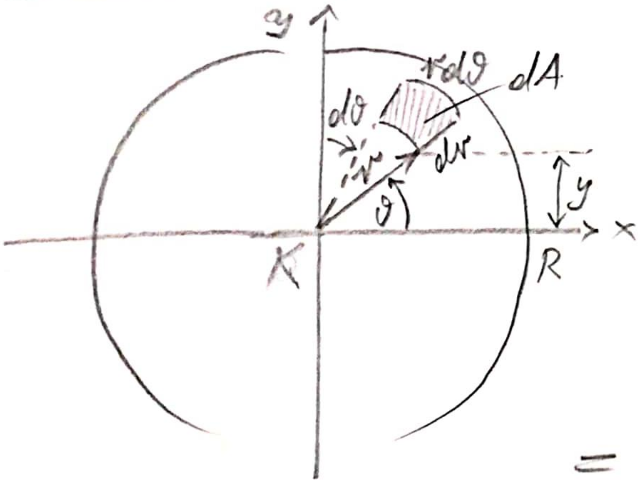
1) Κυκλική επιφάνεια



$$I_{xx} = \int_A y^2\,dA =$$

$$= \iint (r\sin\theta)^2\, r\,d\theta\,dr =$$

$$= \int_0^R r^3\,dr \int_0^{2\pi} \sin^2\vartheta\,d\vartheta = \quad (*)$$

$$= \frac{r^4}{4}\Big|_0^R \int_0^{2\pi} \frac{1-\cos 2\vartheta}{2}\,d\vartheta =$$

$$= \frac{R^4}{8}\left(\int_0^{2\pi} d\vartheta - \int_0^{2\pi}\cos 2\vartheta\,d\vartheta\right) = \frac{R^4}{8}\left(2\pi - \cancel{\frac{\sin 2\vartheta}{2}\Big|_0^{2\pi}}^{\,0}\right) \Rightarrow$$

$$\Rightarrow \boxed{I_{xx} = \frac{\pi R^4}{4} \underset{\text{συμμ.}}{=} I_{yy}} \quad (25)$$

$\sin^2\vartheta = \sin^2\vartheta - \cos^2\vartheta + \cos^2\vartheta = -\cos 2\vartheta + \cos^2\vartheta = -\cos 2\vartheta + 1 - \sin^2\vartheta \Rightarrow$

$\Rightarrow 2\sin^2\vartheta = 1 - \cos 2\vartheta \Rightarrow \boxed{\sin^2\vartheta = \frac{1-\cos 2\vartheta}{2}} \quad (*)$

$$\left\{\begin{array}{l} \underline{\text{Εάν τεταρτοκύκλιο:}} \\ \underline{I_{xx}} = \dfrac{R^4}{8}\left(\displaystyle\int_0^{\frac{\pi}{2}} d\vartheta - \int_0^{\frac{\pi}{2}} \cos 2\vartheta\,d\vartheta\right) = \\ \qquad = \dfrac{R^4}{8}\left(\dfrac{\pi}{2} - \cancel{\dfrac{\sin 2\vartheta}{2}\Big|_0^{\frac{\pi}{2}}}^{\,0}\right) = \underline{\dfrac{R^4\pi}{16}} \end{array}\right\}$$