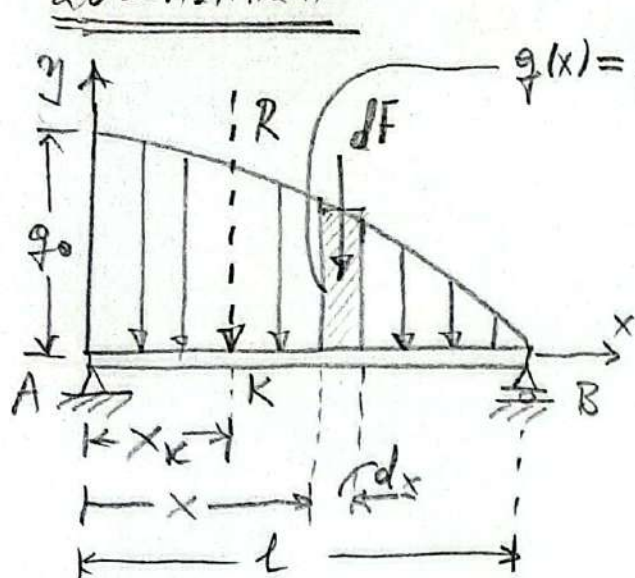


(4-4)

20^η ΑΣΚΗΣΗ

$$q(x) = q_0 \cos\left(\frac{\pi x}{2l}\right)$$

Να υπολογισθεί το μέτρο R , και
 το σημείο εφαρμογής της
 συνισταμένης δύναμης λόγω της
συμπριονομεδών, καταρτιζόμενα πρώτα

$$dF = q(x) dx$$

$$\boxed{R = \int dF = \int_0^l q(x) dx = q_0 \int_0^l \cos\left(\frac{\pi x}{2l}\right) dx =}$$

$$= q_0 \left[\frac{1}{\frac{\pi}{2l}} \sin \frac{\pi x}{2l} \right]_0^l = \frac{2q_0 l}{\pi} \left(\sin \frac{\pi}{2} - \sin 0 \right) = \boxed{\frac{2q_0 l}{\pi}}$$

$$\boxed{x_k = \frac{\int_0^l x q(x) dx}{\int_0^l q(x) dx}}$$

(Από θεωρία)

$$\int_0^l x q(x) dx = \int_0^l x \cdot q_0 \cos \frac{\pi x}{2l} dx = q_0 \int_0^l \underbrace{x}_{u} \underbrace{\left(\cos \frac{\pi x}{2l} \right)}_{dv} dx$$

$$u := x \Rightarrow du = dx$$

$$dv := \cos\left(\frac{\pi x}{2l}\right) dx \Rightarrow v = \frac{1}{\frac{\pi}{2l}} \sin\left(\frac{\pi x}{2l}\right) = \frac{2l}{\pi} \sin \frac{\pi x}{2l}$$

$$\text{Ολοκλήρωση κατά παράγοντες: } \int_0^l u dv = [uv]_0^l - \int_0^l v du$$

$$\begin{aligned} q_0 \int_0^l x \cos \frac{\pi x}{2l} dx &= q_0 \left[x \cdot \frac{2l}{\pi} \sin \frac{\pi x}{2l} \right]_0^l - q_0 \int_0^l \frac{2l}{\pi} \sin \frac{\pi x}{2l} dx = \\ &= q_0 \frac{2l^2}{\pi} \cdot 1 - q_0 \frac{2l}{\pi} \left[-\frac{1}{\frac{\pi}{2l}} \cos \frac{\pi x}{2l} \right]_0^l = \underbrace{\frac{2q_0 l^2}{\pi} + q_0 \frac{4l^2}{\pi^2} (-1)} \end{aligned}$$

$$\boxed{x_k} = \left(\frac{\cancel{2l^2}}{\pi} - \frac{\cancel{4l^2}}{\pi^2} \right) \cdot \frac{\cancel{\pi}}{\cancel{2l^2}} = \boxed{\frac{\pi - 2}{\pi} l}$$

(4-5)