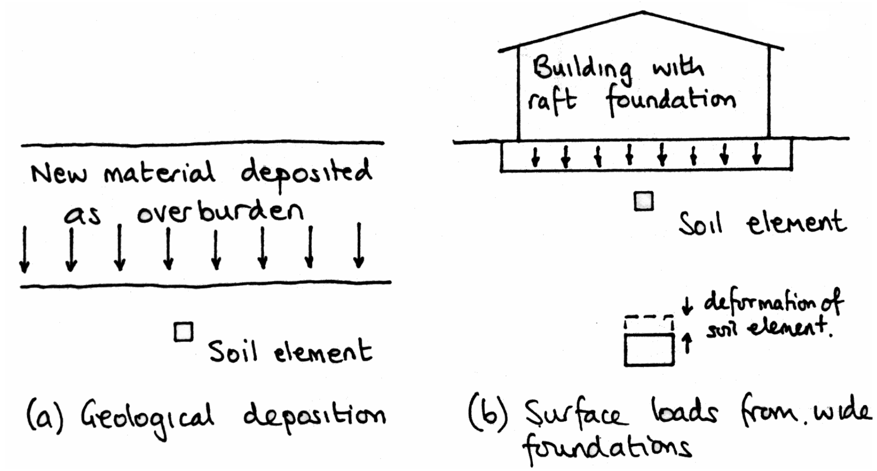
Coupled pore pressure-deformation (consolidation) analysis

Reminder: What is consolidation?

Consolidation: Time-dependent compression of soil under constant total stress that is accompanied by dissipation of excess pore pressure.

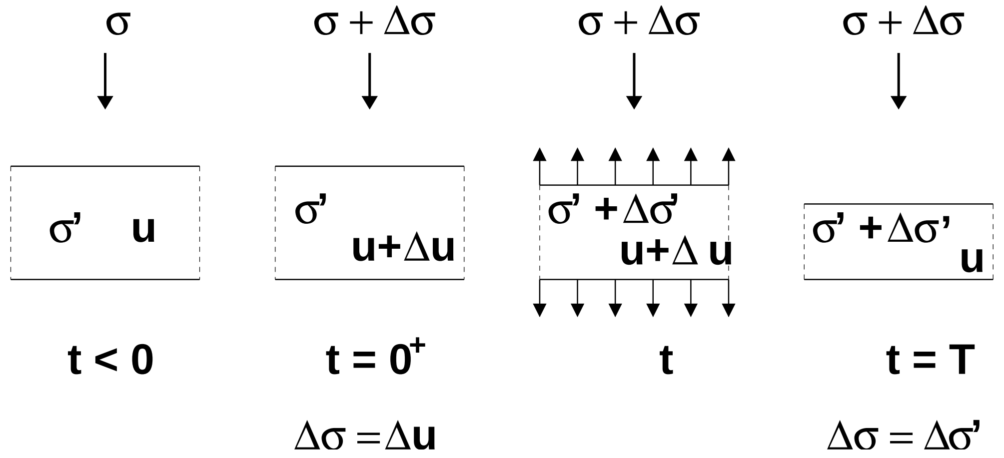
You have already discussed, in your undergraduate studies, **one dimensional** compression and consolidation.

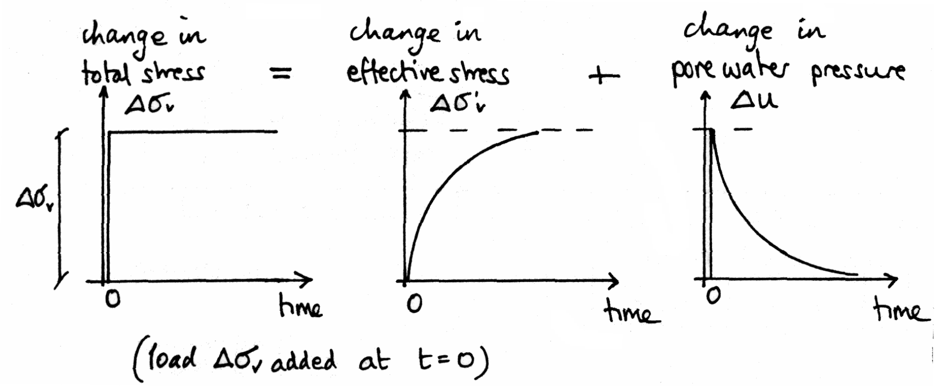
- Vertical but no lateral deformation; vertical strain ϵ_z equal to volumetric ϵ_v.
- Variations only with depth.
- Simplified approach; applicable to e.g. the soil under a shallow foundation.



The process of consolidation is time-dependent because:

- Soil volume can change only if soil particles rearrange, changing the volume of voids.
- If the voids are full of water, flow needs to take place to accommodate compression.
- Flow needs time to occur, as soil permeability is finite.

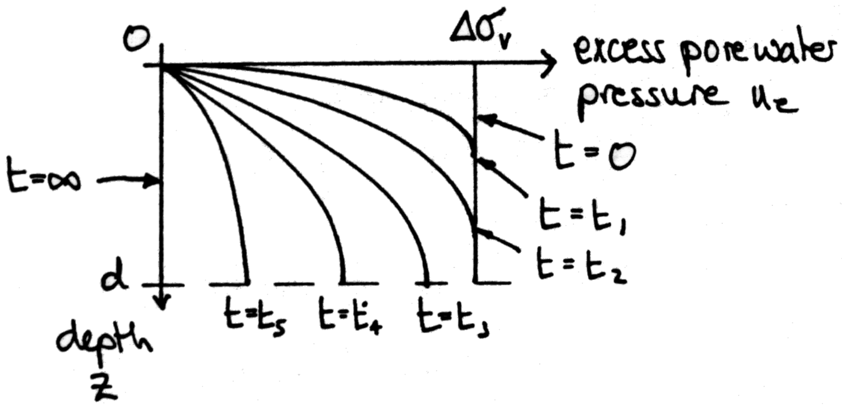




- The applied load increases total stress by $\Delta\sigma$.
- The skeleton can't take more load till particles rearrange; **excess** pore pressures generated.
- Pore pressure increase creates a hydraulic gradient; flow commences.
- Due to flow, excess pore pressures gradually diffuse; effective stresses gradually increase.
- Finally, the skeleton takes all the extra load; pore pressures back to their equilibrium value.

A line showing the distribution of excess pore pressure at a particular time is called the **isochrone of excess pore pressure** for that time.

- In one-dimensional consolidation the isochrones consist of excess pore pressure plotted vs depth.



Typical isochrones for a consolidating layer that drains from the top only are shown on the left.

In your undergraduate studies you saw how to determine the isochrones at different times, and how to use them to calculate settlements over time.

What if the problem is not one-dimensional?

. . . Previously we assumed that:

- Consolidating layers are relatively thin, so an initially uniform distribution of uniform excess pore pressure with depth can be assumed.
- There is no lateral, only vertical flow.
- The soil is linear elastic, homogeneous and isotropic.
- Soil stiffness and permeability do not vary during the consolidation process.
- Deformations are small.

This allows dealing with simple problems that are not far from one-dimensional. But:

- What if a problem is truly two- or even three-dimensional?
- What if the consolidating layer is not thin?
- What if lateral flow cannot be ignored?
- What if there is material anisotropy, or even multiple material layers?
- (Soil is **not** linear elastic; permeability and stiffness depend on eff. stress.)

Then it is necessary to solve the general consolidation problem numerically.

Formulation of the coupled problem: Equilibrium

Consolidation involves both soil skeleton deformation and fluid flow. Therefore we will solve the equations of equilibrium and of continuity of fluid flow.
We need however to take into account that soil deformation is governed by increments of the effective stress $\boldsymbol{\sigma}'$, rather than total stress $\boldsymbol{\sigma}$.

Equilibrium:

- $\mathbf{L}^\top \boldsymbol{\sigma} + \mathbf{b} = \mathbf{0}$
- $\boldsymbol{\sigma}' = \boldsymbol{\sigma} + \mathbf{m}^\top p_w$ is the effective stress, where $\mathbf{m}^\top = \{1\;1\;1\;0\;0\;0\}$
- $\boldsymbol{\sigma}' = \mathbf{D}' \cdot \boldsymbol{\epsilon}$
- $\boldsymbol{\epsilon} = \mathbf{L} \cdot \mathbf{u}$

where $\mathbf{L}$ is the usual differential operator, p_w the pore pressure and $\mathbf{D}'$ the (drained) elastic moduli. The boundary conditions are:

- $\mathbf{u} = \mathbf{u_0}$ on S_u (the part of S where displacements are prescribed.)
- $\boldsymbol{\Sigma} \cdot \mathbf{n} = \mathbf{t}$ on S_σ (the part of S where stresses are applied.)

Note that the definition of effective stress assumes that positive stress is *tensile* while positive pore pressure is *compressive*.

We have already worked out the weak form to be:

$$\int_V \delta\boldsymbol{\epsilon}^\top \boldsymbol{\sigma}\, dV = \int_V \delta\mathbf{u}^\top \mathbf{b}\, dV + \int_S \delta\mathbf{u}^\top \mathbf{t}\, dS$$

where $\delta\mathbf{u}$ a virtual displacement field and $\delta\boldsymbol{\epsilon}$ the corresponding virtual strain. Substituting the definition of effective stress we obtain:

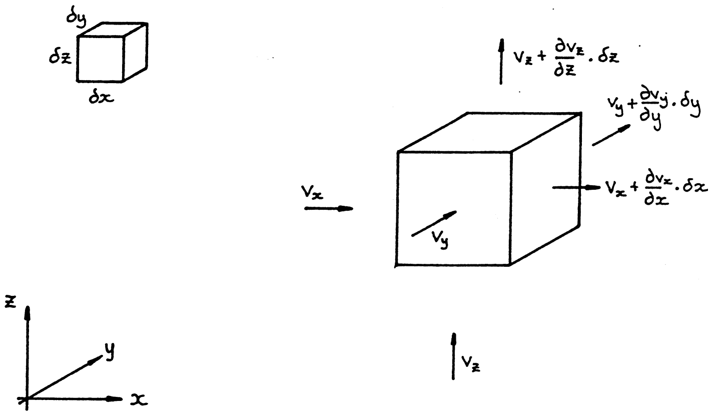
$$\int_V \delta\boldsymbol{\epsilon}^\top \boldsymbol{\sigma}'\, dV - \int_V \delta\boldsymbol{\epsilon}^\top \mathbf{m}\, p_w\, dV = \int_V \delta\mathbf{u}^\top \mathbf{b}\, dV + \int_S \delta\mathbf{u}^\top \mathbf{t}\, dS$$

and finally the weak form of the equations of equilibrium is written as:

$$\boxed{\int_V \delta\boldsymbol{\epsilon}^\top \mathbf{D}'\boldsymbol{\epsilon}\, dV - \int_V \delta\boldsymbol{\epsilon}^\top \mathbf{m}\, p_w\, dV = \int_V \delta\mathbf{u}^\top \mathbf{b}\, dV + \int_S \delta\mathbf{u}^\top \mathbf{t}\, dS}$$

We note that the equation contains *two* unknown fields: the displacement $\mathbf{u}$ (through its derivatives $\boldsymbol{\epsilon}$) and the pore pressure p_w.

Formulation of the coupled problem: Continuity



Flow through inf/mal cube $V_0 = dx \times dy \times dz$

- "negative" faces: v_x, v_y, v_z.
- "positive" faces: $v_x + \dfrac{\partial v_x}{\partial x} dx$, $v_y + \dfrac{\partial v_y}{\partial y} dy$, $v_z + \dfrac{\partial v_z}{\partial z} dz$

In: $q_{in} = v_x \, dydz + v_y \, dxdz + v_z \, dxdy$

Out: $q_{out} = (v_x + \dfrac{\partial v_x}{\partial x} dx) \, dydz + (v_y + \dfrac{\partial v_y}{\partial y} dy) \, dxdz + (v_z + \dfrac{\partial v_z}{\partial z} dz) \, dxdy$

Assume that the reference volume V_0 changes (increases) by dV during a time interval dt.

Also assume conservation of mass and incompressibility. Then:

$$q_{in} = q_{out} + \frac{dV}{dt} \Longrightarrow \left(\nabla^\top \mathbf{v}\right) V_0 = \frac{dV}{dt} \Longrightarrow \nabla^\top \cdot \mathbf{v} = \frac{\partial \epsilon_v}{\partial t} \Longrightarrow \boxed{\nabla^\top \cdot \mathbf{v} = \mathbf{m}^\top \cdot \frac{\partial \boldsymbol{\epsilon}}{\partial t}}$$

where $\mathbf{v}$ the seepage velocity, ϵ_v the volumetric strain and $\epsilon_v < 0$ for compression.

The total head at a point is defined as: $h = s + \frac{p_w}{\rho_w g}$, where:

- p_w: the total pore pressure.
- s: the distance from the datum for head measurement.
- ρ_w: the density of the fluid (water.)
- g: the acceleration of gravity.

Darcy's Law: $\mathbf{v} = -\mathbf{K} \cdot \nabla h$, where $\mathbf{K}$ the permeability matrix

Defining the gravity acceleration vector as $\mathbf{g} = -g(\nabla s)$ and substituting the definition of total head and Darcy's law into the equation of continuity we obtain:

$$\frac{1}{\rho_w g} \nabla^\top \{\mathbf{K} (\nabla p_w - \rho_w \mathbf{g})\} = -\mathbf{m}^\top \cdot \frac{\partial \boldsymbol{\epsilon}}{\partial t}$$

Continuity: weak form

We can determine the weak form of the flow equation by the usual procedure. Assume a virtual total pore pressure field δp_w consistent with prescribed boundary pore pressures. Then:

$$\int_V \left[\frac{1}{\rho_w g}\nabla^\top \left\{\mathbf{K}\left(\nabla p_w - \rho_w \mathbf{g}\right)\right\}\right]\delta p_w \, dV = -\int_V \mathbf{m}^\top \cdot \frac{\partial \boldsymbol{\epsilon}}{\partial t}\delta p_w \, dV$$

and carrying out the usual algebraic manipulations:

$$\frac{1}{\rho_w g}\int_V \left(\nabla^\top \delta p_w\right)\mathbf{K}\left(\nabla p_w\right) dV = \frac{1}{g}\int_V \left(\nabla^\top \delta p_w\right)\mathbf{K}\cdot\mathbf{g}\, dV$$

$$+\int_V \mathbf{m}^\top \cdot \frac{\partial \boldsymbol{\epsilon}}{\partial t}\cdot \delta p_w \, dV - \int_S \delta p_w \cdot \mathbf{v}^\top \cdot \mathbf{n}\, dS$$

We interpolate displacements and pore pressure as:

- $u = \mathbf{N_u} \cdot \hat{\mathbf{u}}$
- $p_w = \mathbf{N_p} \cdot \hat{\mathbf{p}}$

where $\mathbf{N_u}$ and $\mathbf{N_p}$ shape functions and $\hat{\mathbf{u}}$ and $\hat{\mathbf{p}}$ degrees of freedom.

We also define the matrices: $\mathbf{B_u} = \mathbf{L} \cdot \mathbf{N_u}$ and $\mathbf{B_p} = \nabla \mathbf{N_p}$ and remind that $\boldsymbol{\epsilon} = \mathbf{L} \cdot \mathbf{u}$.

The weak forms of the equilibrium and continuity equations can then be discretised as follows:

$$\int_V \delta\boldsymbol{\epsilon}^\top \mathbf{D}'\boldsymbol{\epsilon}\, dV - \int_V \delta\boldsymbol{\epsilon}^\top \mathbf{m}\, p_w\, dV = \int_V \delta\mathbf{u}^\top \mathbf{b}\, dV + \int_S \delta\mathbf{u}^\top \mathbf{t}\, dS \longrightarrow$$

$$\int_V \mathbf{B_u}^\top \mathbf{D}'\mathbf{B_u}\, dV \cdot \hat{\mathbf{u}} - \int_V \mathbf{B_u}^\top \mathbf{m}\, \mathbf{N_p}\, dV \cdot \hat{\mathbf{p}} = \int_V \mathbf{N_u}^\top \mathbf{b}\, dV + \int_S \mathbf{N_u}^\top \mathbf{t}\, dS \longrightarrow$$

$$\boxed{\mathbf{K_{uu}} \cdot \hat{\mathbf{u}} + \mathbf{K_{up}} \cdot \hat{\mathbf{p}} = \mathbf{f_u}}$$

where:

$\mathbf{K_{uu}} = \int_V \mathbf{B_u}^\top \mathbf{D}'\mathbf{B_u}\, dV$, $\mathbf{K_{up}} = -\int_V \mathbf{B_u}^\top \mathbf{m}\, \mathbf{N_p}\, dV$, and

$\mathbf{f_u} = \int_V \mathbf{N_u}^\top \mathbf{b}\, dV + \int_S \mathbf{N_u}^\top \mathbf{t}\, dS$

Discretised form of continuity

$$\frac{1}{\rho_w g}\int\limits_V \left(\nabla^\top \delta p_w\right)\mathbf{K}\left(\nabla p_w\right)dV = \frac{1}{g}\int\limits_V \left(\nabla^\top \delta p_w\right)\mathbf{K}\cdot\mathbf{g}\,dV$$

$$+\int\limits_V \delta p_w\,\mathbf{m}^\top\cdot\frac{\partial\boldsymbol{\epsilon}}{\partial t}dV - \int\limits_S \delta p_w\cdot\mathbf{v}^\top\cdot\mathbf{n}\,dS \longrightarrow$$

$$-\int\limits_V {\mathbf{N_p}}^\top\mathbf{m}^\top\mathbf{B_u}dV\cdot\frac{\partial\hat{\mathbf{u}}}{\partial t} + \frac{1}{\rho_w g}\int\limits_V {\mathbf{B_p}}^\top\mathbf{K}\mathbf{B_p}dV\cdot\hat{\mathbf{p}} =$$

$$\frac{1}{g}\int\limits_V {\mathbf{B_p}}^\top\mathbf{K}\cdot\mathbf{g}\,dV - \int\limits_S {\mathbf{N_p}}^\top\cdot\mathbf{v}^\top\cdot\mathbf{n}\,dS \longrightarrow$$

$$\boxed{\mathbf{K_{pu}} \cdot \dot{\hat{\mathbf{u}}} + \mathbf{K_{pp}} \cdot \hat{\mathbf{p}} = \mathbf{f_p}}$$

where:

$$\mathbf{K_{pp}} = \frac{1}{\rho_w g} \int_V \mathbf{B_p}^\top \mathbf{K} \mathbf{B_p} dV,$$

$\mathbf{K_{pu}} = \mathbf{K_{up}}^\top$, and

$$\mathbf{f_p} = \frac{1}{g} \int_V \mathbf{B_p}^\top \mathbf{K} \cdot \mathbf{g} \, dV - \int_S \mathbf{N_p}^\top \cdot \mathbf{v}^\top \cdot \mathbf{n} \, dS$$

We arrived at two equations that are coupled:

$$\mathbf{K_{uu}} \cdot \hat{\mathbf{u}} + \mathbf{K_{up}} \cdot \hat{\mathbf{p}} = \mathbf{f_u} \qquad \mathbf{K_{pu}} \cdot \dot{\hat{\mathbf{u}}} + \mathbf{K_{pp}} \cdot \hat{\mathbf{p}} = \mathbf{f_p}$$

Nevertheless they cannot be solved directly because the second one contains the *rate* of the displacement $\dot{\hat{\mathbf{u}}}$, rather than the displacement $\hat{\mathbf{u}}$.

We need to discretise in time as well as space!

- We assume that we know the solution $(\hat{\mathbf{u}}_\mathbf{n}, \hat{\mathbf{p}}_\mathbf{n})$ at the end of timestep n, corresponding to time t.
- We want to determine $(\hat{\mathbf{u}}_\mathbf{n+1} = \hat{\mathbf{u}}_\mathbf{n} + \Delta\hat{\mathbf{u}}, \hat{\mathbf{p}}_\mathbf{n+1} = \hat{\mathbf{p}}_\mathbf{n} + \Delta\hat{\mathbf{p}})$ at the end of timestep $n+1$, corresponding to time $t + \Delta t$.
  - ☐ Determine $(\Delta\hat{\mathbf{u}}, \Delta\hat{\mathbf{p}})$ for Δt.

Writing the first equation at both t and $t + \Delta t$ and subtracting:

$$\boxed{\mathbf{K_{uu}} \cdot \boldsymbol{\Delta}\hat{\mathbf{u}} + \mathbf{K_{up}} \cdot \boldsymbol{\Delta}\hat{\mathbf{p}} = \boldsymbol{\Delta}\mathbf{f_u}}$$

To discretise the second one, we assume the following approximation in time:

$$\hat{\mathbf{u}}_{\mathbf{n+1}} = \hat{\mathbf{u}}_{\mathbf{n}} + \Delta t \left((1-\theta)\dot{\hat{\mathbf{u}}}_{\mathbf{n}} + \theta \dot{\hat{\mathbf{u}}}_{\mathbf{n+1}} \right)$$

i.e. that the average derivative of $\hat{\mathbf{u}}$ over Δt is a weighted sum of the derivatives at the beginning ($\dot{\hat{\mathbf{u}}}_{\mathbf{n}}$) and the end ($\dot{\hat{\mathbf{u}}}_{\mathbf{n+1}}$) of the interval.

- $\theta = 0$: (Unstable) explicit scheme $\hat{\mathbf{u}}_{\mathbf{n+1}} = \hat{\mathbf{u}}_{\mathbf{n}} + \Delta t \dot{\hat{\mathbf{u}}}_{\mathbf{n}}$; only uses information available at time t.
- $\theta = 1$: (Stable) backward difference $\hat{\mathbf{u}}_{\mathbf{n+1}} = \hat{\mathbf{u}}_{\mathbf{n}} + \Delta t \dot{\hat{\mathbf{u}}}_{\mathbf{n+1}}$; implicit scheme so the solution must be determined iteratively.
- To ensure stability, $\theta \geq 1/2$. For $\theta = 1/2$: Crank-Nicholson scheme.

Writing the second equation at both t and $t + \Delta t$, premultiplied by $(1-\theta)$ and θ respectively:

$$\mathbf{K_{pu}} \cdot \theta \dot{\hat{\mathbf{u}}}_{\mathbf{n+1}} + \mathbf{K_{pp}} \cdot \theta \hat{\mathbf{p}}_{\mathbf{n+1}} = \theta \mathbf{f_{p,n+1}}$$
$$\mathbf{K_{pu}} \cdot (1-\theta)\, \dot{\hat{\mathbf{u}}}_{\mathbf{n}} + \mathbf{K_{pp}} \cdot (1-\theta)\, \hat{\mathbf{p}}_{\mathbf{n}} = (1-\theta)\, \mathbf{f_{p,n}}$$

Adding up yields:

$$\mathbf{K_{pu}} \cdot \frac{\boldsymbol{\Delta}\hat{\mathbf{u}}}{\Delta t} + \mathbf{K_{pp}} \cdot \left(\theta \hat{\mathbf{p}}_{\mathbf{n+1}} + (1-\theta)\, \hat{\mathbf{p}}_{\mathbf{n}}\right) = \theta \mathbf{f_{p,n+1}} + (1-\theta)\, \mathbf{f_{p,n}} \longrightarrow$$
$$\mathbf{K_{pu}} \cdot \boldsymbol{\Delta}\hat{\mathbf{u}} + \theta \Delta t \mathbf{K_{pp}} \cdot \Delta \hat{\mathbf{p}} = \left(\theta \mathbf{f_{p,n+1}} + (1-\theta)\, \mathbf{f_{p,n}} - \mathbf{K_{pp}} \cdot \hat{\mathbf{p}}_{\mathbf{n}}\right) \Delta t$$

In the following we consider the case $\theta = 1$, which is implemented in ABAQUS:

$$\boxed{\mathbf{K_{pu}} \cdot \boldsymbol{\Delta}\hat{\mathbf{u}} + \Delta t \mathbf{K_{pp}} \cdot \Delta \hat{\mathbf{p}} = \left(\mathbf{f_{p,n+1}} - \mathbf{K_{pp}} \cdot \hat{\mathbf{p}}_{\mathbf{n}}\right) \Delta t}$$

Finally the system of equations to solve becomes:

$$\begin{bmatrix} \mathbf{K_{uu}} & \mathbf{K_{up}} \\ \mathbf{K_{up}}^\top & \Delta t \mathbf{K_{pp}} \end{bmatrix} \cdot \begin{Bmatrix} \Delta \hat{\mathbf{u}} \\ \Delta \hat{\mathbf{p}} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{\Delta} \mathbf{f_u} \\ (\mathbf{f_{p,n+1}} - \mathbf{K_{pp}} \cdot \hat{\mathbf{p}}_\mathbf{n}) \Delta t \end{Bmatrix}$$

$\mathbf{u}$ and p_w are interpolated independently. However:

- p_w is driven by changes in volumetric strain, i.e. derivatives of $\mathbf{u}$.
- At the undrained limit $\mathbf{K} = \mathbf{0} \longrightarrow \mathbf{K_{pp}} = \mathbf{0}$ a solution exists only if $\mathbf{u}$ has more degrees of freedom than p_w: $n_u > n_p$

For these reasons typical elements that perform well interpolate the pore pressure with a polynomial one order lower than the displacement.

- E.g. biquadratic quadrilateral for $\mathbf{u}$ and bilinear quadrilateral for p_w

- Use a transient *SOILS analysis step.
- Use elements that interpolate the displacement and the pore pressure.
  - Typically (bi)quadratic displacement, (bi)linear pore pressure.
- Supply displacement and pore pressure boundary conditions. Potentially time-dependent, using *AMPLITUDE.
- Prescribe boundary loads and seepage conditions. Potentially time-dependent, using *AMPLITUDE.
  - May be complex, e.g. drainage-only: see the Seepage section.
- Prescribe initial conditions of void ratio, pore pressure and effective stress.
  - This can be done in CAE under *Loads / Create Predefined Field*.
  - Select *Mechanical* for initial (geostatic) stress.
  - Select *Other* for initial void ratio and pore pressure.
  - Select the *Initial* step, or the relevant options may not be available.
- Run a *GEOSTATIC step before *SOILS to obtain equilibrium.

In a transient analysis you need to define some timestepping parameters:

- Initial, minimum and maximum timestep.
- Total time period to be modelled. (Alternatively you can require the analysis to run till steady-state is reached.)
- Maximum pore pressure change during the increment: defines the tolerance with which the solution is obtained.

There are two different ways of deciding timestep size.

- Timesteps of fixed size.
  - Generally not a good idea unless done for specific reasons and you already know that the step size you set is small enough.
- Timesteps automatically adjusted by ABAQUS.
  - Should be preferred. Allows automatic adjustment of timestep size to optimise accuracy vs. computational time.

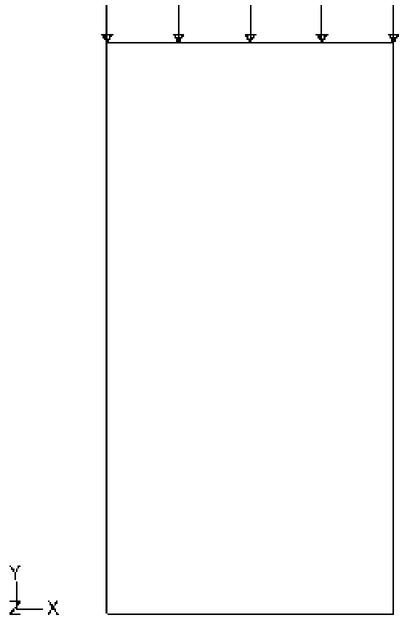
Last but not least: the units you use must be consistent.

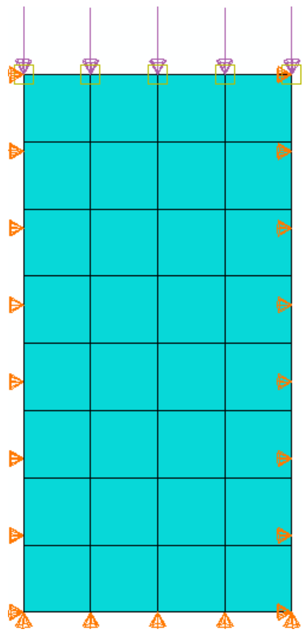
Example: 1D consolidation

A $2m$-thick soil layer undergoes 1D consolidation under a constant load of $10kPa$, applied instantaneously at $t = 0$. The soil has permeability $k = 10^{-7}m/sec$ and is linear elastic ($E = 10MPa$ and $\nu = 0.3$).

- Determine the settlement vs time curve of the layer.
- Plot profiles of excess pore pressure vs depth.

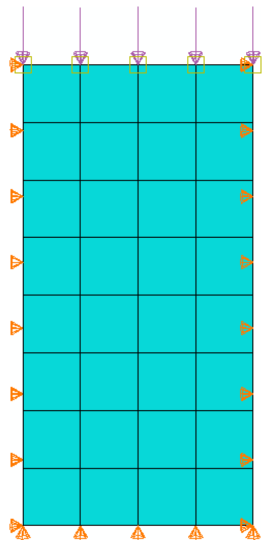
For convenience, assume that the initial pore pressure is zero throughout.





Setting up the analysis:

- Geometry and material properties are trivial to enter.
- So is the mesh and the supports.
- We also prescribe a uniform initial void ratio (e.g. $e = 1.0$).
- Parameters requiring further thought:
  - ☐ Application of load? Drainage/pore pressure?



If the load is applied instantaneously, uniform excess pore pressure should develop throughout the sample. If we want to reproduce this response and "set up" excess pore pressure before we allow it to dissipate, we can:

- Run a first analysis step where the load is applied but drainage is prohibited. Then the pore pressure should rise to match the applied load.
- Continue with a second analysis step where we change one of the boundary conditions to allow drainage. As excess pore pressure is present, consolidation will start.

- Create a first step of the *Geostatic type.
  - Create supports.
  - Apply vertical load.
  - Solve for equilibrium; this sets up excess pore pressures.
- Create a second step of the *Soils (transient) type.
  - This step inherits all loads and supports.
  - Additional boundary condition: zero pore pressure at the top.
  - This step will solve the consolidation problem.

We use fully-integrated, biquadratic displacement - bilinear pore pressure quadrilaterals.

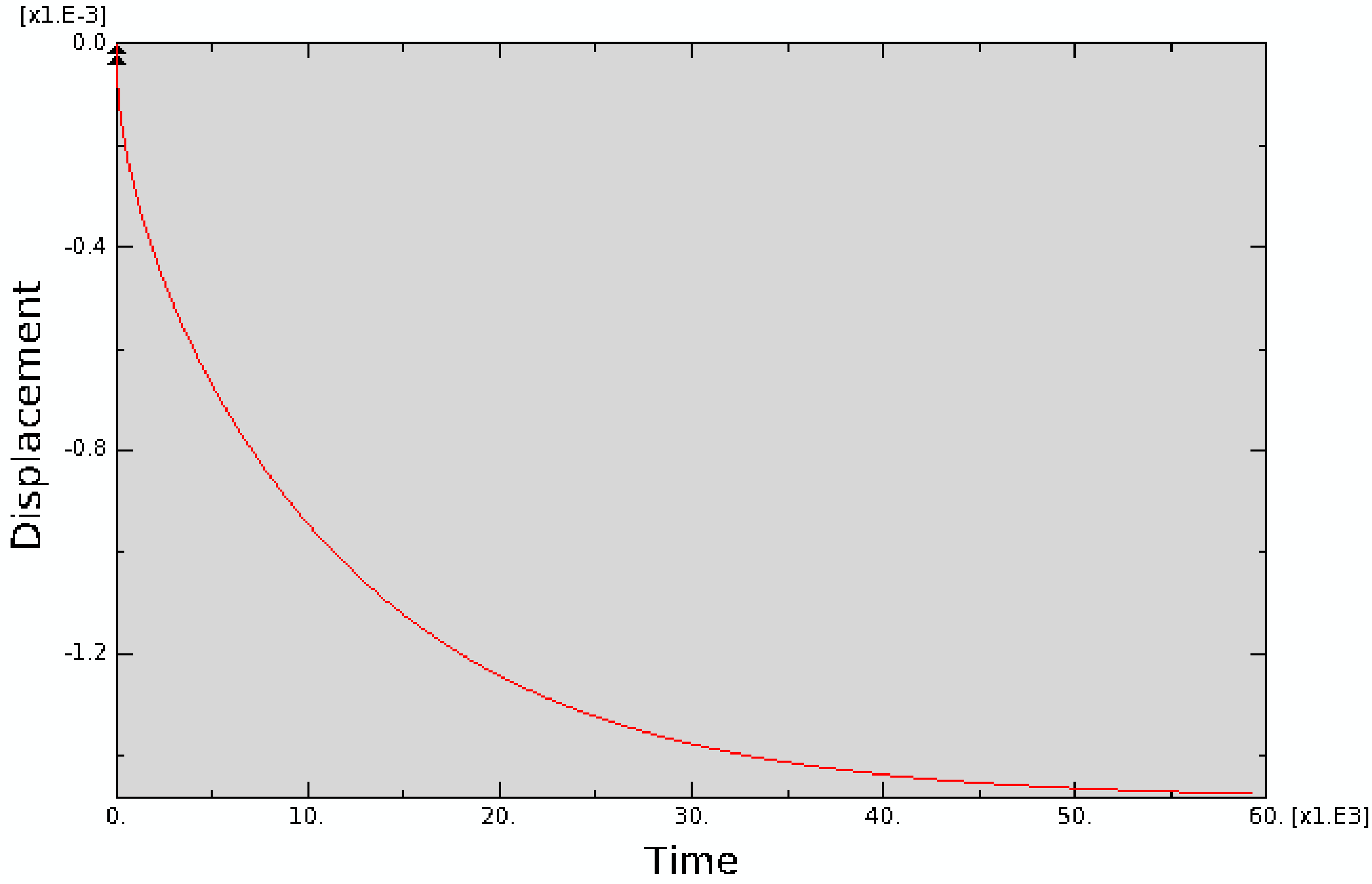
Time incrementation parameters:

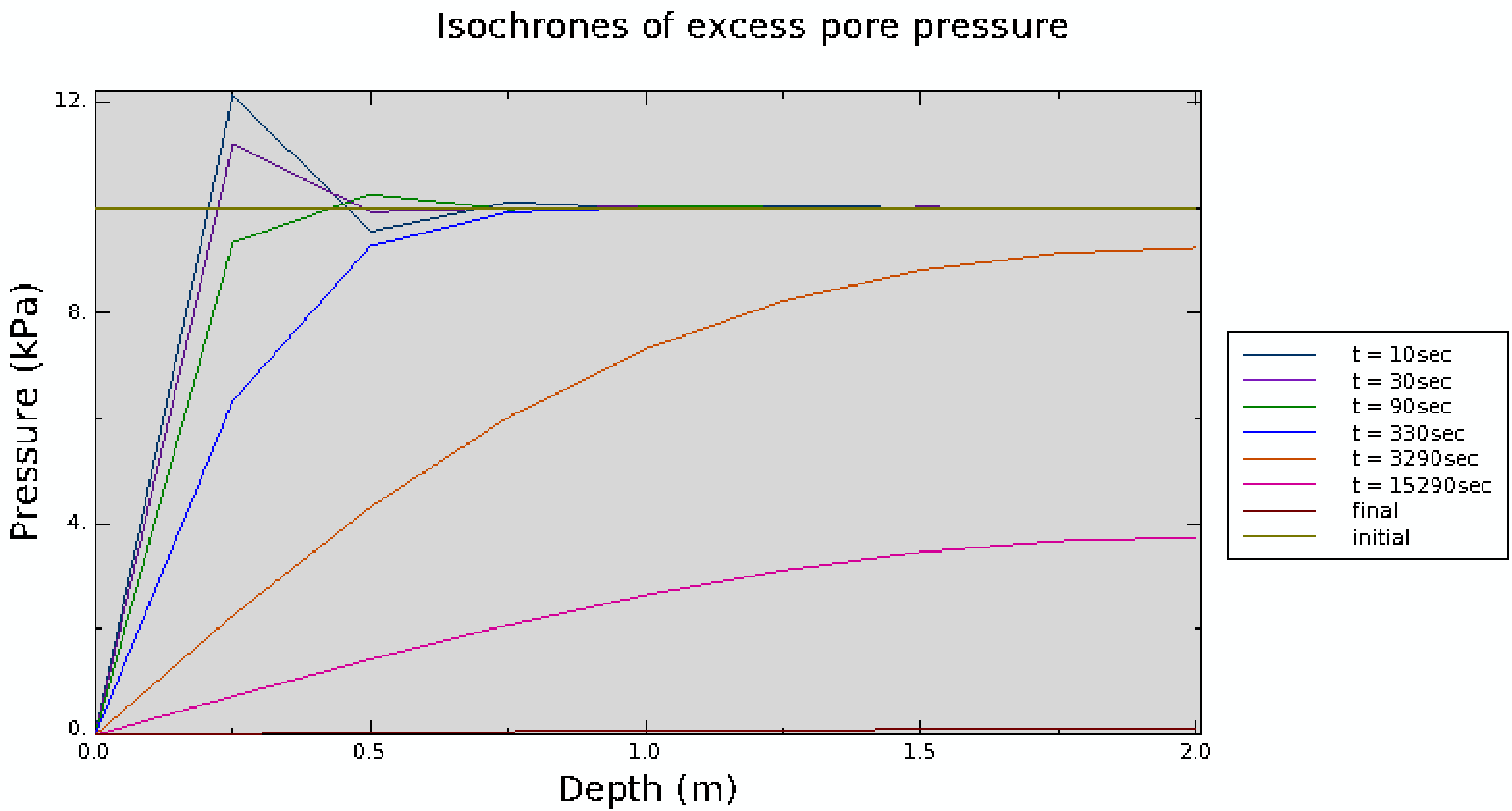
- Opt for automatic incrementation.
- Aim for a maximum modelled time period $t_{max} = 100,000sec$ – in reality we expect the phenomenon to end much earlier.
- Set the initial timestep $\Delta t_{ini} = 10sec$, and set $1 \leq \Delta t \leq 1000$.
- If $\Delta p_w/\Delta t < 10^{-5}$ assume steady state has been reached and stop.
- Only accept $\Delta p_w \leq 10$ for each increment – this criterion controls the accuracy of the flow solution.

The backward-difference scheme used for time integration is *unconditionally stable*, i.e. the error will be bounded. Nevertheless it may be significant!

- Too large timesteps: may lead to inaccuracies, although not cumulative.
- Too small timesteps: may lead to spurious pore pressure oscillations.
  - Problematic if soil plasticity models are used (see next few lectures!)
- Generally $\Delta t \geq \dfrac{\gamma_w (\Delta l)^2}{6 \cdot E \cdot k}$, where Δl a representative element length.

The respective model is called “Column” and is in the “Consolidation.cae” database. The file is available on Helios for you to download and play with.


[x1.E-3]
0.0
-0.4
-0.8
-1.2
Displacement
0.
10.
20.
30.
40.
50.
60. [x1.E3]
Time
settlement



Isochrones of excess pore pressure
Pressure (kPa)
Depth (m)
12.
8.
4.
0.
0.0
0.5
1.0
1.5
2.0
t = 10sec
t = 30sec
t = 90sec
t = 330sec
t = 3290sec
t = 15290sec
final
initial

The oscillations could be due to the initial timestep being too small:

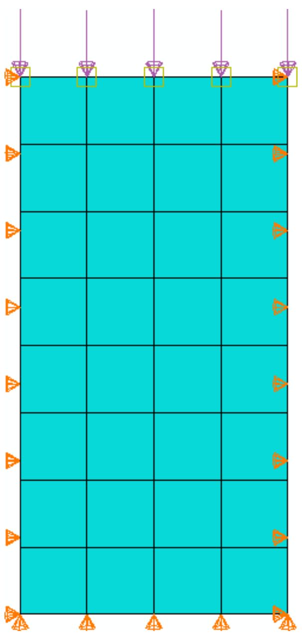
$$\Delta t \geq \frac{\gamma_w \left(\Delta l\right)^2}{6 \cdot E \cdot k} \longrightarrow \Delta t \geq \frac{10 \cdot 0.25^2}{6 \cdot 10000 \cdot 10^{-7}} \longrightarrow \Delta t \geq 104sec$$

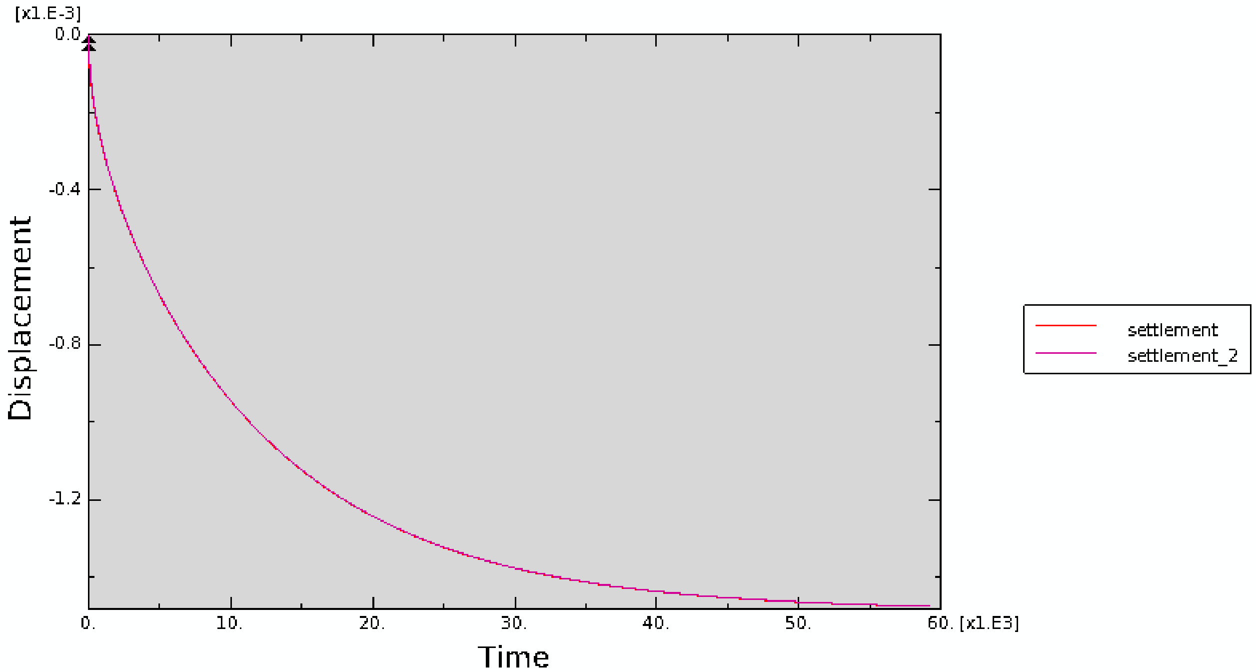
while we used $\Delta t_{ini} = 10sec$.

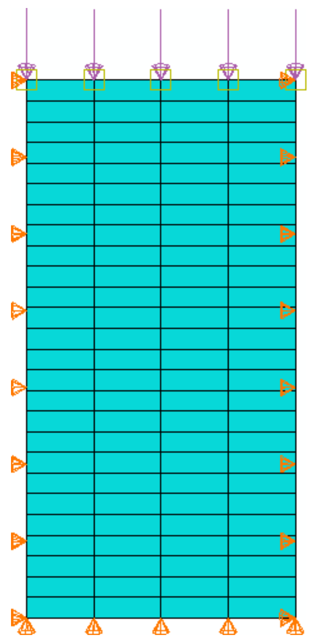
To eliminate the oscillations we need to either increase the initial timestep or refine the mesh.

Let us assume that we indeed want to use $\Delta t_{ini} = 10sec$ for the initial parts of the consolidation process, to have results available. Then from the above $\Delta l \leq 7.75 \cdot 10^{-2}m$, i.e. we need at least 26 elements along the depth of $2m$.

Results from a model with refined mesh 4×26 (named "Column_2") are presented in the following.

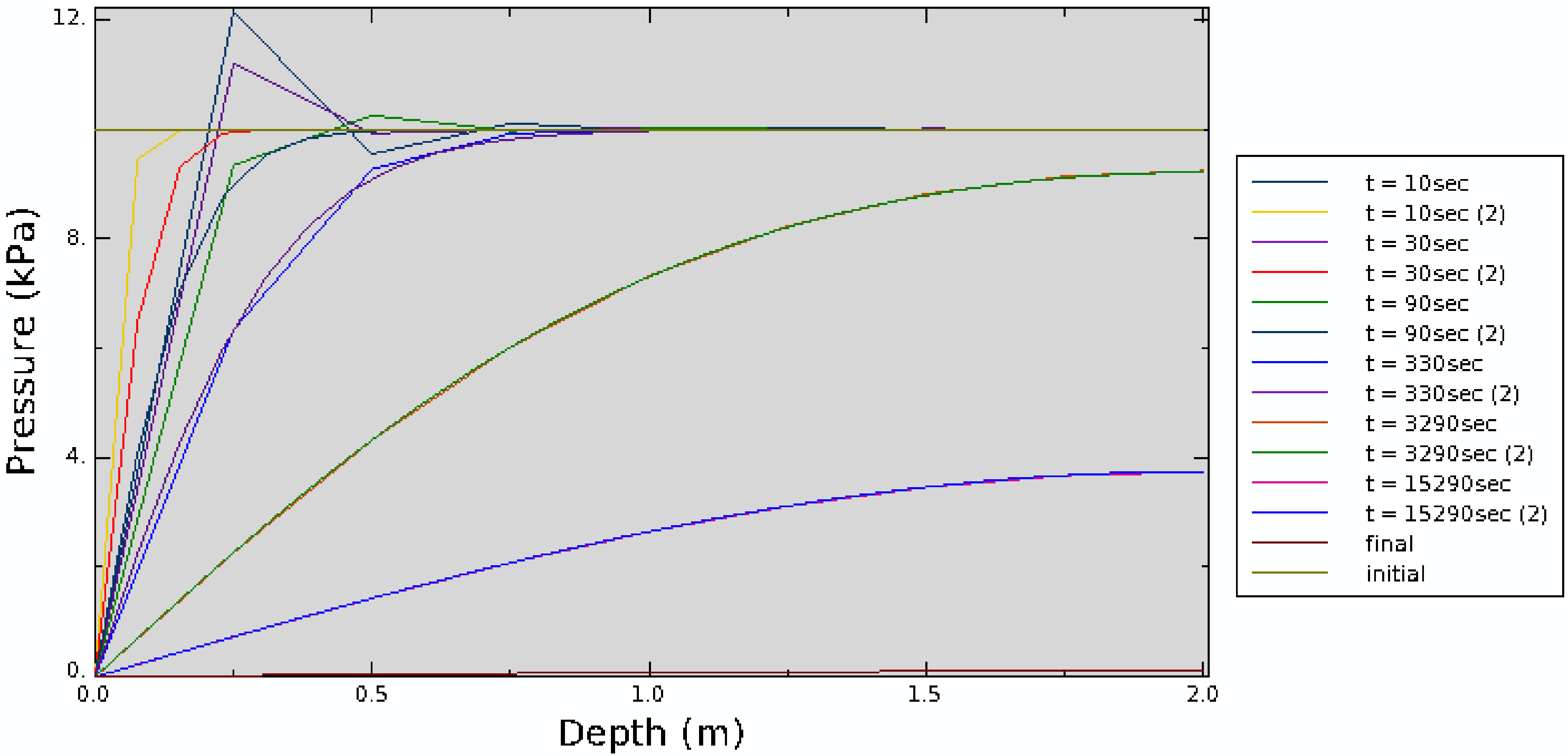






No change in the predicted settlement vs. time.

Isochrones of excess pore pressure



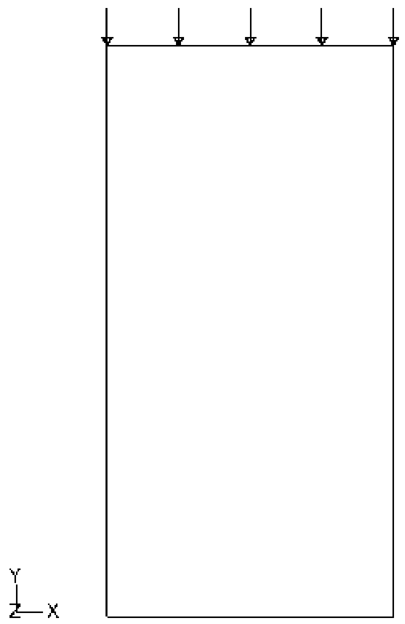
Early isochrones no longer exhibit oscillations.

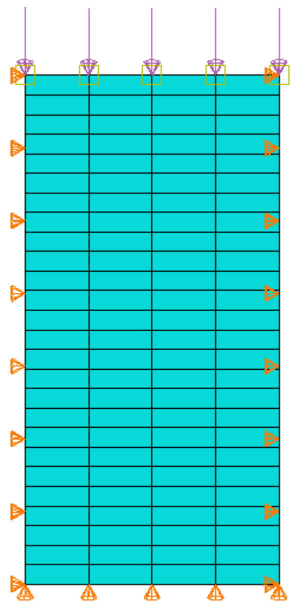
Example: 1D consolidation revisited

A $2m$-thick soil layer undergoes 1D consolidation under a load of $100kPa$, which is applied gradually over the first $2000sec$. The soil has permeability $k = 10^{-7}m/sec$ and is linear elastic ($E = 10MPa$ and $\nu = 0.3$).

- Determine the settlement vs time curve of the layer.
- Plot profiles of excess pore pressure vs depth.

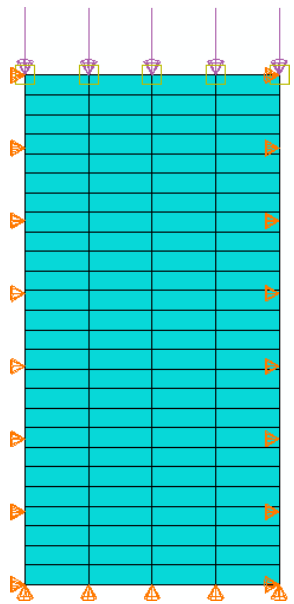
For convenience, assume that the initial pore pressure is zero throughout.





Setting up the analysis:

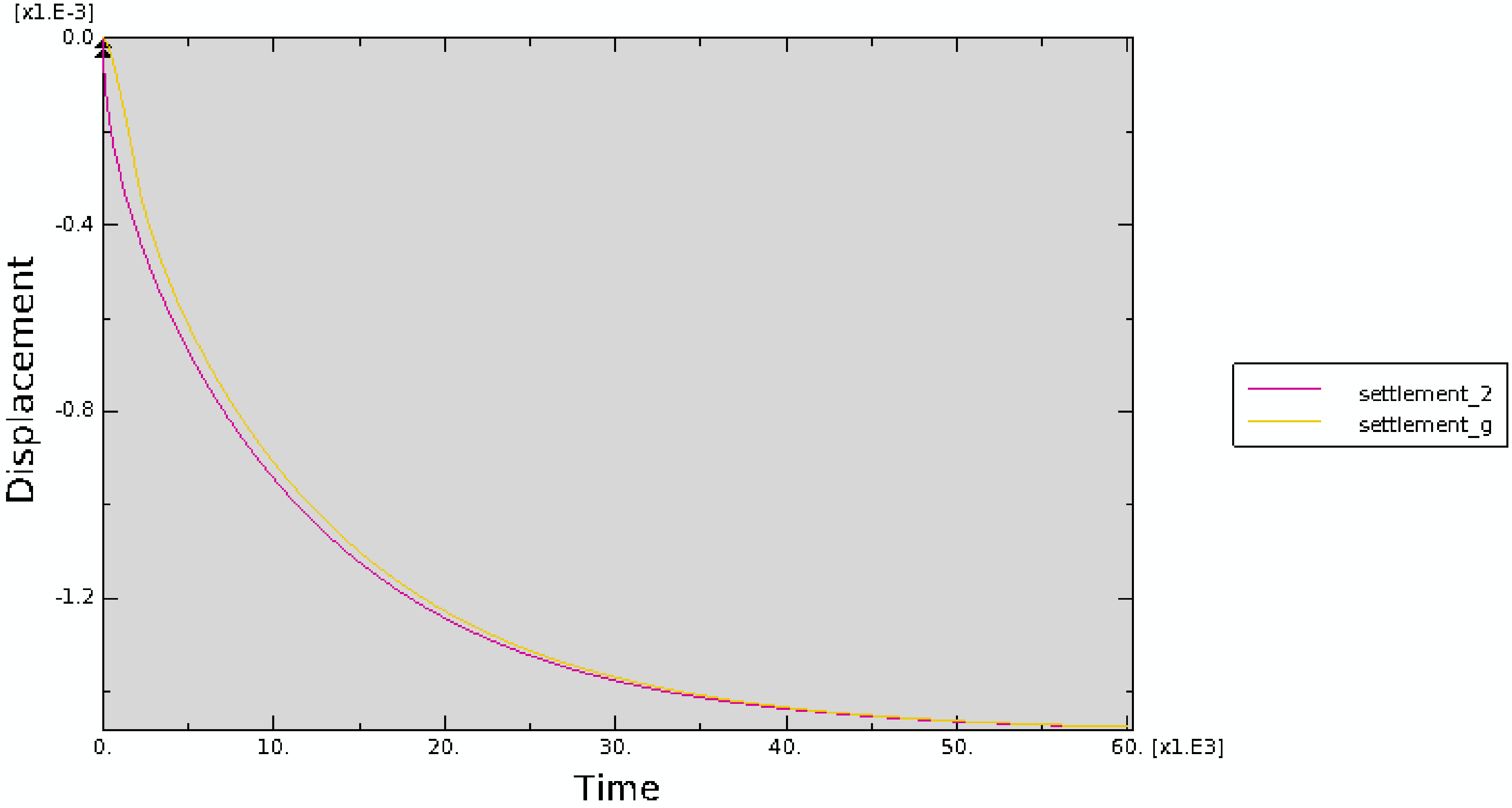
- Geometry and material properties are trivial to enter.
- So is the mesh and the supports.
- We also prescribe a uniform initial void ratio (e.g. $e = 1.0$).
- Parameters requiring further thought:
  - Application of load? Drainage/pore pressure?



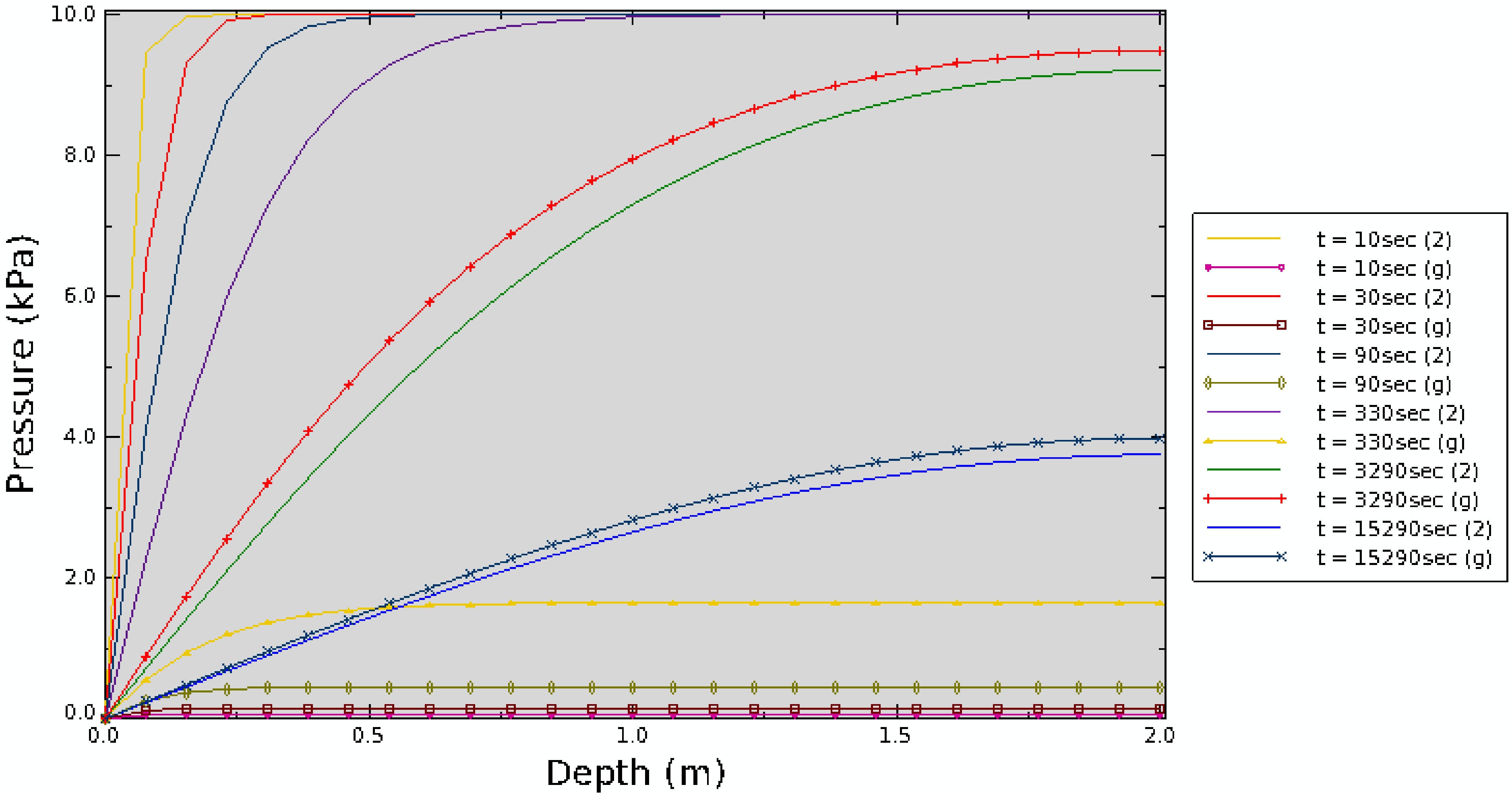
The external load increases gradually, as consolidation takes place.

- No need for a first analysis step to set up uniform excess pore pressure.
- The drainage boundary condition should be active from the outset.
- The load should be applied with an *AMPLITUDE to tell ABAQUS that it should be ramped up over $2000 sec$.

We use the same fine mesh as in "Column_2". Results from the new model, named "Column_gradual", are presented in the following.



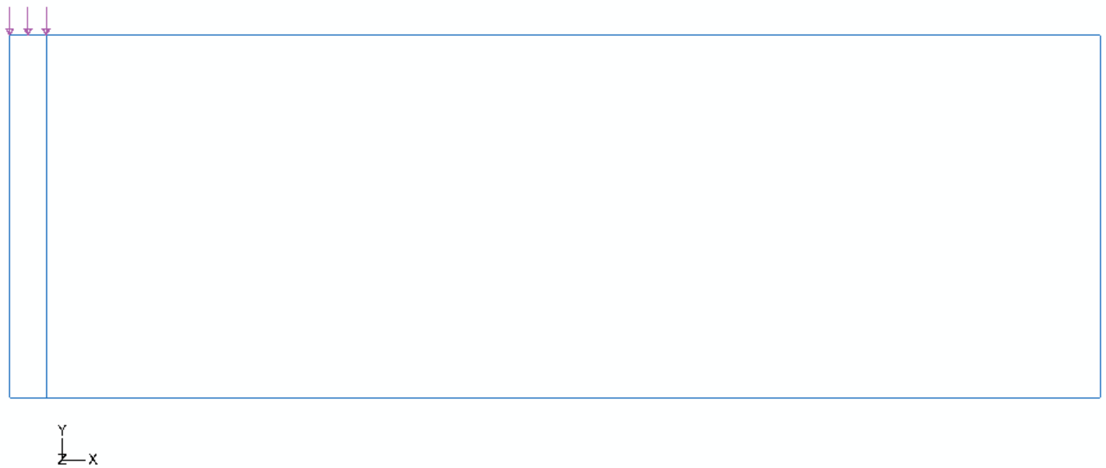
More gradual development of settlement vs. time.



Isochrones show slow ramping up of pore pressure, attainment of a maximum and dissipation.

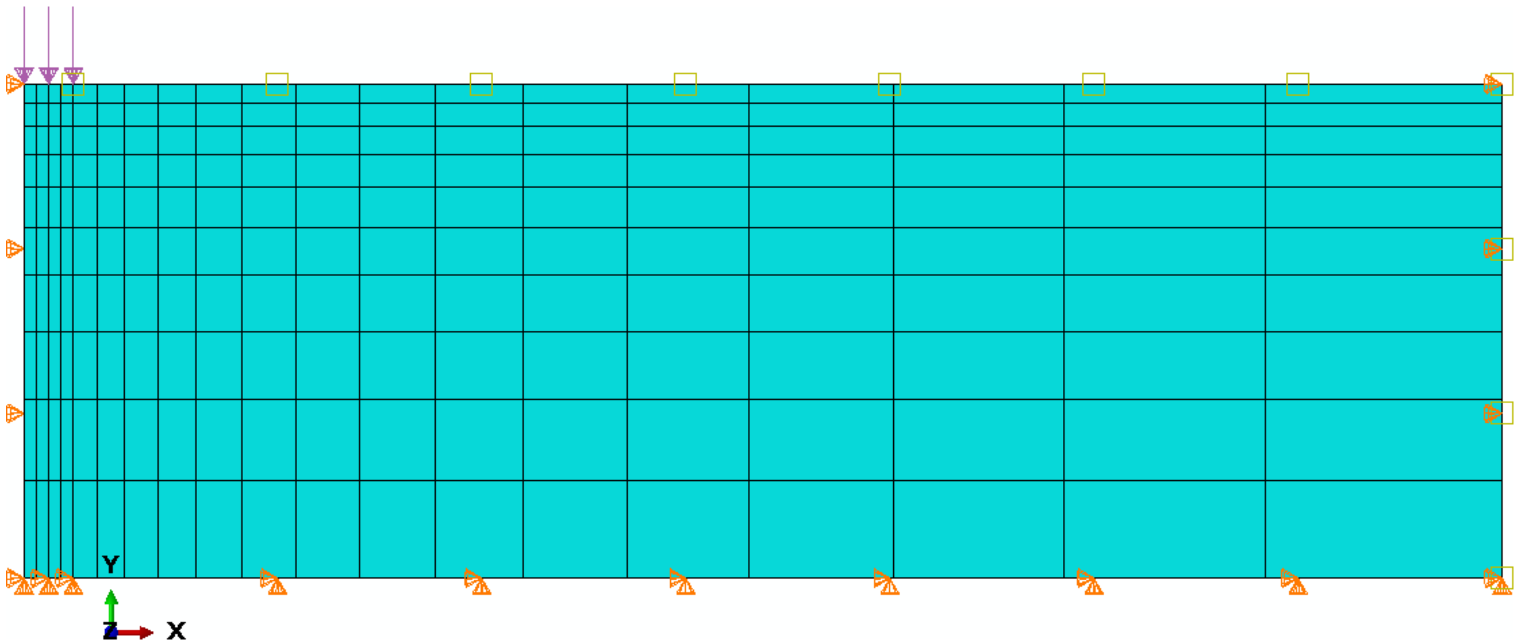
Assume a $2m$-wide footing on a $10m$-thick layer of soil consolidating under a load of $100kPa$. The soil is underlain by impermeable bedrock. The load is applied over the first 5 days and remains constant thereafter. The soil has permeability $k = 10^{-9}m/sec = 8.64 \cdot 10^{-5}m/day$ and is linear elastic with $E = 50MPa$ and $\nu = 0.3$. Pore pressures are initially hydrostatic.

- Determine the settlement vs time curve of the layer.
- Plot key contours of excess pore pressure.



Setting up the analysis (model “Footing” in the “Consolidation.cae” database):

- Geometry, material properties, mesh and supports are trivial to enter.
  - Due to symmetry only the right part of the geometry is modelled.
- For convenience we assume weightless soil and zero initial pore pressures and effective stresses: This assumption does not affect the results.
  - We calculate excess pore pressures and increments of effective stress.
- Application of load? Drainage/pore pressure?
  - Apply load using an *AMPLITUDE: ramp up from zero over 5 days.
  - A boundary condition should ensure that excess pore pressures remain zero at the ground surface and “far enough” from the footing.
    - The soil layer will be draining from the top as well as laterally.



Mesh, load and boundary conditions.

- Arrows correspond to displacement boundary conditions in their direction.
- Squares correspond to pore pressure boundary conditions.
- A biquadratic displacement, bilinear pore pressure quadrilateral element was used.

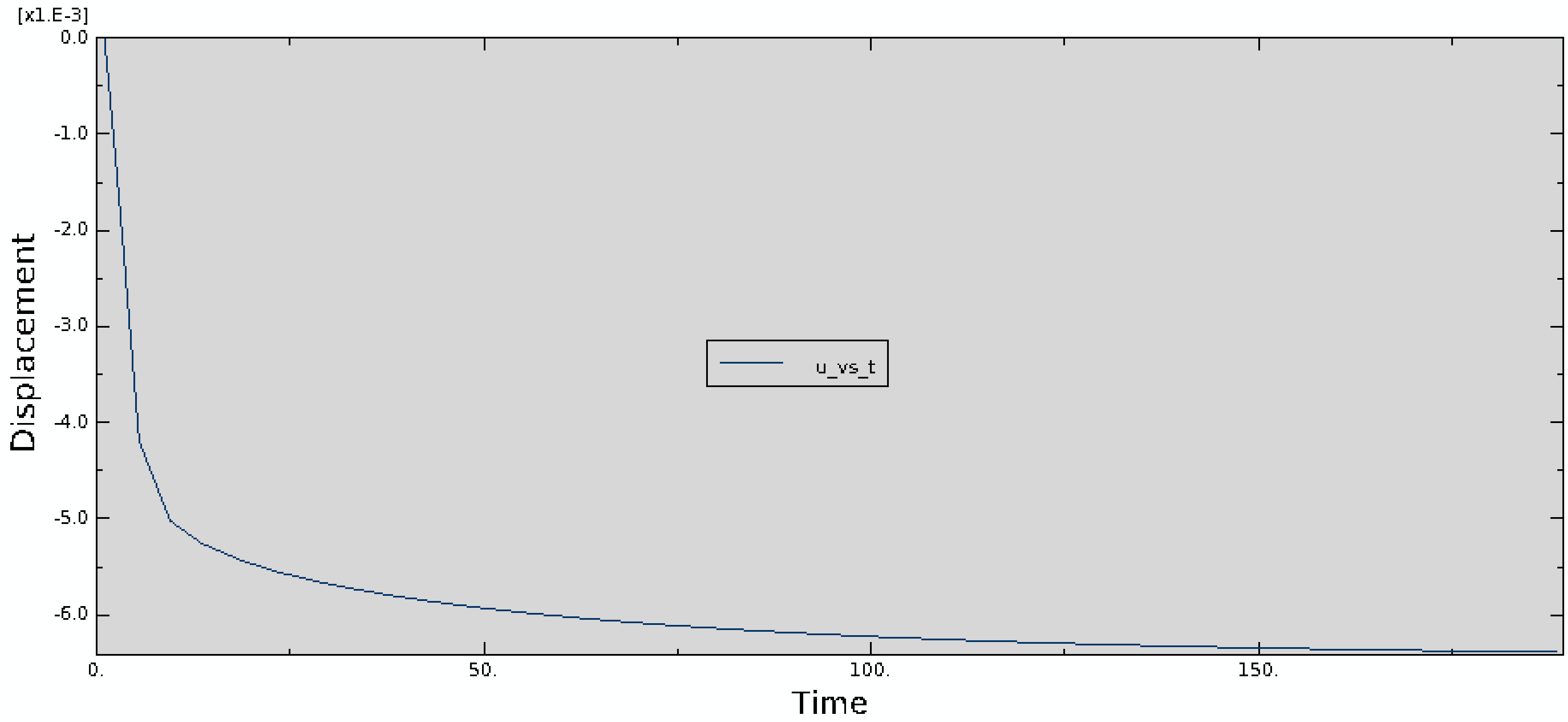
The analysis can be set up in two steps as follows:

- Introduce supports.
- Run a *Geostatic step.

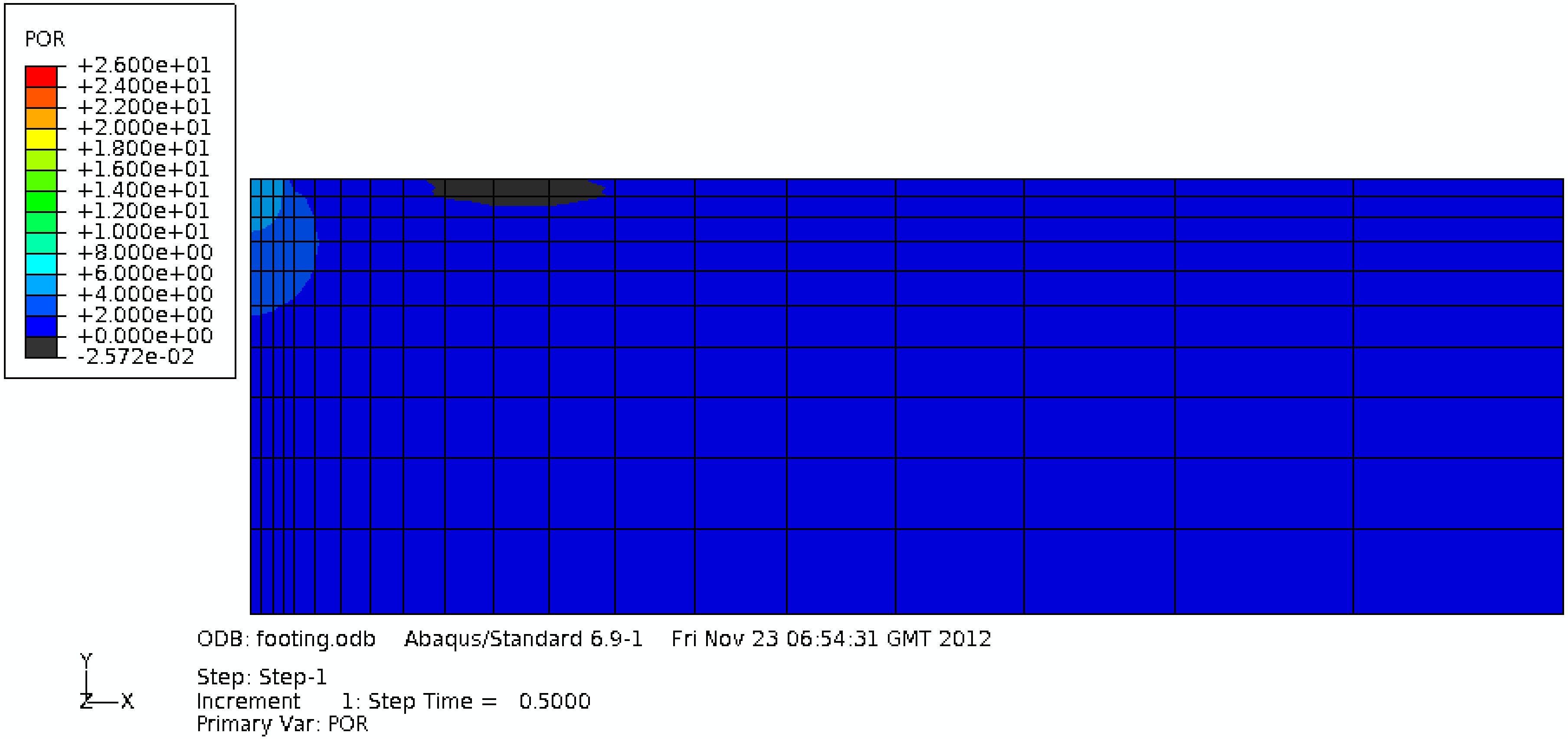
At the end of this step deformation and pore pressure should be zero, as no loads or initial conditions of pore pressure have been prescribed. Then:

- Introduce the load.
- Fix far-field pore pressure to its current (zero) value.
- Fix top-surface pore pressure to its current (zero) value.
- Run a *Soils step to carry out the consolidation analysis.

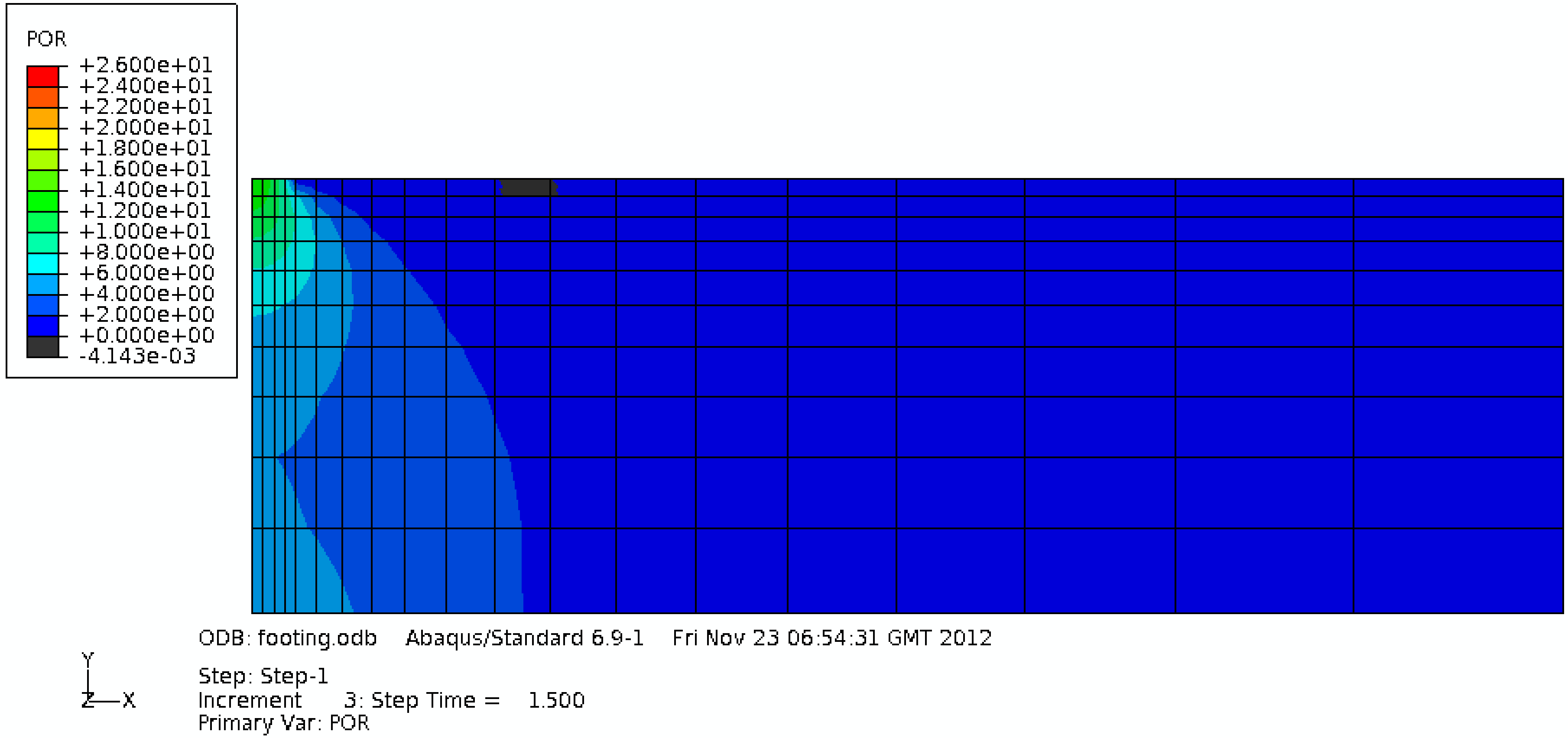
In this case the *Geostatic step could have been omitted. However including it provides a way of checking that the analysis indeed starts from zero, and besides having a first step to set up initial conditions (even trivial ones) is generally a good habit.



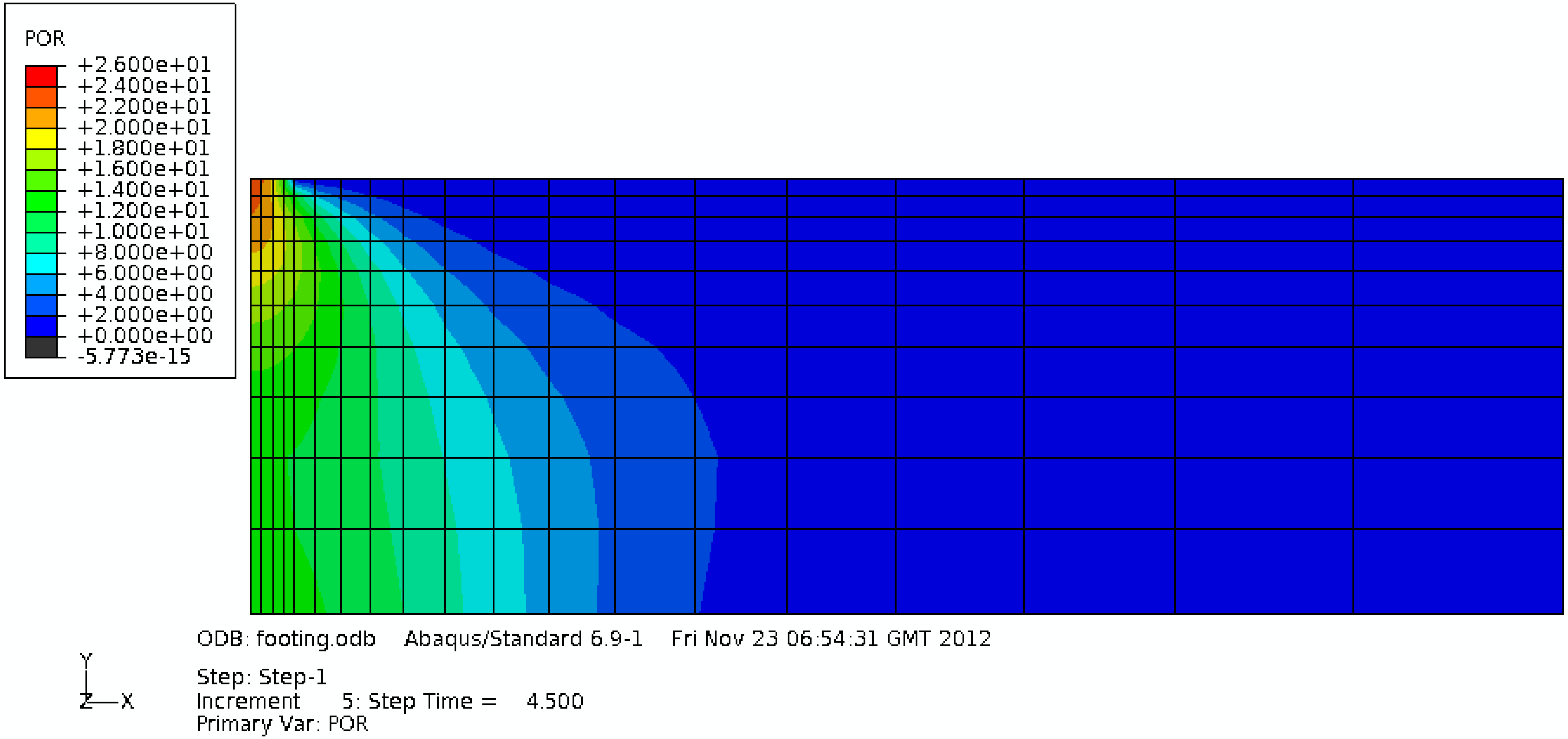
Settlement vs. time curve for the footing.



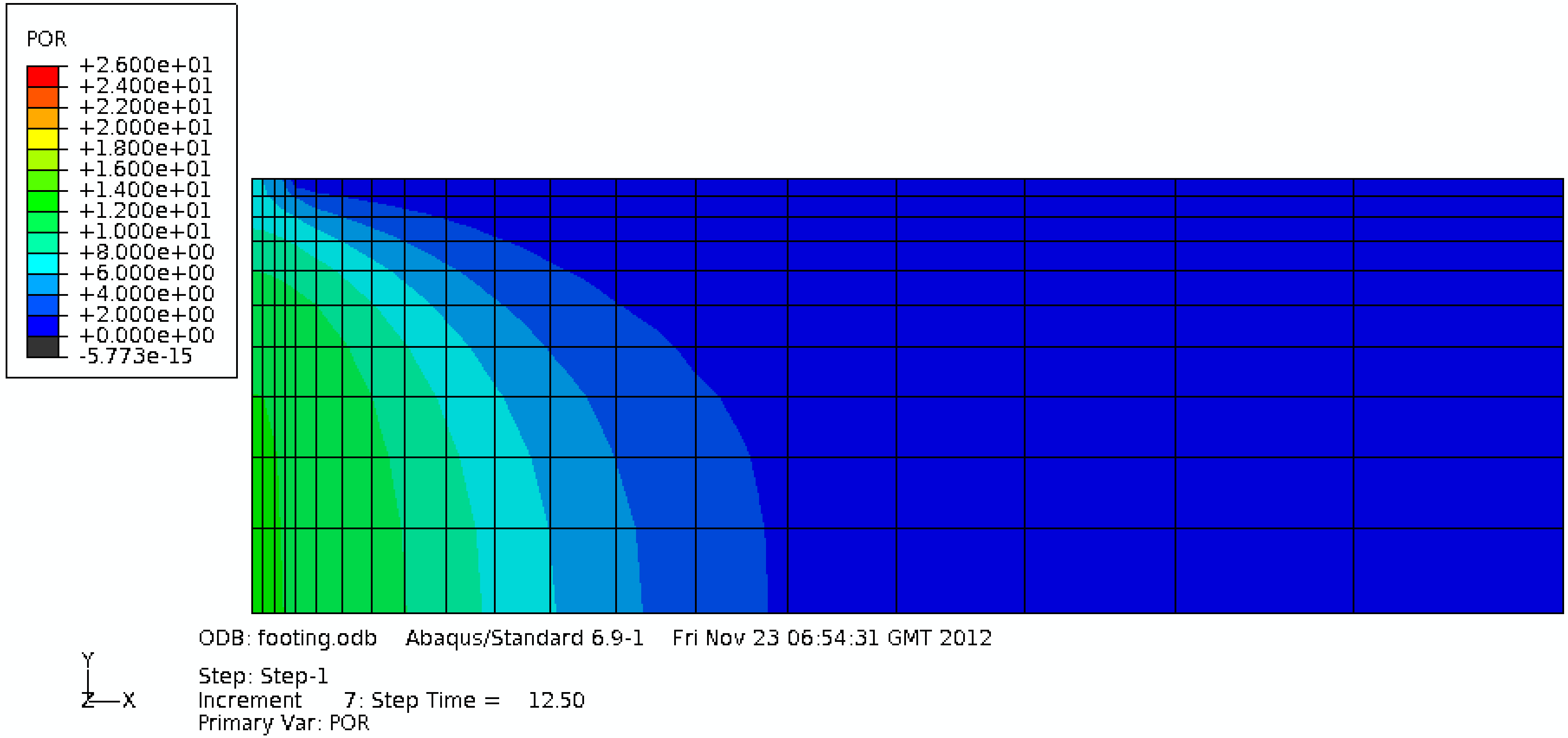
Excess pore pressure contours at t = 0.5 days.



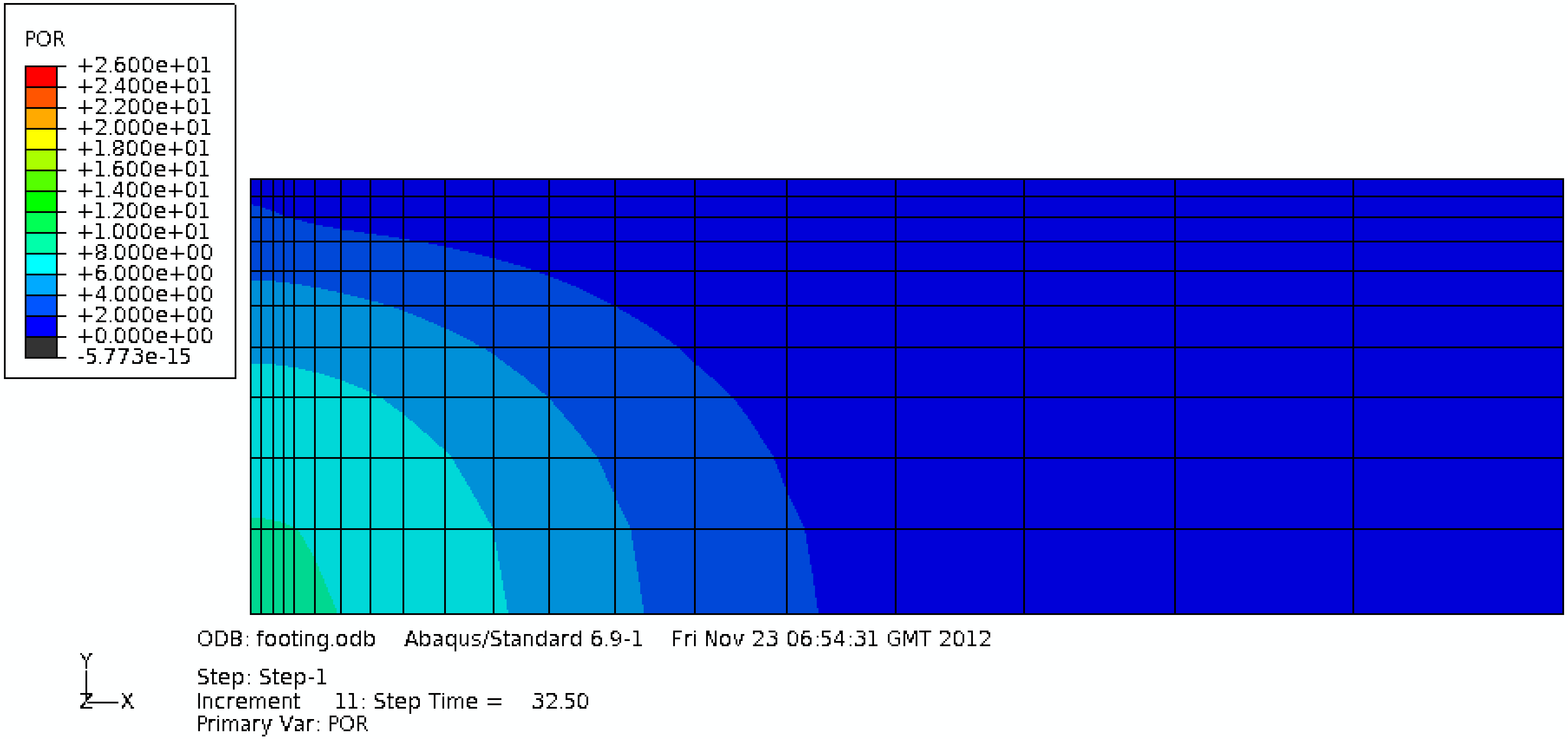
Excess pore pressure contours at t = 1.5 days.



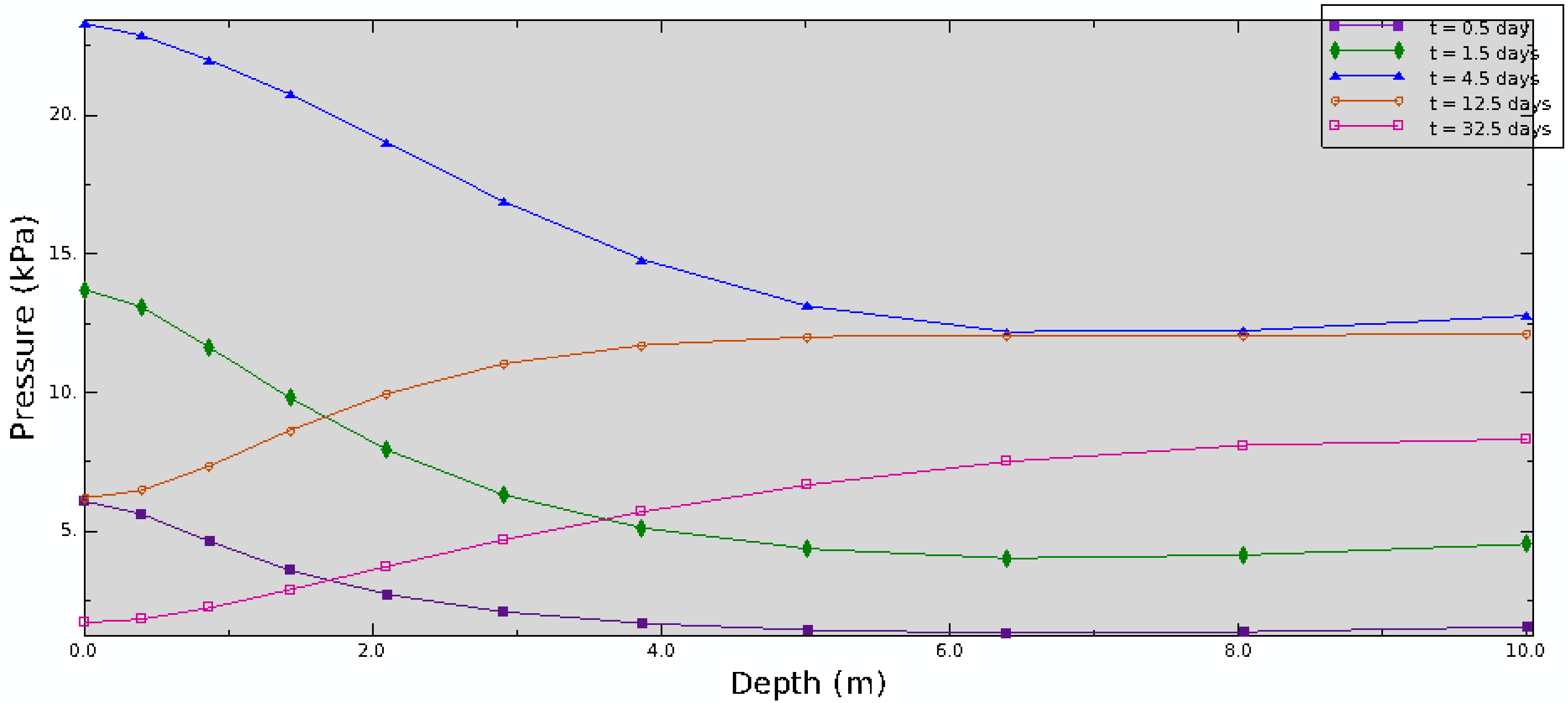
Excess pore pressure contours at t = 4.5 days.



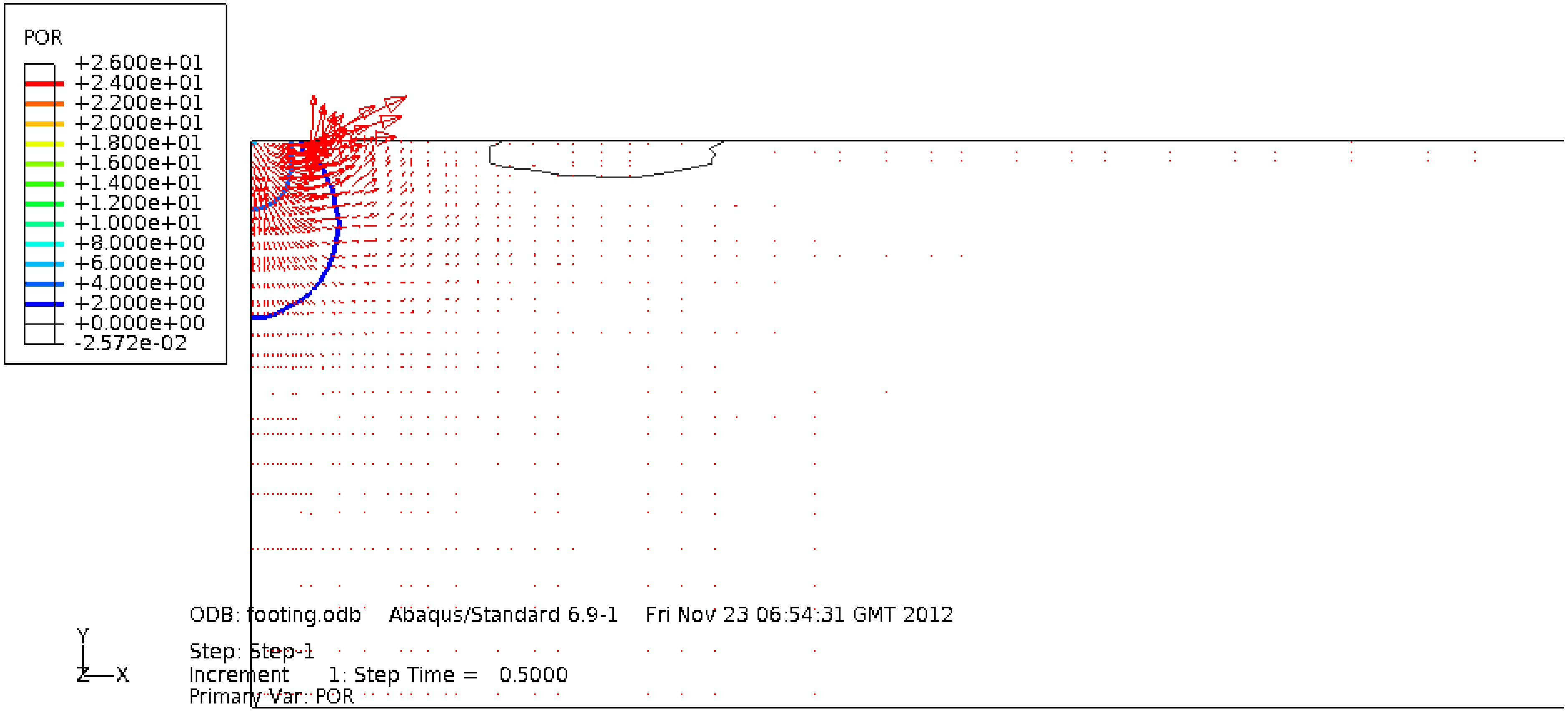
Excess pore pressure contours at t = 12.5 days.



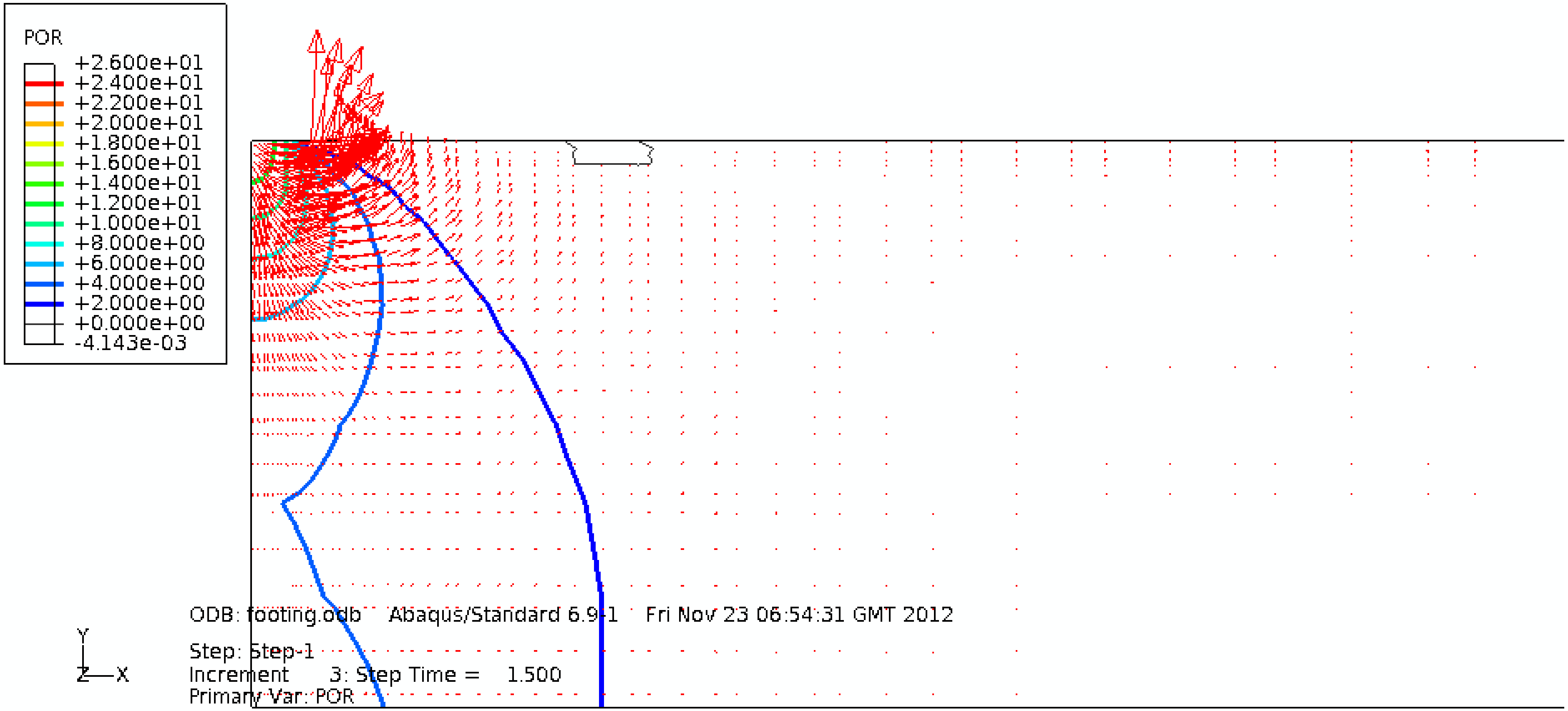
Excess pore pressure contours at t = 32.5 days.



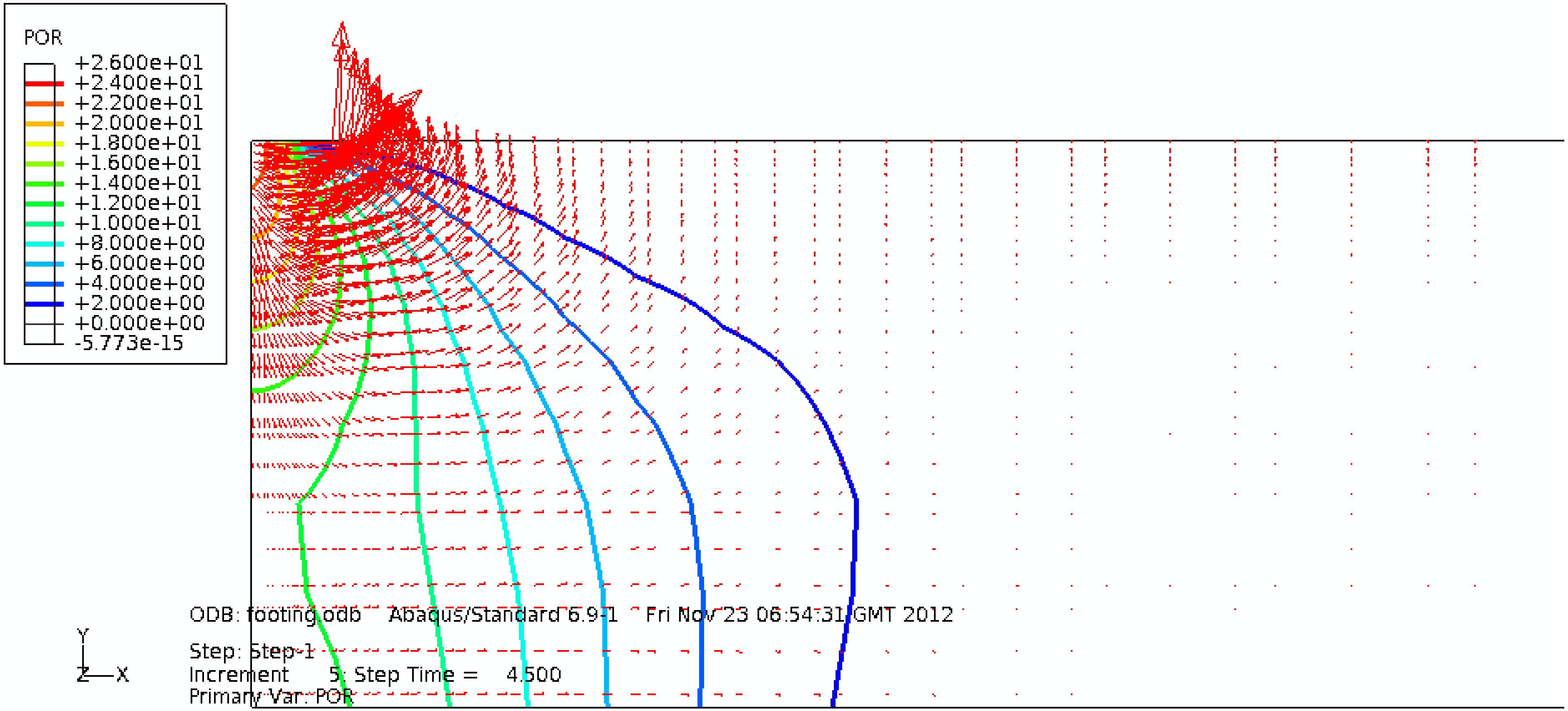
Excess pore pressure vs. depth under the centre at 0.5, 1.5, 4.5, 12.5 and 32.5 days.



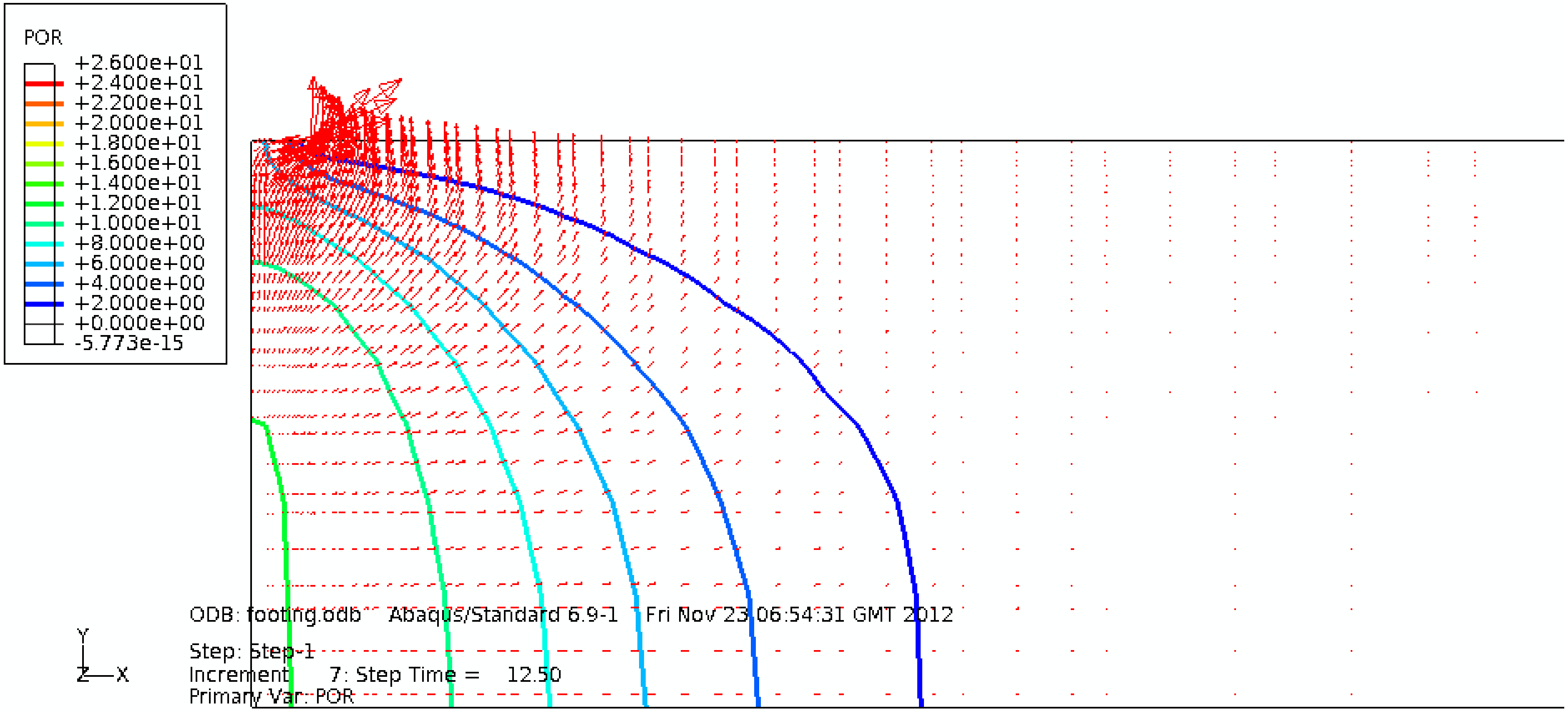
Excess pore pressure contours and flowrates at t = 0.5 days.



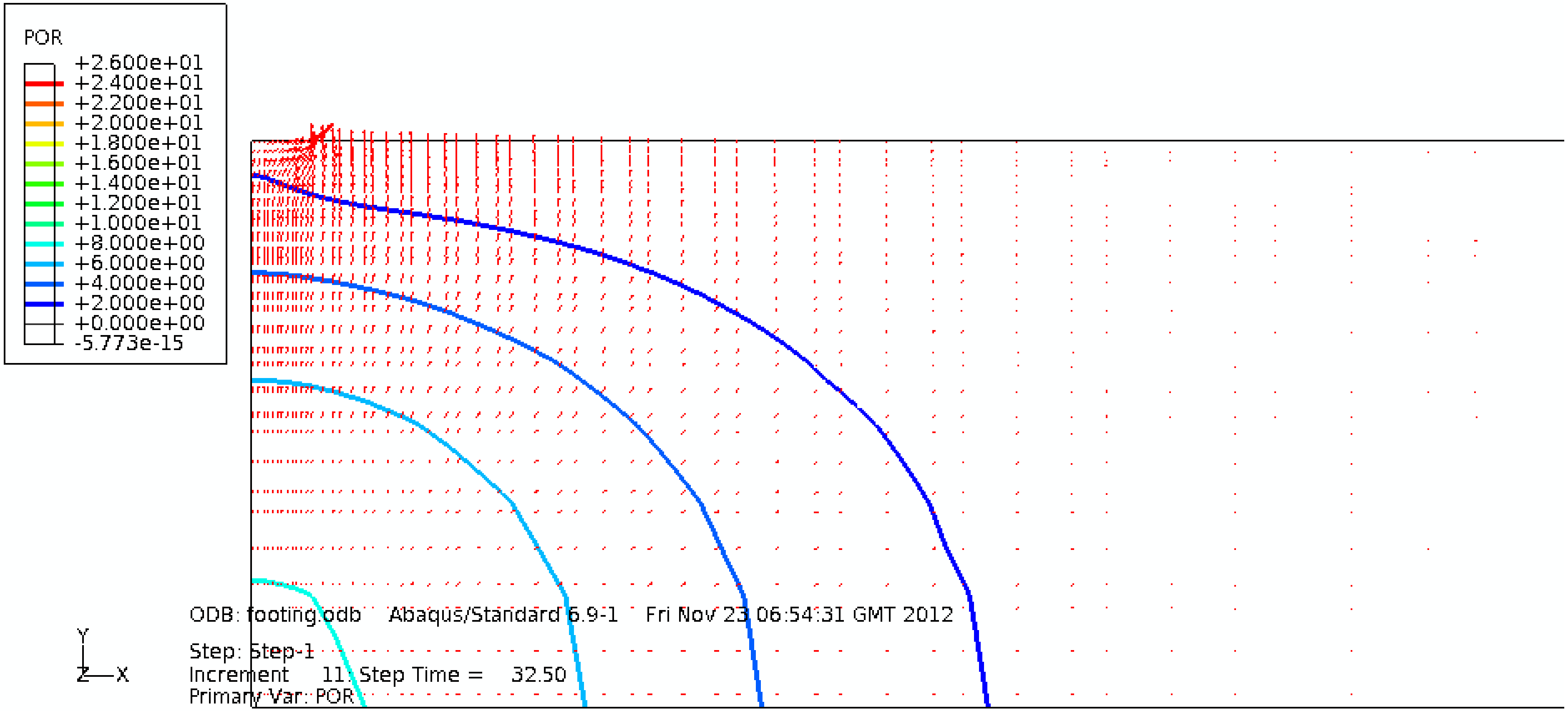
Excess pore pressure contours and flowrates at t = 1.5 days.



Excess pore pressure contours and flowrates at t = 4.5 days.



Excess pore pressure contours and flowrates at t = 12.5 days.



Excess pore pressure contours and flowrates at t = 32.5 days.