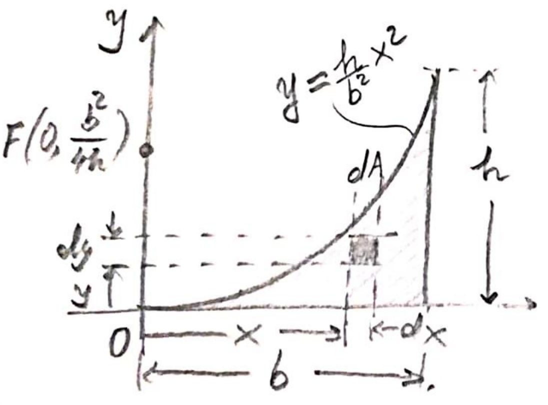
3) Παραβολικός Τομέας

(Παραβολή: $y=\dfrac{x^2}{2p}$, με $F(0,\frac{p}{2})$, άρα εδώ

$\dfrac{h}{b^2}=\dfrac{1}{2p} \Rightarrow \dfrac{p}{2}=\dfrac{b^2}{4h}$)



$$I_{xx}=\int_A y^2\,dA = \int_{x=0}^{x=b}\left(\int_{y=0}^{y=\frac{h}{b^2}x^2} y^2\,dy\right)dx =$$

$(dA=dx\,dy)$

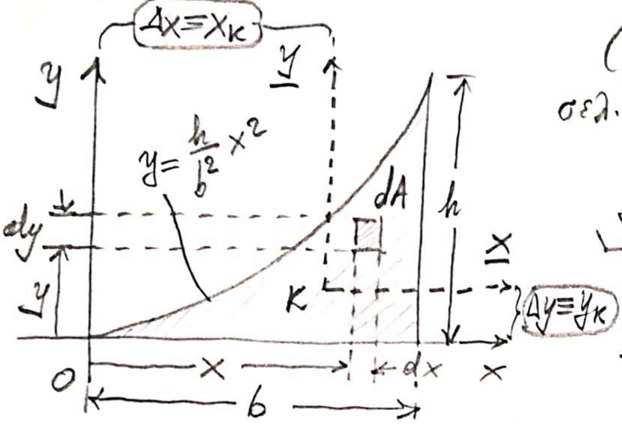
$$=\int_{x=0}^{x=b}\left(\frac{y^3}{3}\right)_{y=0}^{y=\frac{h}{b^2}x^2}dx = \frac{1}{3}\int_{x=0}^{x=b}\frac{h^3}{b^6}x^6\,dx=\frac{h^3}{3b^6}\frac{x^7}{7}\Big|_0^b=$$

$$=\frac{h^3b^7}{21b^6} \Rightarrow \boxed{I_{xx}=\frac{bh^3}{21}} \quad (27.\alpha)$$

$$I_{yy}=\int_A x^2\,dA = \int_{x=0}^{x=b}x^2\left(\int_{y=0}^{y=\frac{h}{b^2}x^2}dy\right)dx=\int_{x=0}^{x=b}\frac{h}{b^2}x^4\,dx=\frac{h}{b^2}\frac{x^5}{5}\Big|_0^b \Rightarrow \boxed{I_{yy}=\frac{hb^3}{5}} \quad (27.\beta)$$

$$I_{xy}=\int_A xy\,dA=\int_{x=0}^{x=b}x\left(\int_{y=0}^{y=\frac{h}{b^2}x^2}y\,dy\right)dx=\int_{x=0}^{x=b}x\left(\frac{y^2}{2}\right)_{y=0}^{y=\frac{h}{b^2}x^2}dx=\frac{1}{2}\int_{x=0}^{x=b}\frac{h^2}{b^4}x^5\,dx=\frac{h^2}{2b^4}\frac{x^6}{6}\Big|_0^b=\boxed{I_{xy}=\frac{h^2b^2}{12}} \quad (27.\gamma)$$

Για τον υπολογισμό του I_{ij} ως προς σύστημα στο κεντροειδές Κ, χρήση του Θεωρήματος Παραλλήλων Αξόνων. Πρώτα υπολογισμός $K(x_K, y_K)$:



(8): $x_K = \dfrac{\int_A x\,dA}{\int_A dA}$, $y_K = \dfrac{\int_A y\,dA}{\int_A dA}$ $(dA = dx\,dy)$

σελ. 8

$$\int_A dA = \int_{x=0}^{x=b}\left(\int_{y=0}^{y=\frac{h}{b^2}x^2} dy\right)dx = \int_{x=0}^{x=b}\frac{h}{b^2}x^2\,dx = \frac{h}{b^2}\frac{x^3}{3}\Big|_0^b = \frac{h}{b^2}\frac{b^3}{3} = \frac{hb}{3} = A$$

$$\int_A x\,dA = \int_{x=0}^{x=b} x\left(\int_{y=0}^{y=\frac{h}{b^2}x^2} dy\right)dx = \int_{x=0}^{x=b}\frac{h}{b^2}x^3\,dx = \frac{h}{b^2}\frac{x^4}{4}\Big|_0^b = \frac{hb^2}{4}$$

$$\int_A y\,dA = \int_{x=0}^{x=b}\left(\int_{y=0}^{y=\frac{h}{b^2}x^2} y\,dy\right)dx = \int_{x=0}^{x=b}\left(\frac{y^2}{2}\right)\Big|_{y=0}^{y=\frac{h}{b^2}x^2} dx = \frac{1}{2}\int_{x=0}^{x=b}\frac{h^2}{b^4}x^4\,dx = \frac{h^2}{2b^4}\frac{x^5}{5}\Big|_0^b = \frac{h^2 b}{10}$$

$$x_K = \frac{hb^2}{4}\frac{3}{hb} \Rightarrow \boxed{x_K = \frac{3b}{4}}\ , \quad y_K = \frac{h^2 b}{10}\frac{3}{hb} \Rightarrow \boxed{y_K = \frac{3h}{10}}$$

$$(24.\alpha):\ I_{\underline{xx}} = I_{xx} - \Delta y^2 A \overset{(27.\alpha)}{=} \frac{bh^3}{21} - \left(\frac{3h}{10}\right)^2\frac{hb}{3} = \frac{bh^3}{21} - \frac{\cancel{3}\cdot 3\cdot h^3 b}{\cancel{3}\cdot 100} =$$

$$= bh^3\,\frac{100 - 3\cdot 21}{21\cdot 100} \Rightarrow \boxed{I_{\underline{xx}} = \frac{37}{2100}bh^3} \quad (28.\alpha)$$

$$(24.\beta):\ I_{\underline{yy}} = I_{yy} - \Delta x^2 A \overset{(27.\beta)}{=} \frac{hb^3}{5} - \left(\frac{3b}{4}\right)^2\frac{hb}{3} = \frac{hb^3}{5} - \frac{\cancel{3}\cdot 3\cdot hb^3}{\cancel{3}\cdot 16} =$$

$$= hb^3\,\frac{16 - 15}{5\cdot 16} \Rightarrow \boxed{I_{\underline{yy}} = \frac{hb^3}{80}} \quad (28.\beta)$$

$$(24.\gamma):\ I_{\underline{xy}} = I_{xy} - \Delta x\,\Delta y\,A \overset{(27.\gamma)}{=} \frac{h^2 b^2}{12} - \left(+\frac{3b}{4}\right)\left(+\frac{3h}{10}\right)\frac{hb}{3} = \frac{h^2 b^2}{12} - \frac{\cancel{3}\cdot 3\,h^2 b^2}{\cancel{3}\cdot 40} =$$

$$= h^2 b^2\,\frac{40 - 36}{12\cdot 40} \Rightarrow \boxed{I_{\underline{xy}} = \frac{h^2 b^2}{120} > 0} \quad (28.\gamma)$$