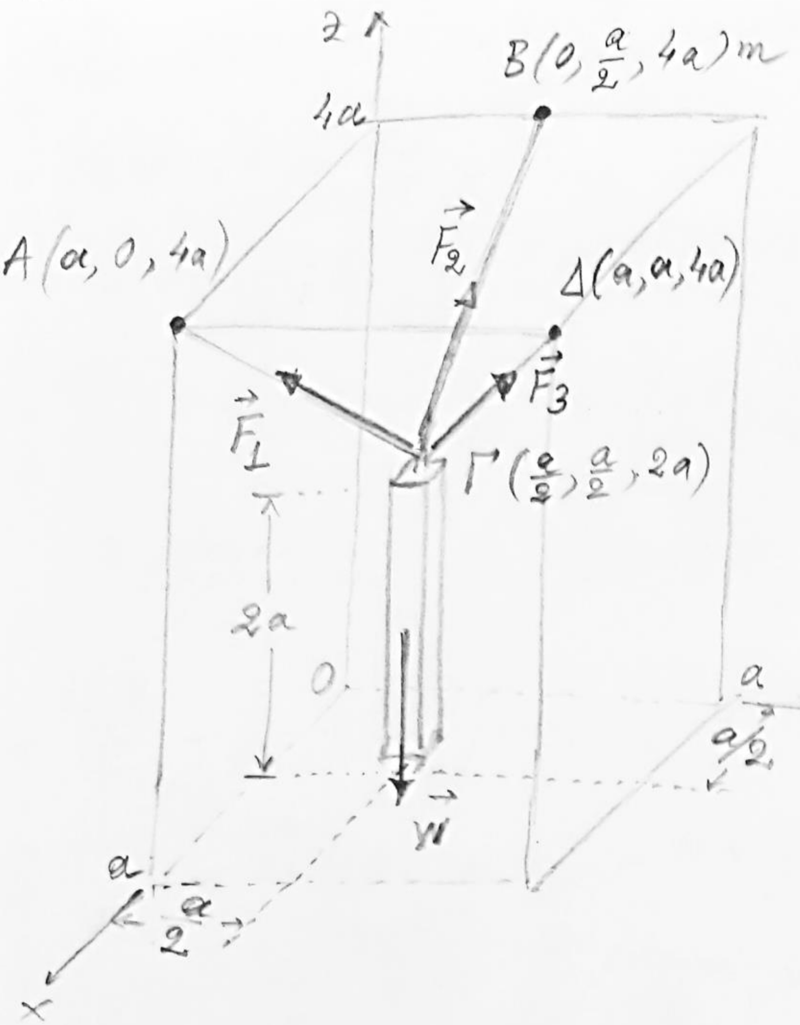
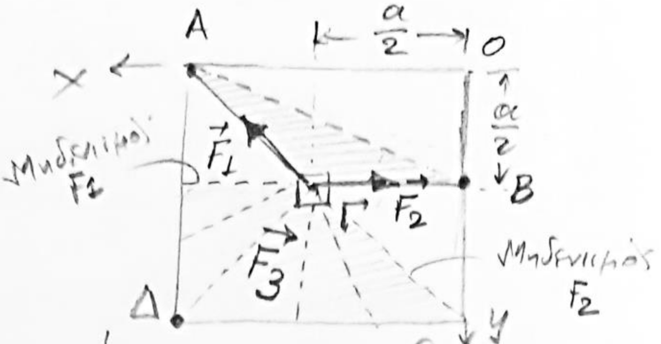
1η ΑΣΚΗΣΗ

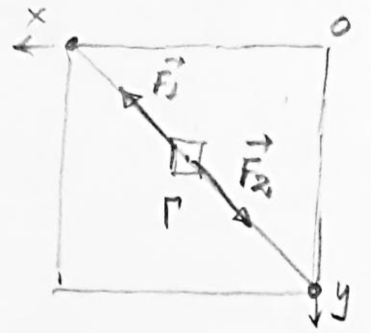
Δίνονται: Γεωμετρία και W

Ζητούνται: F_1, F_2, F_3





όχι ισορροπία διότι η συνισταμένη τους όχι σε κατακόρυφο επίπεδο όπου βρίσκεται το βάρος $\vec{W}$



μία περίπτωση ισορροπίας

$$\vec{\Gamma A} = \left(a-\frac{a}{2}\right)\vec{i} + \left(0-\frac{a}{2}\right)\vec{j} + (4a-2a)\vec{k} = \frac{a}{2}\vec{i} - \frac{a}{2}\vec{j} + 2a\vec{k}$$

$$\Gamma A = \sqrt{\left(\frac{a}{2}\right)^2 + \left(-\frac{a}{2}\right)^2 + (2a)^2} = \sqrt{\frac{a^2}{4} + \frac{a^2}{4} + 4a^2} = \frac{3a}{\sqrt{2}}$$

$$\vec{e}_{\Gamma A} = \frac{\vec{\Gamma A}}{\Gamma A} = \frac{\sqrt{2}}{3\cancel{a}}\left(\frac{\cancel{a}}{2}\vec{i} - \frac{\cancel{a}}{2}\vec{j} + 2\cancel{a}\vec{k}\right) = \frac{\sqrt{2}}{6}\vec{i} - \frac{\sqrt{2}}{6}\vec{j} + \frac{\overset{4}{\cancel{2}}\sqrt{2}}{\cancel{3}6}\vec{k}$$

$$\vec{F}_1 = F_1\vec{e}_{\Gamma A} = \frac{F_1\sqrt{2}}{6}\left(\vec{i} - \vec{j} + 4\vec{k}\right)$$

$$\vec{\Gamma B} = \left(0-\frac{a}{2}\right)\vec{i} + \left(\frac{a}{2}-\frac{a}{2}\right)\vec{j} + (4a-2a)\vec{k} = -\frac{a}{2}\vec{i} + 2a\vec{k}$$

$$\Gamma B = \sqrt{\left(-\frac{a}{2}\right)^2 + (2a)^2} = \sqrt{\frac{a^2}{4} + 4a^2} = \frac{a\sqrt{17}}{2}$$

$$\vec{e}_{\Gamma B} = \frac{\vec{\Gamma B}}{\Gamma B} = \frac{2}{\cancel{a}\sqrt{17}}\left(-\frac{\cancel{a}}{2}\vec{i} + 2\cancel{a}\vec{k}\right) = \frac{\cancel{2}}{\sqrt{17}}\left(-\frac{\vec{i}}{\cancel{2}} + \frac{\overset{4}{\cancel{2}}\vec{k}}{\cancel{2}}\right) = \frac{1}{\sqrt{17}}\left(-\vec{i} + 4\vec{k}\right)$$

$$\vec{F}_2 = F_2\vec{e}_{\Gamma B} = \frac{F_2}{\sqrt{17}}\left(-\vec{i} + 4\vec{k}\right)$$

$\Delta(a, a, 4a)$

$\vec{\Gamma\Delta} = (a - \frac{a}{2})\vec{i} + (a - \frac{a}{2})\vec{j} + (4a - 2a)\vec{k} = \frac{a}{2}\vec{i} + \frac{a}{2}\vec{j} + 2a\vec{k}$

$\Gamma\Delta \equiv \Gamma A = \frac{3a}{\sqrt{2}}$

$\vec{e}_{\Gamma\Delta} = \frac{\vec{\Gamma\Delta}}{\Gamma\Delta} = \frac{\sqrt{2}}{3\cancel{a}}\left(\frac{\cancel{a}}{2}\vec{i} + \frac{\cancel{a}}{2}\vec{j} + 2\cancel{a}\vec{k}\right) = \frac{\sqrt{2}}{6}(\vec{i} + \vec{j} + 4\vec{k})$

$\vec{F}_3 = F_3 \vec{e}_{\Gamma\Delta} = \frac{F_3\sqrt{2}}{6}(\vec{i} + \vec{j} + 4\vec{k})$

<u>Ισορροπία:</u> $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{W} = \vec{0} \Rightarrow$

$\Rightarrow \left(\frac{F_1\sqrt{2}}{6} - \frac{F_2}{\sqrt{17}} + \frac{F_3\sqrt{2}}{6}\right)\vec{i} + \left(-\frac{F_1\sqrt{2}}{6} + 0F_2 + \frac{F_3\sqrt{2}}{6}\right)\vec{j} + \left(\frac{2F_1\sqrt{2}}{3} + \frac{4F_2}{\sqrt{17}} + \frac{2F_3\sqrt{2}}{3}\right)\vec{k} =$

$= -\left(0\vec{i} + 0\vec{j} - W\vec{k}\right) \Rightarrow$ (1 διανυσματική εξίσωση στον χώρο $\Rightarrow$ 3 αλγεβρικές εξισώσεις)

$$\Rightarrow \begin{cases} \frac{F_1\sqrt{2}}{6} - \frac{F_2}{\sqrt{17}} + \frac{F_3\sqrt{2}}{6} = 0 & (1) \\ \frac{F_1\sqrt{2}}{6} + 0F_2 - \frac{F_3\sqrt{2}}{6} = 0 \Rightarrow F_1 = F_3 & (2) \quad \text{λογικό λόγω γεωμετρίας} \\ \frac{2F_1\sqrt{2}}{3} + \frac{4F_2}{\sqrt{17}} + \frac{2F_3\sqrt{2}}{3} = W & (3) \end{cases}$$

$(1),(2) \Rightarrow \frac{F_1\sqrt{2}}{3} - \frac{F_2}{\sqrt{17}} = 0 \quad (4)$

$(3),(2) \Rightarrow \frac{\cancel{4}F_1\sqrt{2}}{3} + \frac{\cancel{4}F_2}{\sqrt{17}} = W/4$

$\xrightarrow{(+)} \frac{2F_1\sqrt{2}}{3} = \frac{W}{4} \Rightarrow$

$$\boxed{F_1 = F_3 = \frac{3}{8\sqrt{2}}W \cong 0.27\,W} \quad (5)$$

$(4) \Rightarrow F_2 = \frac{\sqrt{2}\sqrt{17}}{3}F_1 \overset{(5)}{=} \frac{\sqrt{2}\sqrt{17}}{3} \cdot \frac{3}{8\sqrt{2}}W \Rightarrow \boxed{F_2 = W\sqrt{17} = 4.12\,W} \quad (6)$

$$\underline{\underline{F_2 \cong 15.3\,F_1, F_3}}$$