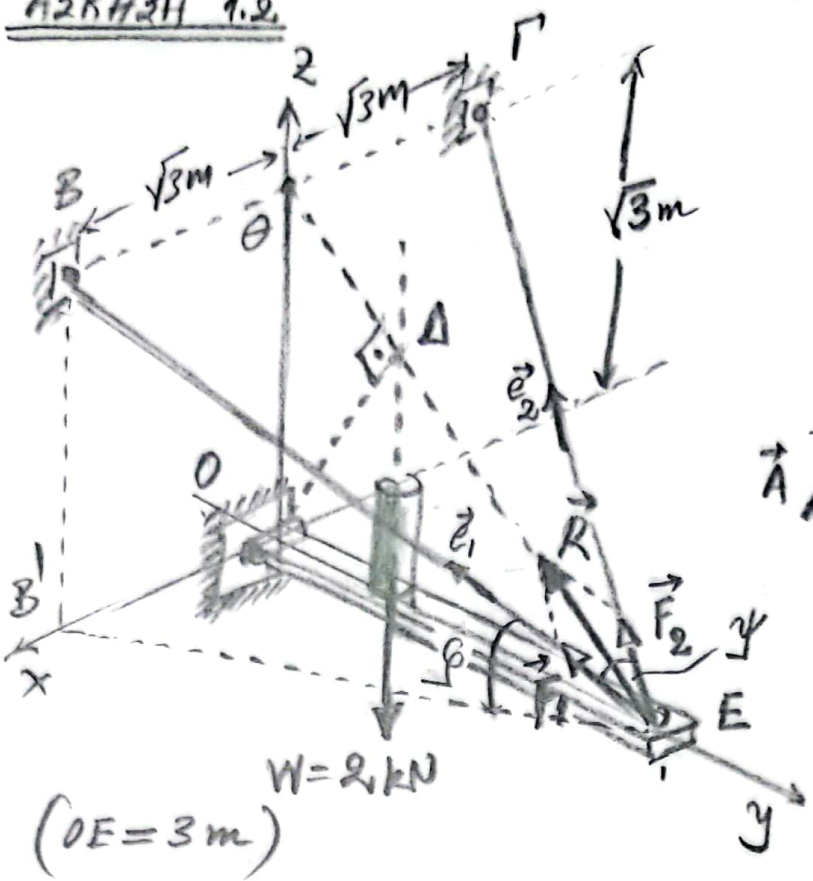
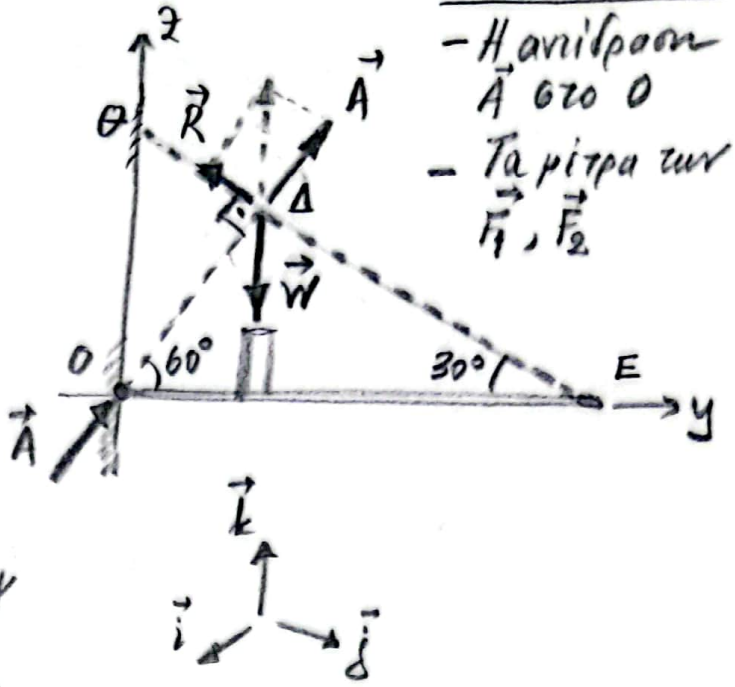
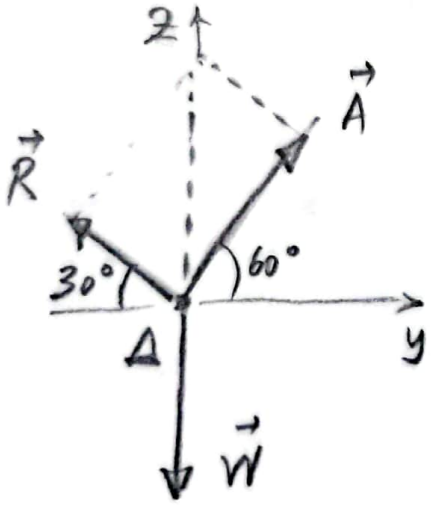
ΑΣΚΗΣΗ 1.2.



Σημειώσεις
- Η ανίδρασα $\vec{A}$ στο O
- Τα μέτρα των $\vec{F_1}$, $\vec{F_2}$



Ισορροπία – Στατική: Διάγραμμα ελεύθερου σώματος



$\Sigma F_z = 0 \Rightarrow A\sin 60° + R\sin 30° = 2$

$\frac{A\sqrt{3}}{2} + \frac{R}{2} = 2 \Rightarrow A\sqrt{3} + R = 4$ (1)

$\Sigma F_y = 0 \Rightarrow A\cos 60° - R\cos 30° = 0$

$\frac{A}{2} - \frac{R\sqrt{3}}{2} = 0 \Rightarrow A = R\sqrt{3}$ (2)

(1), (2) $\Rightarrow 3R + R = 4 \Rightarrow R = 1\,kN$, $\boxed{A = \sqrt{3}\,kN}$

Από συμμετρία, $F_1 = F_2$

Υπολογισμός γωνίας ψ από $\vec{e_1} \cdot \vec{e_2}$

Σαρώθηκε με το CamScanner

$B(\sqrt{3}, 0, \sqrt{3})$, $\Gamma(-\sqrt{3}, 0, \sqrt{3})$, $E(0, 3, 0)$

$\vec{EB} = (\sqrt{3}-0)\vec{i} + (0-3)\vec{j} + (\sqrt{3}-0)\vec{k} = \sqrt{3}\,\vec{i} - 3\vec{j} + \sqrt{3}\,\vec{k}$

$EB = \sqrt{(\sqrt{3})^2 + (-3)^2 + (\sqrt{3})^2} = \sqrt{15}$

$\vec{e}_1 = \frac{\vec{EB}}{EB} = \frac{1}{\sqrt{5}}\vec{i} - \frac{\sqrt{3}}{\sqrt{5}}\vec{j} + \frac{1}{\sqrt{5}}\vec{k} = 0.447\vec{i} - 0.774\vec{j} + 0.447\vec{k}$

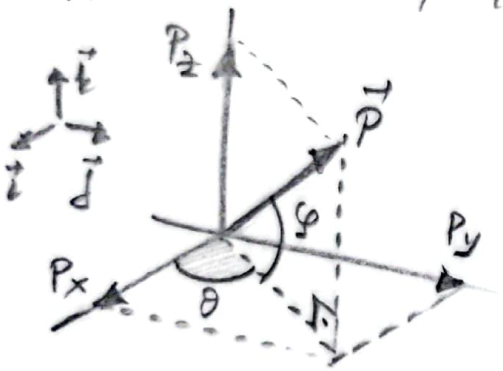
Συμμετρία: $\vec{e}_2 = -\frac{1}{\sqrt{5}}\vec{i} - \frac{\sqrt{3}}{\sqrt{5}}\vec{j} + \frac{1}{\sqrt{5}}\vec{k} = -0.447\vec{i} - 0.774\vec{j} + 0.447\vec{k}$

$\vec{e}_1 \cdot \vec{e}_2 = 1 \cdot 1 \cdot \cos\psi = \left(\frac{1}{\sqrt{5}}, -\frac{\sqrt{3}}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right) \cdot \left(-\frac{1}{\sqrt{5}}, -\frac{\sqrt{3}}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right) = -\frac{1}{5} + \frac{3}{5} + \frac{1}{5} \Rightarrow$

$\Rightarrow \boxed{\psi = \cos^{-1}\left(\frac{3}{5}\right) = 53.13^\circ}$

Εναλλακτικός υπολογισμός $\vec{e}_1, \vec{e}_2$

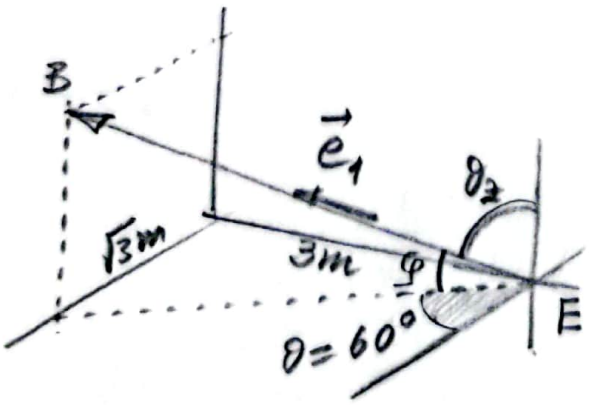
Αν δίνονταν ή μπορούν να υπολογισθούν οι γωνίες φ, θ



$P_x = P\cos\varphi\cos\theta, \quad P_y = P\cos\varphi\sin\theta, \quad P_z = P\sin\varphi$

$\vec{P} = P(\cos\varphi\cos\theta\,\vec{i} + \cos\varphi\sin\theta\,\vec{j} + \sin\varphi\,\vec{k})$

Εδώ είναι:



$\vec{EB} \cdot \vec{k} = \sqrt{15} \cdot 1 \cos\theta_2 = (\sqrt{3}, -3, \sqrt{3}) \cdot (0, 0, 1) = \sqrt{3}$

$\Rightarrow \cos\theta_2 = \frac{\sqrt{3}}{\sqrt{15}} \Rightarrow \theta_2 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right) = 63.44^\circ$

$\underline{\varphi = 90^\circ - 63.44^\circ = 26.56^\circ}$

Οπότε

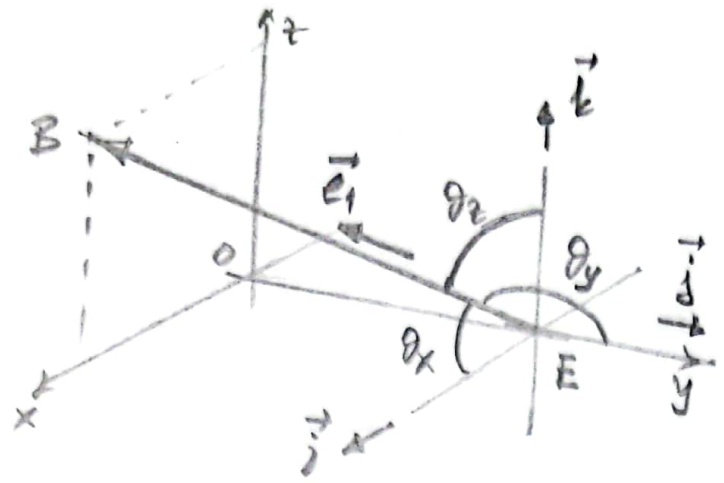
$\vec{e}_1 = 1(\cos 26.56^\circ \cos 60^\circ \vec{i} -$

$- \cos 26.56^\circ \sin 10^\circ \vec{j} + \sin 26.56^\circ \vec{k}) =$

$= 0.447\vec{i} - 0.774\vec{j} + 0.447\vec{k}$

Σαρώθηκε με το CamScanner

Άλλιως



$\vec{EB}\cdot\vec{i} = \sqrt{15}\cdot 1\cdot\cos\vartheta_x =$

$= (\sqrt{3}, -3, \sqrt{3})\cdot(1,0,0) = \sqrt{3} \Rightarrow$

$\Rightarrow \cos\vartheta_x \equiv l = \frac{\sqrt{3}}{\sqrt{15}} = \frac{1}{\sqrt{5}}$

$\vec{EB}\cdot\vec{j} = \sqrt{15}\cdot 1\cdot\cos\vartheta_y =$

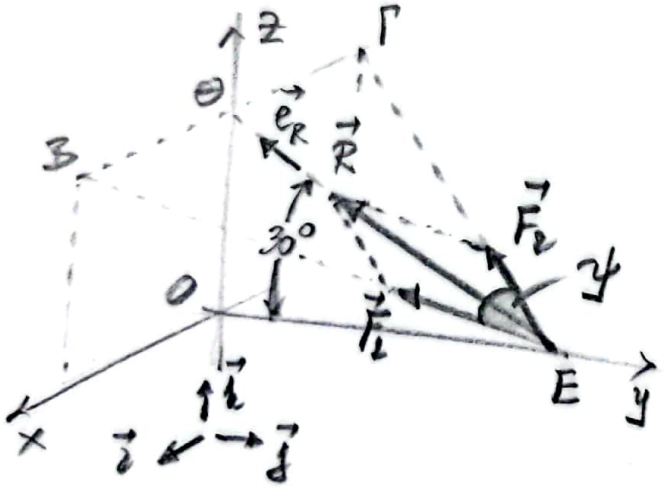
$= (\sqrt{3}, -3, \sqrt{3})\cdot(0,1,0) = -3 \Rightarrow$

$\Rightarrow \cos\vartheta_y \equiv m = -\frac{3}{\sqrt{15}} = -\frac{\sqrt{3}}{\sqrt{5}}$

$\vec{EB}\cdot\vec{k} = \sqrt{15}\cdot 1\cdot\cos\vartheta_z = (\sqrt{3}, -3, \sqrt{3})\cdot(0,0,1) = \sqrt{3} \Rightarrow \cos\vartheta_z \equiv n = \frac{1}{\sqrt{5}}$

$\vec{e_1} = l\vec{i} + m\vec{j} + n\vec{k} = \frac{1}{\sqrt{5}}\vec{i} - \frac{\sqrt{3}}{\sqrt{5}}\vec{j} + \frac{1}{\sqrt{5}}\vec{k}$

Υπολογισμοί F_1, F_2 ($F_1 = F_2$)



1) Διανυσματικά / αλγεβρικά

$\vec{e_R} = 0\vec{i} - \cos 30^\circ\vec{j} + \sin 30^\circ\vec{k} =$

$= -\frac{\sqrt{3}}{2}\vec{j} + \frac{1}{2}\vec{k}$

$\vec{F_1} + \vec{F_2} = \vec{R}$ $\quad F_1 = F_2 \Rightarrow$

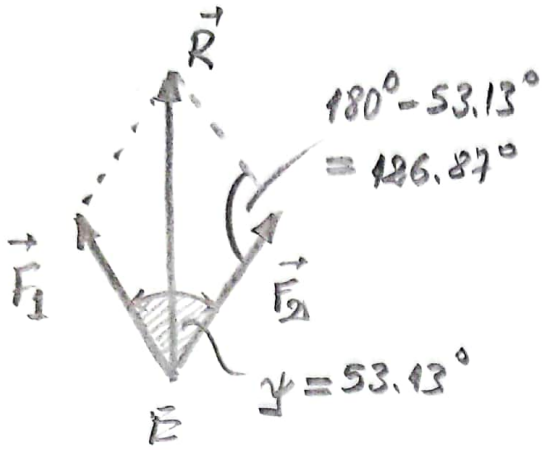
$F_1(\vec{e_1} + \vec{e_2}) = R\vec{e_R}$ $\quad R = 1\ kN \Rightarrow$

$\Rightarrow F_1\left(\frac{\vec{i}}{\sqrt{5}} - \frac{\sqrt{3}}{\sqrt{5}}\vec{j} + \frac{\vec{k}}{\sqrt{5}} - \frac{\vec{i}}{\sqrt{5}} - \frac{\sqrt{3}}{\sqrt{5}}\vec{j} + \frac{\vec{k}}{\sqrt{5}}\right) = -\frac{\sqrt{3}}{2}\vec{j} + \frac{\vec{k}}{2} \Rightarrow$

$\Rightarrow \begin{cases} (\vec{j}): & \frac{2\sqrt{3}}{\sqrt{5}}F_1 = \frac{\sqrt{3}}{2} \\ (\vec{k}): & \frac{2}{\sqrt{5}}F_1 = \frac{1}{2} \end{cases} \Rightarrow \boxed{F_1 = F_2 = \frac{\sqrt{5}}{4} = 0.56\ kN}$

Σαρώθηκε με το CamScanner

2) Θεώρημα



Νόμος συνημιτόνων

$$R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos 126.87^\circ \Rightarrow$$

$$\Rightarrow 1^2 = 2F_1^2 - 2F_1^2(-0.6) = 3.2F_1^2$$

$$\Rightarrow \boxed{F_1 = F_2 = \sqrt{\frac{1}{3.2}} = 0.56\ \text{kN}}$$

Σαρώθηκε με το CamScanner