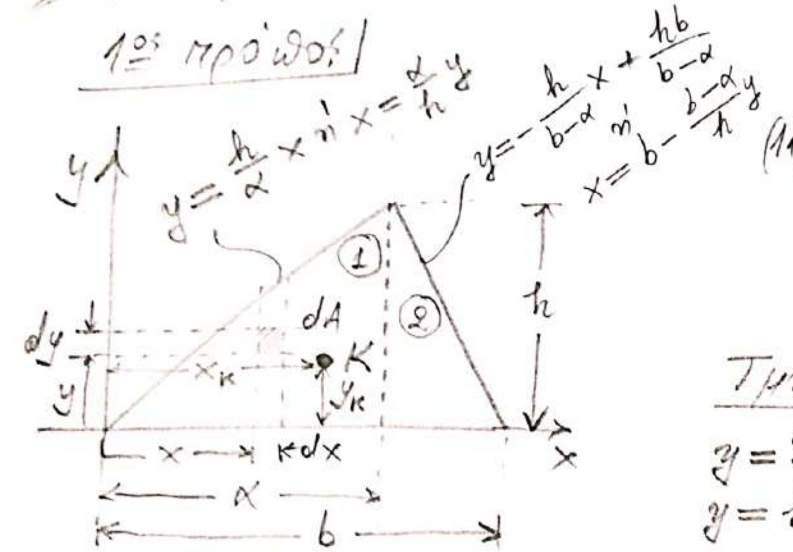
ΕΦΑΡΜΟΓΕΣ ΣΤΑ (11) ΓΕΩΜΕΤΡΙΚΑ ΚΕΝΤΡΑ

1) Κεντροειδές Τριγωνικής Επιφάνειας

1ος τρόπος



(11):
$$\boxed{x_K=\frac{x_{K_1}A_1+x_{K_2}A_2}{A_1+A_2}} \quad (1.1)$$

$$\boxed{y_K=\frac{y_{K_1}A_1+y_{K_2}A_2}{A_1+A_2}} \quad (1.2)$$

Τμήμα (1)

$$\left.\begin{aligned} y&=\lambda x\\ y&=h,\ x=\alpha \end{aligned}\right\} \rightarrow h=\lambda\alpha \rightarrow \lambda=\frac{h}{\alpha}$$

$$\boxed{y=\frac{h}{\alpha}x}$$

$$A_1=\alpha h/2 \quad (1.3)$$

$$x_{K_1}A_1=\int_{A_1} x\,dA=\int_0^\alpha x\left(\int_{y=0}^{y=\frac{h}{\alpha}x} dy\right)dx=\int_0^\alpha x\frac{h}{\alpha}x\,dx=\frac{h}{\alpha}\frac{x^3}{3}\Big|_0^\alpha=\frac{h\alpha^2}{3} \quad (1.4)$$

Τμήμα (2)

$$\left.\begin{aligned} y&=\lambda x+\beta\\ y&=h,\ x=\alpha \end{aligned}\right\} \rightarrow h=\lambda\alpha+\beta \Rightarrow \beta=h-\lambda\alpha \rightarrow \left.\begin{aligned} y&=\lambda x+h-\lambda\alpha\\ y&=0,\ x=b\end{aligned}\right\} \rightarrow 0=\lambda b+h-\lambda\alpha \rightarrow \lambda=-\frac{h}{b-\alpha}$$

$$y=-\frac{h}{b-\alpha}x+h+\frac{h}{b-\alpha}\alpha=-\frac{h}{b-\alpha}x+\frac{hb-\cancel{h\alpha}+\cancel{h\alpha}}{b-\alpha} \rightarrow \boxed{y=-\frac{h}{b-\alpha}x+\frac{hb}{b-\alpha}}$$

$$A_2=\frac{b-\alpha}{2}h \quad (1.5)$$

$$x_{K_2}A_2=\int_{A_2} x\,dA=\int_\alpha^b x\left(\int_{y=0}^{y=-\frac{h}{b-\alpha}x+\frac{hb}{b-\alpha}} dy\right)dx=\int_0^b x\left(-\frac{h}{b-\alpha}x+\frac{hb}{b-\alpha}\right)dx=$$

$$=\frac{hb}{b-\alpha}\frac{x^2}{2}\Big|_\alpha^b-\frac{h}{b-\alpha}\frac{x^3}{3}\Big|_\alpha^b=\frac{hb^3}{2(b-\alpha)}-\frac{hb\alpha^2}{2(b-\alpha)}-\frac{hb^3}{3(b-\alpha)}+\frac{h\alpha^3}{3(b-\alpha)}=$$

$$=\frac{3hb^3-3hb\alpha^2-2hb^3+2h\alpha^3}{6(b-\alpha)}=\frac{hb(b^2-\alpha^2)\mp 2h\alpha^2(\alpha-b)}{6(b-\alpha)}=$$

$$=\frac{h}{6}\left(b^2+\alpha b-2\alpha^2\right) \quad (1.6)$$

(1.1), (1.3), (1.4), (1.5), (1.6) ⇒

$$\boxed{x_K=\frac{\cancel{\frac{h\alpha^2}{3}}+\frac{hb^2}{6_3}+\frac{h\alpha b}{6_3}-\cancel{\frac{h\alpha^2}{3}}}{\cancel{\frac{\alpha h}{2}}+\frac{bh}{2}-\cancel{\frac{\alpha h}{2}}}=\frac{b+\alpha}{3}} \quad (1.7)$$

$\underline{y_{K_1}A_1} = \int_{A_1} y\,dA = \int_0^h y\left(\int_{x=\frac{\alpha}{h}y}^{x=\alpha} dx\right)dy = \int_0^h y\left(\alpha-\frac{\alpha}{h}y\right)dy =$

$= \alpha\frac{y^2}{2}\Big|_0^h - \frac{\alpha}{h}\frac{y^3}{3}\Big|_0^h = \frac{\alpha h^2}{2} - \frac{\alpha h^{\cancel{3}2}}{3\cancel{h}} = \underline{\frac{\alpha h^2}{6}}$ (1.8)

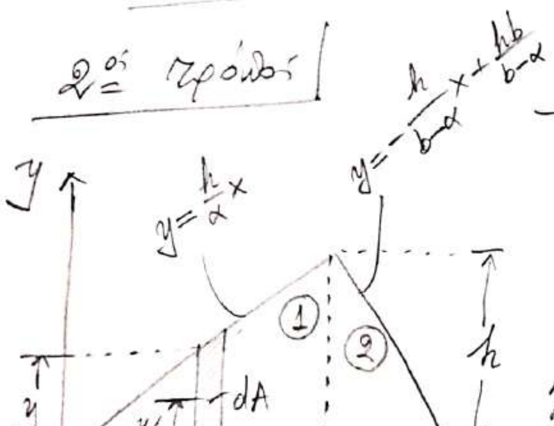
$\underline{y_{K_2}A_2} = \int_{A_2} y\,dA = \int_0^h y\left(\int_{x=\alpha}^{x=b-\frac{b-\alpha}{h}y} dx\right)dy = \int_0^h y\left(b-\frac{b-\alpha}{h}y-\alpha\right)dy =$

$= (b-\alpha)\frac{y^2}{2}\Big|_0^h - \frac{b-\alpha}{h}\frac{y^3}{3}\Big|_0^h = \frac{b-\alpha}{2}h^2 - \frac{b-\alpha}{3\cancel{h}}h^{\cancel{3}2} = \underline{\frac{b-\alpha}{6}h^2}$ (1.9)

(1.2), (1.3), (1.5), (1.8), (1.9) $\Rightarrow$

$$\boxed{y_K = \frac{\frac{\alpha h^2}{6}+\frac{b-\alpha}{6}h^2}{\frac{\alpha h}{2}+\frac{b-\alpha}{2}h} = \frac{\frac{\cancel{\alpha h^2}+bh^2-\cancel{\alpha h^2}}{\cancel{6}\,3}}{\frac{\cancel{\alpha h}+bh-\cancel{\alpha h}}{\cancel{2}}} = \frac{h}{3}} \quad (1.10)$$

2ος τρόπος



Τμήμα ① | $dA = y\,dx$

$\underline{x_{K_1}A_1} = \int_{A_1} x\,dA = \int_0^\alpha x y\,dx = \int_0^\alpha x\frac{h}{\alpha}x\,dx =$

$= \frac{h}{\alpha}\frac{x^3}{3}\Big|_0^\alpha = \frac{h\alpha^3}{3\alpha} = \underline{\frac{h\alpha^2}{3}} = (1.4)$ ✓

Τμήμα ②

$\underline{x_{K_2}A_2} = \int_{A_2} x\,dA = \int_\alpha^b x y\,dx = \int_\alpha^b x\left(\frac{-h}{b-\alpha}x+\frac{hb}{b-\alpha}\right)dx$

$= \frac{hb}{b-\alpha}\frac{x^2}{2}\Big|_\alpha^b - \frac{h}{b-\alpha}\frac{x^3}{3}\Big|_\alpha^b = \frac{hb}{b-\alpha}\frac{b^2-\alpha^2}{2} - \frac{h}{b-\alpha}\frac{b^3-\alpha^3}{3}$

$= \underline{\dots\ (1.6)}$ ✓

Τμήμα ① |

$\underline{y_{K_1}A_1} = \int_{A_1}\frac{y}{2}dA = \int_0^\alpha \frac{y}{2}y\,dx = \frac{1}{2}\int_0^\alpha y^2\,dx = \frac{1}{2}\int_0^\alpha \frac{h^2}{\alpha^2}x^2\,dx = \frac{h^2}{2\alpha^2}\frac{x^3}{3}\Big|_0^\alpha =$

$= \frac{h^2\alpha^3}{6\alpha^2} = \underline{\frac{\alpha h^2}{6}} = (1.8)$ ✓

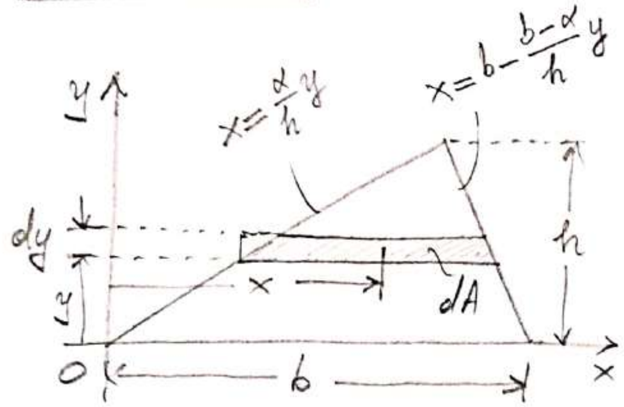
$\underline{y_{K_2}A_2} = \int_{A_2}\frac{y}{2}dA = \int_\alpha^b \frac{y}{2}y\,dx = \frac{1}{2}\int_\alpha^b \left[\frac{h}{b-\alpha}(b-x)\right]^2 dx = \frac{h^2}{2(b-\alpha)^2}\int_\alpha^b (b^2+x^2-2bx)\,dx =$

$= \frac{h^2}{2(b-\alpha)^2}\left[b^2(b-\alpha) - \frac{2b}{2}(b^2-\alpha^2) + \frac{b^3-\alpha^3}{3}\right] = \frac{h^2}{2(b-\alpha)^2}\left[b^2\cancel{(b-\alpha)} - b\cancel{(b-\alpha)}(b+\alpha) + \frac{\cancel{(b-\alpha)}(\alpha^2+\alpha b+b^2)}{3}\right] =$

$$\underbrace{y_{K_2} A_2} = \frac{h^2}{2(b-\alpha)}\left(\cancel{b^2}-\cancel{b^2}-\frac{3}{3}\alpha b+\frac{\alpha^2}{3}+\frac{\alpha b}{3}+\frac{b^2}{3}\right)=\frac{h^2}{6(b-\alpha)}\left(\alpha^2+b^2-2\alpha b\right)=$$

$$=\frac{h^2}{6(b-\alpha)}(b-\alpha)^2=\underbrace{\frac{b-\alpha}{6}h^2}\equiv(1.9)\ ✓$$

3ος Τρόπος



$$\underbrace{dA}=\left(b-\frac{b-\alpha}{h}y-\frac{\alpha}{h}y\right)dy=\underbrace{\left(b-\frac{b}{h}y\right)dy}\quad(*)$$

$$\underbrace{x}=\frac{1}{2}\left(\frac{\alpha}{h}y+b-\frac{b-\alpha}{h}y\right)=\underbrace{\frac{1}{2}\left(b-\frac{b-2\alpha}{h}y\right)}$$

$$x_K \overset{(8)}{=} \frac{\int_A x\,dA}{A} \quad \xRightarrow{\ } \quad A=bh/2$$

$$\Rightarrow \underbrace{x_K}=\frac{2}{bh}\,\frac{1}{2}\int_{y=0}^{y=h}\left(b-\frac{b-2\alpha}{h}y\right)\left(b-\frac{b}{h}y\right)dy=$$

$$=\frac{1}{bh}\int_0^h\left(b^2-\cancel{\frac{b^2}{h}y}-b\frac{b-2\alpha}{h}y+b\frac{b-2\alpha}{h^2}y^2\right)dy=$$

$$=\frac{1}{bh}\left(b^2h-2b\frac{b-\alpha}{h}\frac{y^2}{2}\Big|_0^h+b\frac{b-2\alpha}{h^2}\frac{y^3}{3}\Big|_0^h\right)=$$

$$=\frac{1}{bh}\left[b^2h-b(b-\alpha)h+\frac{b(b-2\alpha)}{3}h\right]=$$

$$=\frac{1}{bh}\left(\cancel{b^2h}-\cancel{b^2h}+\frac{3}{3}\alpha bh+\frac{b^2h}{3}-\frac{2\alpha bh}{3}\right)=\frac{1}{\cancel{bh}}\left(\frac{\alpha bh}{3}+\frac{b^2h}{3}\right)=\underbrace{\frac{\alpha+b}{3}}\quad(1.7)\ ✓$$

$$\underbrace{y_K}\overset{(8)}{=}\frac{\int_A y\,dA}{A}=\frac{2}{bh}\int_{y=0}^{y=h}y\left(b-\frac{b}{h}y\right)dy=\frac{2}{\cancel{b}h}\left(b\frac{y^2}{2}\Big|_0^h-\frac{b}{h}\frac{y^3}{3}\Big|_0^h\right)=$$

$$=\frac{2}{\cancel{h}}\left(\frac{h^{\cancel{2}}}{2}-\frac{h^{\cancel{2}}}{3}\right)=\frac{2h}{6}=\underbrace{\frac{h}{3}}\equiv(1.10)\ ✓$$