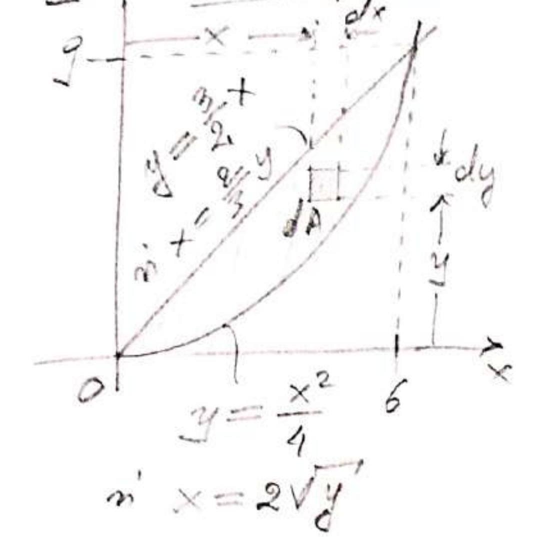
4) Κεντροειδές Επιφάνειας μεταξύ διδομένων καμπυλών

1ος Τρόπος)



$$dA = dx\,dy$$

$$\int_A dA = \int_{x=0}^{x=6}\left(\int_{y=\frac{x^2}{4}}^{y=\frac{3}{2}x} dy\right)dx =$$

$$= \int_0^6\left(\frac{3}{2}x-\frac{x^2}{4}\right)dx = \frac{3}{2}\frac{x^2}{2}\Big|_0^6 - \frac{1}{4}\frac{x^3}{3}\Big|_0^6 =$$

$$= \frac{3}{4}36 - \frac{1}{12}216 = 27-18 = \underline{9} \quad (4.1)$$

Ή, αλλάζοντας τη σειρά ολοκλήρωσης:

$$\int_A dA = \int_{y=0}^{y=9}\left(\int_{x=\frac{2}{3}y}^{x=2\sqrt{y}} dx\right)dy = \int_0^9\left(2\sqrt{y}-\frac{2}{3}y\right)dy =$$

$$= 2\,\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\Big|_0^9 - \frac{2}{3}\frac{y^2}{2}\Big|_0^9 = \frac{4}{3}27 - \frac{\overset{81}{3\cdot 27}}{3} = \frac{27}{3} = \underline{9} = (4.1)$$

Και

$$\int_A x\,dA = \int_{x=0}^{x=6} x\left(\int_{y=\frac{x^2}{4}}^{y=\frac{3}{2}x} dy\right)dx = \int_0^6 x\left(\frac{3}{2}x-\frac{x^2}{4}\right)dx =$$

$$= \frac{3}{2}\frac{x^3}{3}\Big|_0^6 - \frac{1}{4}\frac{x^4}{4}\Big|_0^6 = \frac{1}{2}216 - \frac{1}{16}1.296 = 108-81 = \underline{27} \quad (4.2)$$

Ή, αλλάζοντας τη σειρά ολοκλήρωσης:

$$\int_A x\,dA = \int_{y=0}^{y=9}\left(\int_{x=\frac{2}{3}y}^{x=2\sqrt{y}} x\,dx\right)dy = \int_{y=0}^{y=9}\left(\frac{x^2}{2}\Big|_{x=\frac{2}{3}y}^{x=2\sqrt{y}}\right)dy =$$

$$= \frac{1}{\cancel{2}}\int_{y=0}^{y=9}\left(\overset{2}{\cancel{4}}y - \frac{\overset{2}{\cancel{4}}}{9}y^2\right)dy = 2\,\frac{y^2}{2}\Big|_0^9 - \frac{2}{9}\frac{y^3}{3}\Big|_0^9 = 81 - \frac{2\cdot\cancel{9}\cdot\overset{3}{\cancel{9}}\cdot 9}{\cancel{9}\cdot\cancel{3}} = \underline{27} = (4.2)$$

Οπότε

$$\boxed{x_K = \frac{\int_A x\,dA}{\int_A dA} \underset{(4.1)}{\overset{(4.2)}{=}} \frac{27}{9} = 3} \quad (4.3)$$

Και

$$\underline{\int_A y\,dA} = \int_{y=0}^{y=9} y\left(\int_{x=\frac{2}{3}y}^{x=2\sqrt{y}} dx\right)dy = \int_0^9 y\left(2\sqrt{y}-\frac{2}{3}y\right)dy =$$

$$=2\int_0^9\left(y^{\frac{3}{2}}-\frac{y^2}{3}\right)dy = 2\left(\frac{y^{\frac{5}{2}}}{\frac{5}{2}}\Bigg|_0^9 - \frac{1}{3}\,\frac{y^3}{3}\Bigg|_0^9\right) = \frac{4}{5}243-\frac{2}{9}729=$$

$$= 194.4-162 = \underline{32.4} \quad (4.4)$$

Ή, αλλάζοντας τη σειρά ολοκλήρωσης :

$$\underline{\int_A y\,dA} = \int_{x=0}^{x=6}\left(\int_{y=\frac{x^2}{4}}^{y=\frac{3}{2}x} y\,dy\right)dx = \int_0^6\left(\frac{y^2}{2}\right)\Bigg|_{\frac{x^2}{4}}^{\frac{3}{2}x} dx =$$

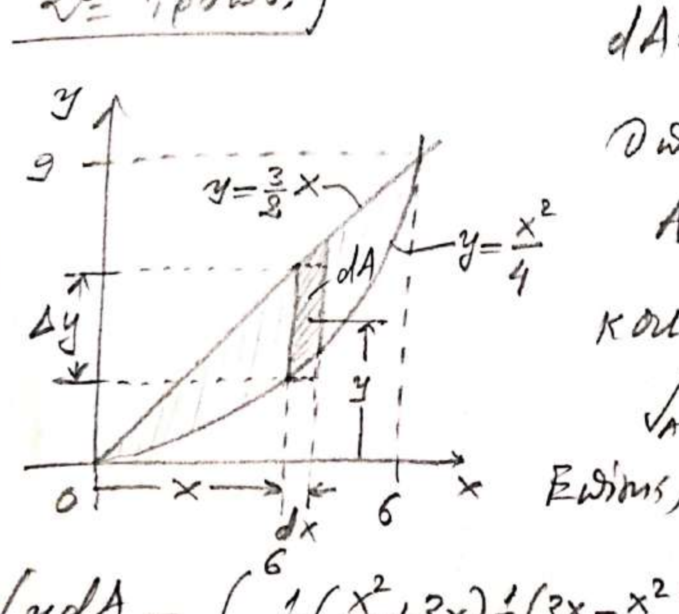
$$= \frac{1}{2}\int_0^6\left(\frac{9}{4}x^2-\frac{x^4}{4\cdot 4}\right)dx = \frac{1}{8}\left(9\frac{x^3}{3}\Bigg|_0^6 - \frac{1}{4}\,\frac{x^5}{5}\Bigg|_0^6\right) = \frac{1}{8}\left(3\cdot 6^3-\frac{6^5}{20}\right)=$$

$$= \frac{1}{8}(648-388{,}8) = \underline{32.4} = (4.4) \checkmark$$

Οπότε $\boxed{\bar{y}_K = \frac{\int_A y\,dA}{\int_A dA} \overset{(4.4)}{\underset{(4.1)}{=}} \frac{32.4}{9} = \underline{3.6}}$

2ος Τρόπος)

$$dA = \Delta y\,dx = \left(\frac{3}{2}x-\frac{x^2}{4}\right)dx$$



Οπότε

σελ (18)

$$A = \int_A dA = \int_0^6\left(\frac{3}{2}x-\frac{x^2}{4}\right)dx = 9 \checkmark$$

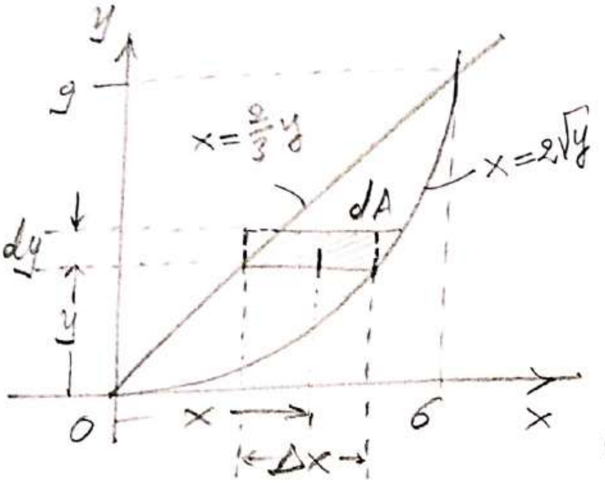
και

$$\int_A x\,dA = \int_0^6 x\left(\frac{3}{2}x-\frac{x^2}{4}\right)dx = 27 \checkmark$$

Επίσης, τότε $y = \frac{1}{2}\left(\frac{x^2}{4}+\frac{3}{2}x\right) = \frac{1}{4}\left(\frac{x^2}{2}+3x\right)$

$$\int_A y\,dA = \int_{x=0}^6 \frac{1}{4}\left(\frac{x^2}{2}+3x\right)\frac{1}{2}\left(3x-\frac{x^2}{2}\right)dx = \frac{1}{8}\int_0^6\left(9x^2-\frac{x^4}{4}\right)dx,$$ που ομοιάζει ακριβώς στον αμέσως προηγούμενο τρόπο, οπότε <u>ok</u>.

3ος Τρόπος



$$dA = \Delta x \, dy = \left(2y^{1/2} - \frac{2}{3}y\right) dy = 2\left(y^{1/2} - \frac{y}{3}\right) dy$$

$$\bar{x} = \frac{1}{2}\left(\frac{2}{3}y + 2y^{1/2}\right) = \frac{y}{3} + y^{1/2}$$

$$\int_A \bar{x}\, dA = 2\int_0^9 \left(y - \frac{y^2}{9}\right) dy,$$ που ομοιάζει ακριβώς στην τελευταία β, σελ. 18

$$\int_A \bar{y}\, dA = \int_0^9 2\left(y^{\frac{3}{2}} - \frac{y^2}{3}\right) dy,$$ που ομοιάζει ακριβώς στην πρώτη β, σελ. 19.