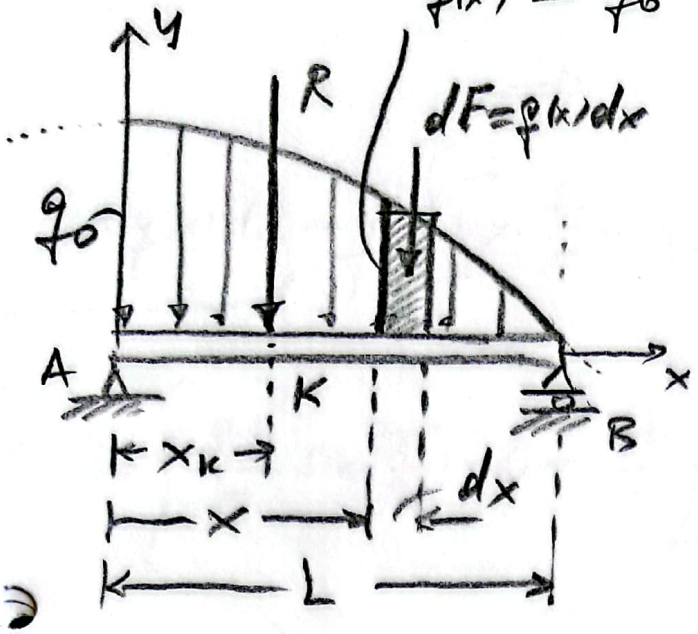
ΑΣΚΗΣΗ 4-3

$$q(x) = q_0 \cos\left(\frac{\pi}{2}\frac{x}{L}\right)$$



Συνεκτικά συνημιτονοειδούς φορτίου

$$q(0) = q_0 \cos 0 = q_0$$

$$q(L) = q_0 \cos\left(\frac{\pi}{2}\frac{L}{L}\right) = 0$$

Θεωρία

$$\boxed{R = \int_0^L q(x)\,dx} = \int_0^L q_0 \cos\left(\frac{\pi}{2}\frac{x}{L}\right) dx = q_0\left(\frac{1}{\frac{\pi}{2L}}\sin\left(\frac{\pi}{2L}x\right)\right)\Bigg|_0^L =$$

$$= \frac{2q_0L}{\pi}\left(\cancelto{1}{\sin\frac{\pi}{2}} - \cancelto{0}{\sin 0}\right) = \boxed{\frac{2q_0L}{\pi}}$$

Θεωρία

$$\boxed{x_K = \frac{\int_0^L x\,q(x)\,dx}{\int_0^L q(x)\,dx}} \qquad \left(\int_0^L q(x)\,dx = \frac{2q_0L}{\pi}\right)$$

$$\int_0^L x\,q(x)\,dx = q_0\int_0^L \underbrace{x}_{u}\,\underbrace{\cos\left(\frac{\pi}{2L}x\right)dx}_{dv}$$

Ολοκλήρωση κατά παραγοντες

$$u := x \Rightarrow du = dx$$

$$dv := \cos\left(\frac{\pi}{2L}x\right)dx \overset{\int}{\Rightarrow} v = \frac{1}{\frac{\pi}{2L}}\sin\left(\frac{\pi}{2L}x\right)$$

Σαρώθηκε με το CamScanner

$$\int_0^L u\,dv = [u\cdot v]_0^L - \int_0^L v\,du$$

$$\Rightarrow$$

$$\int_0^L x\,q(x)\,dx = q_0\left\{\left[x\cdot\frac{2L}{\pi}\sin\left(\frac{\pi}{2L}x\right)\right]_0^L - \frac{2L}{\pi}\int_0^L \sin\left(\frac{\pi}{2L}x\right)dx\right\} =$$

$$= q_0\left\{\frac{2L^2}{\pi}\sin\left(\frac{\pi}{2}\right) - 0 - \frac{2L}{\pi}\left[-\frac{1}{\frac{\pi}{2L}}\cos\left(\frac{\pi}{2L}x\right)\right]_0^L\right\} =$$

$$= q_0\left\{\frac{2L^2}{\pi} + \frac{4L^2}{\pi^2}\left[\cos\left(\frac{\pi}{2}\right) - \cos(0)\right]\right\} = \frac{2q_0L^2}{\pi^2}(\pi-2)$$

$$\Rightarrow$$

$$x_R = \frac{2q_0L^2}{\pi^2}(\pi-2)\cdot\frac{\pi}{2q_0L} = \frac{\pi-2}{\pi}L$$

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