} \checkmark$$

$$S_x = \int_A y\, dA = \int_{y=0}^{y=h} y\left(b - \frac{b}{h}y\right) dy =$$

$$= b\left[\frac{y^2}{2}\right]_0^h - \frac{b}{h}\left[\frac{y^3}{3}\right]_0^h = \frac{bh^2}{2} - \frac{bh^2}{3} = \boxed{\frac{bh^2}{6}} \checkmark$$

3ος τρόπος



Θεωρούμε τετράγωνα τα

$A_1 = \frac{\alpha h}{2}$, $x_{K_1} = \frac{2\alpha}{3}$,

$y_{K_1} = \frac{h}{3}$

$A_2 = \frac{b-\alpha}{2}h$, $y_{K_2} = \frac{h}{3}$

$x_{K_2} = \alpha + \frac{b-\alpha}{3} = \frac{b+2\alpha}{3}$

Οπότε χρήση των:

$$x_K = \frac{\Sigma x_{K_i} A_i}{\Sigma A_i}, \quad y_K = \frac{\Sigma y_{K_i} A_i}{\Sigma A_i}$$

$$x_K = \frac{\frac{2\alpha}{3}\left(\frac{\alpha h}{2}\right) + \frac{b+2\alpha}{3}\left(\frac{b-\alpha}{2}h\right)}{\left(\frac{\alpha h}{2}\right) + \left(\frac{b-\alpha}{2}h\right)} = \frac{2\alpha^2 h + (b+2\alpha)(b-\alpha)h}{6\,\frac{bh}{2}} =$$

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$$= \frac{2\alpha^2 h + b^2 h - b\alpha h + 2\alpha b h - 2\alpha^2 h}{3bh} =$$

$$= \frac{bh(b+\alpha)}{3bh} = \boxed{\frac{b+\alpha}{3} = x_k} \checkmark$$

$$\boxed{\eta_k = \frac{\Sigma y_{ki} A_i}{\Sigma A_i}} = \frac{\frac{h}{3}\left(\frac{\alpha h}{2}\right) + \frac{h}{3}\left(\frac{b-\alpha}{2} h\right)}{\left(\frac{\alpha h}{2}\right) + \left(\frac{b-\alpha}{2} h\right)} =$$

$$= \frac{\alpha h^2 + b h^2 - \alpha h^2}{6 \frac{bh}{2}} = \boxed{\frac{h}{3}} \checkmark$$

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