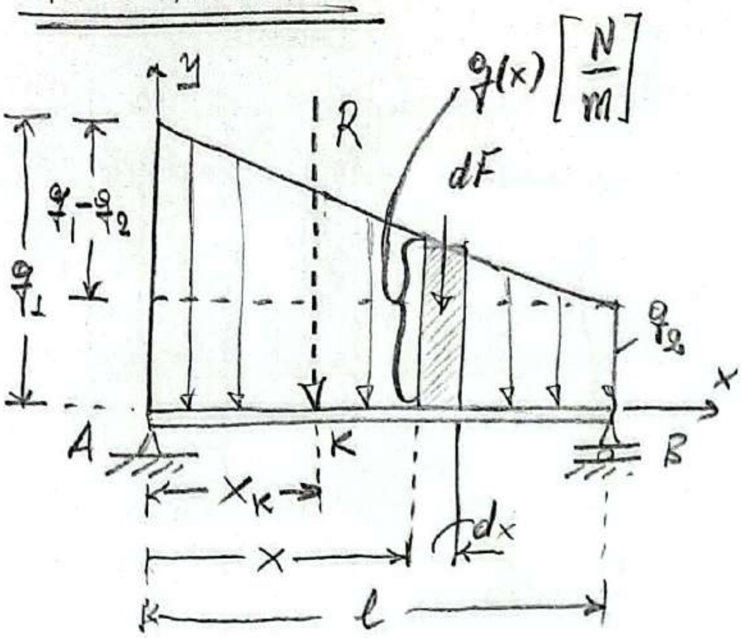
19η ΑΣΚΗΣΗ

Να υπολογισθεί το μέτρο R, και το σημείο εφαρμογής της συνισταμένης δύναμης λόγω του τραπεζοειδούς, κατανεμημένου φορτίου



$$\frac{q(x)-q_2}{l-x} = \frac{q_1-q_2}{l} \Rightarrow$$

$$\Rightarrow q(x) = \frac{q_1-q_2}{l}(l-x)+q_2 = q_1\frac{l-x}{l} - \cancel{q_2} + q_2\frac{x}{l} + \cancel{q_2} \Rightarrow$$

$$\Rightarrow \boxed{q(x) = q_1\left(1-\frac{x}{l}\right)+q_2\frac{x}{l}}$$

$$dF = q(x)\,dx$$

$$\boxed{R = \int dF = \int_0^l q(x)\,dx} =$$

$$= q_1\int_0^l dx - \frac{q_1}{l}\int_0^l x\,dx + \frac{q_2}{l}\int_0^l x\,dx = q_1 x\Big|_0^l - \frac{q_1}{2l}x^2\Big|_0^l + \frac{q_2}{2l}x^2\Big|_0^l =$$

$$= q_1 l - \frac{q_1}{2\cancel{l}}l^{\cancel{2}} + \frac{q_2}{2\cancel{l}}l^{\cancel{2}} = \boxed{\frac{q_1+q_2}{2}l}$$

$$M_A = x_K R = \int_0^l x\,dF \Rightarrow x_K\int_0^l q(x)\,dx = \int_0^l x\,q(x)\,dx \Rightarrow$$

$$\Rightarrow \boxed{x_K = \frac{\int_0^l x\,q(x)\,dx}{\int_0^l q(x)\,dx}}$$ (ή αμέσως από θεωρία)

$$\int_0^l x\,q(x)\,dx = q_1\int_0^l x\,dx - \frac{q_1}{l}\int_0^l x^2\,dx + \frac{q_2}{l}\int_0^l x^2\,dx =$$

$$= \frac{q_1}{2}x^2\Big|_0^l - \frac{q_1}{3l}x^3\Big|_0^l + \frac{q_2}{3l}x^3\Big|_0^l = \frac{q_1 l^2}{2} - \frac{q_1 l^{\cancel{3}\,2}}{3\cancel{l}} + \frac{q_2 l^{\cancel{3}\,2}}{3\cancel{l}} = \frac{q_1+2q_2}{6}l^2$$

$$\boxed{x_K = \left(\frac{q_1+2q_2}{6}l^2\right)\Big/\left(\frac{q_1+q_2}{2}l\right) = \frac{q_1+2q_2}{q_1+q_2}\,\frac{l}{3}}$$