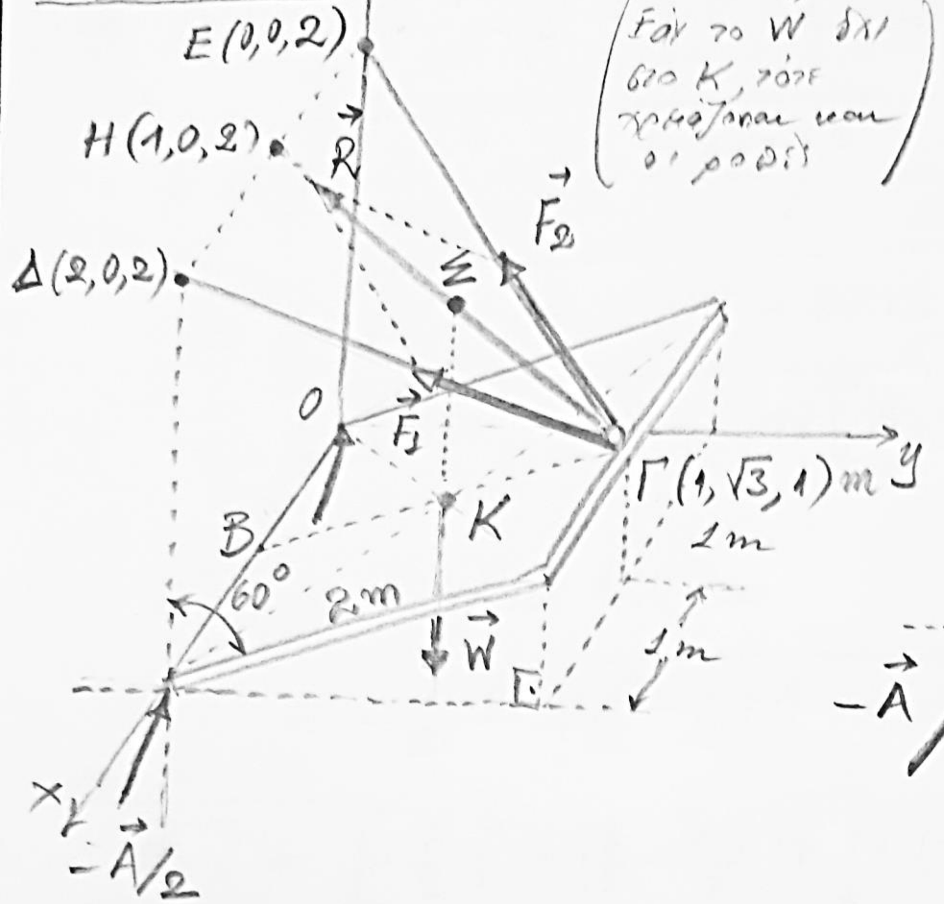
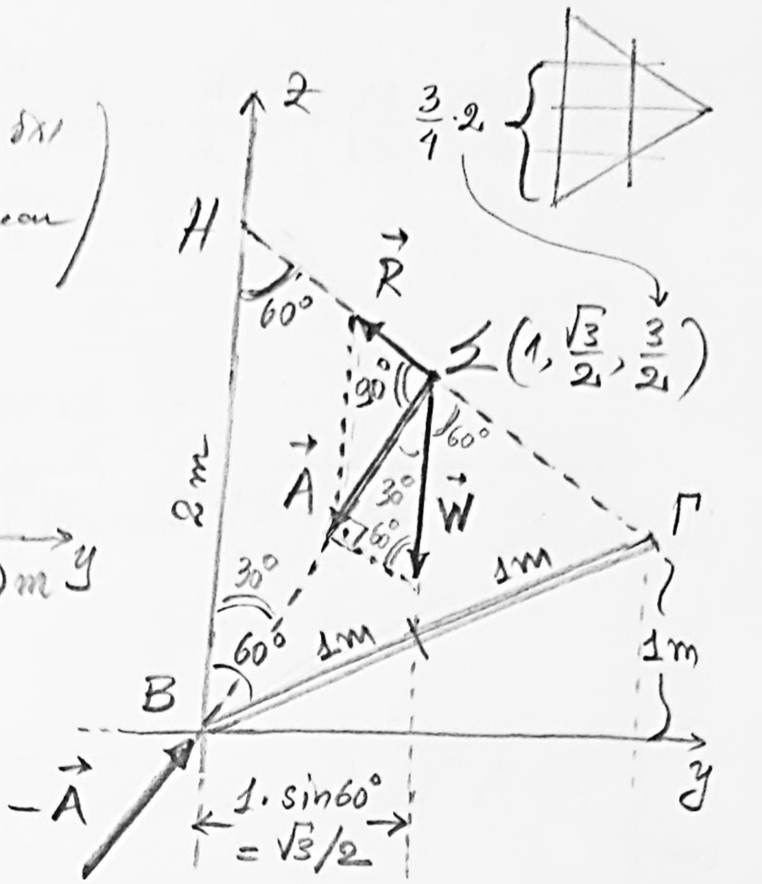
2η ΑΣΚΗΣΗ





Δίνεται $W = 2\ kN$

Ζητούνται (α) η $\vec{A}$, και (β) οι $\vec{F_1}$, $\vec{F_2}$ (λόγω συμμετρίας $F_1=F_2$)

(α) Η $\vec{A}$ πρέπει να διέρχεται από το Σ λόγω ισορροπίας,

άρα $\vec{A} = A\,\vec{e}_{\Sigma B}$

$$\vec{\Sigma B} = 0\vec{i} - \frac{\sqrt{3}}{2}\vec{j} - \frac{3}{2}\vec{k}\ ,\quad \Sigma B = \sqrt{\left(-\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{3}{2}\right)^2} = \sqrt{\frac{3}{4}+\frac{9}{4}} = \sqrt{3}$$

$$\vec{e}_{\Sigma B} = \frac{1}{\sqrt{3}}\left(-\frac{\sqrt{3}}{2}\vec{j} - \frac{3}{2}\vec{k}\right) = -\frac{\vec{j}}{2} - \frac{\sqrt{3}}{2}\vec{k}$$

$$\vec{A} = -\frac{A}{2}\left(\vec{j} + \sqrt{3}\vec{k}\right)$$

1ος Τρόπος υπολογισμού A (περίπλοκος, για εξάσκηση στις πράξεις)

$$\vec{A} = \vec{R} + \vec{W} \Rightarrow \vec{R} = \vec{A} - \vec{W} = -\frac{A}{2}\left(\vec{j} + \sqrt{3}\vec{k}\right) - \left(-2\vec{k}\right) \Rightarrow$$

$$\Rightarrow \vec{R} = -\frac{A}{2}\vec{j} + \left(2 - \frac{A\sqrt{3}}{2}\right)\vec{k} = -\frac{A}{2}\vec{j} + \left(\frac{4}{2} - \frac{A\sqrt{3}}{2}\right)\vec{k}$$

$$R = \sqrt{\left(-\frac{A}{2}\right)^2 + \left(\frac{4}{2} - \frac{A\sqrt{3}}{2}\right)^2} =$$

$$= \frac{1}{2}\sqrt{A^2 + 16 + 3A^2 - 8A\sqrt{3}} =$$

$$= \frac{1}{2}\sqrt{4A^2 + 4\cdot 4 - 2\cdot 4\sqrt{3}A} = \sqrt{A^2 - 2\sqrt{3}A + 4}$$

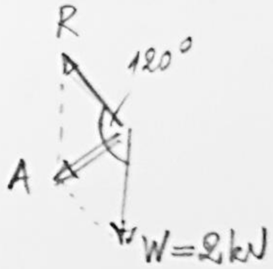
$$\vec{R}\cdot\vec{W} = RW\cos 120^\circ =$$

$$= -\frac{1}{2}\,2\sqrt{A^2 - 2\sqrt{3}A + 4} = -\sqrt{A^2 - 2\sqrt{3}A + 4}$$

και επίσης:

$$\vec{R}\cdot\vec{W} = \left[-\frac{A}{2}\vec{j} + \left(\frac{4}{2} - \frac{A\sqrt{3}}{2}\right)\vec{k}\right]\cdot\left(-2\vec{k}\right) = A\sqrt{3} - 4$$

$$\Rightarrow \sqrt{A^2 - 2\sqrt{3}A + 4} = 4 - A\sqrt{3} \xrightarrow{(\;)^2}$$



$$\Rightarrow A^2 - 2\sqrt{3}A + 4 = 16 + 3A^2 - 8\sqrt{3}A \Rightarrow$$

$$\Rightarrow 2A^2 - 6\sqrt{3}A + 12 = 0 \Rightarrow A^2 - 3\sqrt{3}A + 6 = 0$$

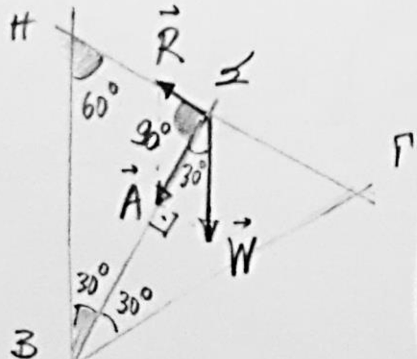
$$\boxed{A_\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3\sqrt{3} \pm \sqrt{3}}{2}$$

(+) $2\sqrt{3} = 3.46$ kN, απορρ.

(−) $\underline{\sqrt{3} = 1.73 \text{ kN}}$ δεκτή

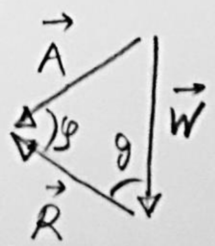
Άρα $\boxed{\vec{A} = -\frac{\sqrt{3}}{2}\left(\vec{j} + \sqrt{3}\vec{k}\right)}$

2ος Τρόπος υπολογισμού A



$$A = W\cos 30^\circ = 2\,\frac{\sqrt{3}}{2} = \underline{\underline{\sqrt{3}\text{ kN}}}$$

(Αν τυχαίο διαμόρφωμα, υπολογισμός από γεωμετρία φ και θ, και νόμος ημιτόνου:



$$\frac{A}{\sin\theta} = \frac{W}{\sin\varphi} \Rightarrow A = \frac{\sin\theta}{\sin\varphi}W$$)

1ος τρόπος

(β) $$\boxed{\vec{R} = \vec{A} - \vec{W} = -\frac{\sqrt{3}}{2}\left(\vec{j} + \sqrt{3}\,\vec{k}\right) - \left(-2\vec{k}\right) = -\frac{\sqrt{3}}{2}\vec{j} + \frac{1}{2}\vec{k} \ \text{kN}}$$

$$R = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1 \ \text{kN}$$

$$\vec{R} = \vec{F}_1 + \vec{F}_2$$

$$\vec{F}_1 = F_1 \vec{e}_{\Gamma\Delta} \ , \quad \vec{e}_{\Gamma\Delta} = \frac{\vec{\Gamma\Delta}}{\Gamma\Delta}$$

$$\vec{\Gamma\Delta} = (2-1)\vec{i} + (0-\sqrt{3})\vec{j} + (2-1)\vec{k} = \vec{i} - \sqrt{3}\vec{j} + \vec{k} \ \text{m}$$

$$\Gamma\Delta = \sqrt{1^2 + (-\sqrt{3})^2 + 1^2} = \sqrt{5} \ \text{m}, \quad \vec{e}_{\Gamma\Delta} = \frac{1}{\sqrt{5}}\left(\vec{i} - \sqrt{3}\vec{j} + \vec{k}\right)$$ Αδιάστατο διάνυσμα

$$\vec{F}_1 = \frac{F_1}{\sqrt{5}}\left(\vec{i} - \sqrt{3}\vec{j} + \vec{k}\right) \ \text{kN}$$

$$\vec{F}_2 = F_2 \vec{e}_{\Gamma E} \ , \quad \vec{e}_{\Gamma E} = \frac{\vec{\Gamma E}}{\Gamma E}$$

$$\vec{\Gamma E} = (0-1)\vec{i} + (0-\sqrt{3})\vec{j} + (2-1)\vec{k} = -\vec{i} - \sqrt{3}\vec{j} + \vec{k} \ \text{m}, \quad \Gamma E = \sqrt{5} \ \text{m}$$

$$\vec{e}_{\Gamma E} = \frac{1}{\sqrt{5}}\left(-\vec{i} - \sqrt{3}\vec{j} + \vec{k}\right), \quad \vec{F}_2 = \frac{F_2}{\sqrt{5}}\left(-\vec{i} - \sqrt{3}\vec{j} + \vec{k}\right) \ \text{kN}$$

$$\underbrace{-\frac{\sqrt{3}}{2}\vec{j} + \frac{1}{2}\vec{k}}_{\vec{R}} = \underbrace{\frac{F_1}{\sqrt{5}}\left(\vec{i} - \sqrt{3}\vec{j} + \vec{k}\right)}_{\vec{F}_1} + \underbrace{\frac{F_2}{\sqrt{5}}\left(-\vec{i} - \sqrt{3}\vec{j} + \vec{k}\right)}_{\vec{F}_2} \Rightarrow$$

$$(\vec{i}): \ \frac{F_1}{\sqrt{5}} - \frac{F_2}{\sqrt{5}} = 0 \Rightarrow F_1 = F_2$$

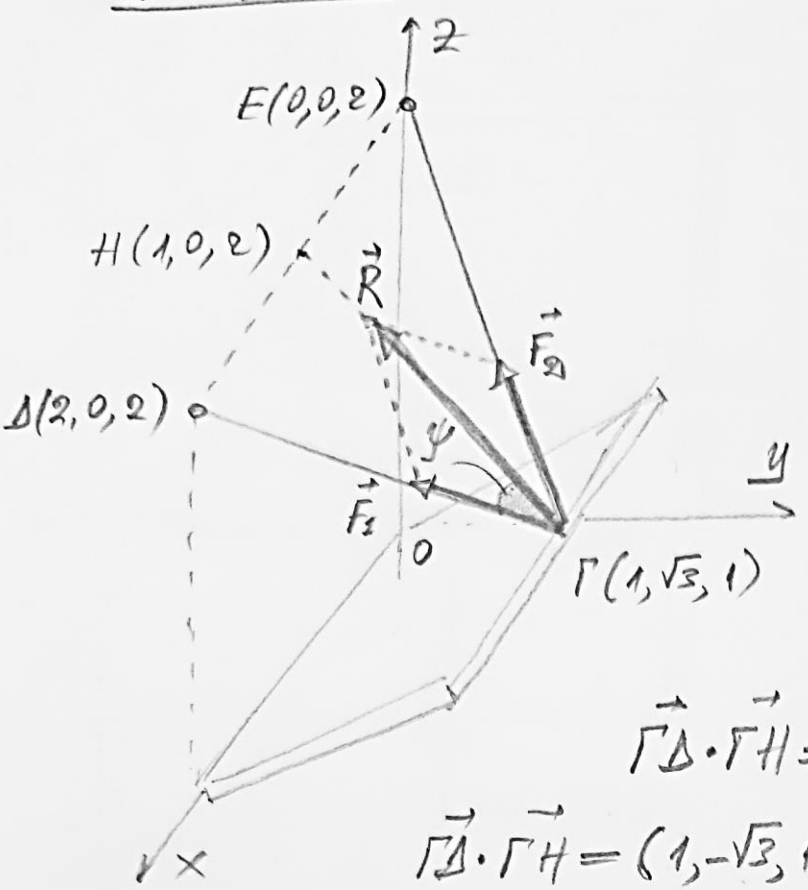
$$(\vec{j}): \ \cancel{-} F_1\sqrt{\frac{\cancel{3}}{5}} \cancel{-} F_2\sqrt{\frac{\cancel{3}}{5}} = \cancel{-}\frac{\cancel{\sqrt{3}}}{2} \Rightarrow \frac{2F_1}{\sqrt{5}} = \frac{1}{2} \Rightarrow \boxed{F_1 = F_2 = \frac{\sqrt{5}}{4} \ \text{kN}}$$

(όπως αναμενόταν λόγω συμμετρίας)

$$\frac{\sqrt{5}}{4} \simeq \underline{0.56 \ \text{kN}}$$

$$(\vec{k}): \ \text{ή} \ \frac{F_1}{\sqrt{5}} + \frac{F_2}{\sqrt{5}} = \frac{1}{2} \Rightarrow \frac{2F_1}{\sqrt{5}} = \frac{1}{2} \Rightarrow \text{το ίδιο}$$

2ος τρόπος



Εκ της συμμετρίας, $F_1 \equiv F_2$

Υπολογισμός γωνίας ψ.

$$\vec{\Gamma\Delta} = \vec{i} - \sqrt{3}\vec{j} + \vec{k} \ \text{m}$$

$$\Gamma\Delta = \sqrt{5} \ \text{m}$$

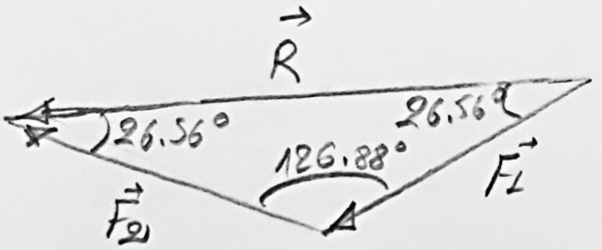
$$\vec{\Gamma H} = (1-1)\vec{i} + (0-\sqrt{3})\vec{j} + (2-1)\vec{k} = -\sqrt{3}\vec{j} + \vec{k} \ \text{m}$$

$$\Gamma H = \sqrt{3+1} = 2 \ \text{m}$$

$$\vec{\Gamma\Delta} \cdot \vec{\Gamma H} = \Gamma\Delta \cdot \Gamma H \cos\psi = 2\sqrt{5}\cos\psi$$

$$\vec{\Gamma\Delta} \cdot \vec{\Gamma H} = (1, -\sqrt{3}, 1) \cdot (0, -\sqrt{3}, 1) = 3 + 1 = 4$$

$$\Rightarrow \cos\psi = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}} \Rightarrow \underline{\psi = 26.56^\circ}$$



Νόμος Ημιτόνων:

$$\frac{F_1}{\sin 26.56^\circ} = \frac{R}{\sin 126.88^\circ} \Rightarrow$$

$$\Rightarrow \boxed{F_1 = F_2 = \frac{\sin 26.56^\circ}{\sin 126.88^\circ} \cdot 1\,\text{kN} = 0.56\ \text{kN}}$$