Functional Analysis I

1st week

Linear spaces, subspaces, basis and dimension of a linear space.

2nd week

Normed spaces, open and closed ball of a normed space. Open and closed sets of a normed space. Continuity of the norm and of addition/scalar multiplication. Convergent and Cauchy sequences. The definition of a Banach space. Examples of Banach spaces: $\mathbb{R}^n$, $C([a, b)]$, l^p for $1 \leq p < \infty$.

3rd/4th week

Finite dimensional normed spaces. Equivalent norms. We proved that all norms on a finite dimensional space are equivalent and then that every finite dimensional space is a Banach space. We then proved that if for a subspace Y of a normed space X we have that $Y^\circ \neq \emptyset$, then $Y = X$. We concluded with the very important Theorem that the dimension of a Banach space is either finite or uncountable. Bounded linear operators.

5th week

Main properties of bounded linear operators. Examples: the Voltera integral operator and the right shift. The norm of a bounded linear operator.