Notation table

Set	Species	~~Prope~~ Subset (property)
E	w, PG, Na, Ca, Cl	EF compartment
E_{ions}	Na, Ca, Cl	EF ions
E_{mo}	$E - \{PG\}$	EF **mo**bile species
I_{in}	w, Na, Ca	**In**dependent IF species
I_{ne}	$\{w\} \cup I_{salts}$	**Ne**utral IF species

E: extrafibrillar phase

I: intrafibrillar phase

EF: extrafibrillar phase

IF: intrafibrillar phase

The effective stress is

$$\underset{\sim}{\sigma}' = \underset{\sim}{\sigma} + P_{eff}\,\underset{\sim}{I} = \underset{\approx}{E} : \underset{\sim}{\varepsilon}$$

← constitutive equation of the medium

where

$\underset{\sim}{\sigma}$: the mechanical stress tensor, ~~ref~~ referring to the solid skeleton of the porous medium

P_{eff}: is the effective pressure that brings in the problem (accounts for) the chemical and electrical effects

$\underset{\sim}{I}$: 2nd order unit tensor

$\underset{\approx}{E}$: 4th order stiffness tensor ~~at the~~ of the solid skelton, at the hypertonic state

$\underset{\sim}{\varepsilon}$: strain tensor

Osmotic pressure is

$$\pi_{osm} = P_{wE} - \tilde{\hat{P}}_{wE}$$

where

P_{wE} : the pressure of fluid in the EF ~~ext~~ phase

$\tilde{\hat{P}}_{wE}$: the pressure in the fictitious bath

The effective pressure is

$$P_{eff} = P_I - \underline{X}$$

where

P_I : is the incompressibility pressure

X : is a chemistry and strain dependent scalar

If $\underline{X} = 0$ then $P_{eff} = P_I$ and we ~~isolate~~ recover the mechanical effect, and write the effective stress as

$$\underset{\sim}{\sigma}' = \underset{\sim}{\sigma} + P_I \underset{\sim}{I}$$

which is the well known incompressibility relation for the purely mechanical stress.

When we have chemical effects entering the problem, $\underline{X}$ is non-zero and equal to

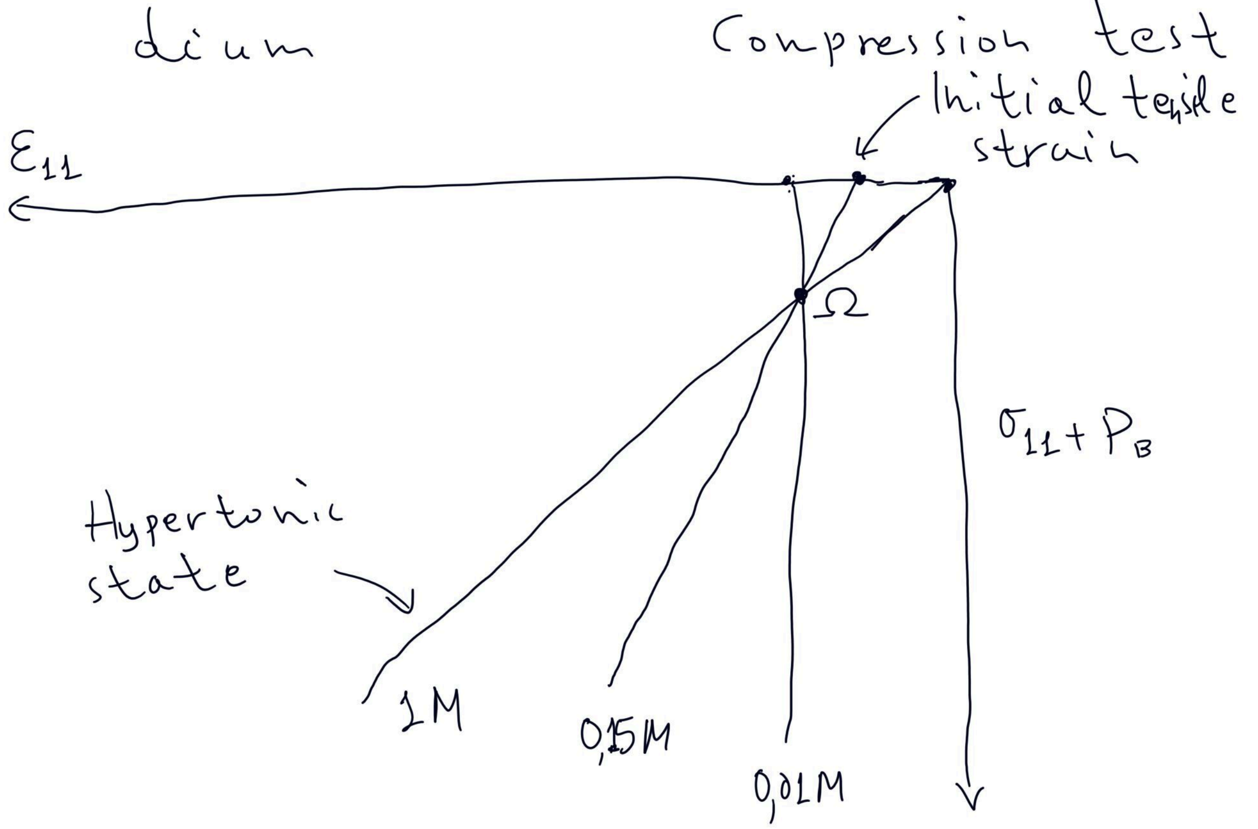
$$\underline{X} = \Lambda(\mathrm{tr}\,\underset{\sim}{\varepsilon} - \mathrm{tr}\,\underset{\sim}{\varepsilon}^{\Omega}) + \pi_{osm} - P_{fw}$$

where

A: slope factor

$\underset{\sim}{\varepsilon}^{\Omega}$: strain tensor at the meeting point of the sress-strain curves, at different concentrations of ions

P_{fw}: pressure regulating the chemical equilibrium of the different fluid phases in the medium



The fluxes equations

The include:

- Darcy's law for seepage of water
- Fick's law for the diffusion of ions
- Ohm's law for the flow of electrical charges

They can be written in matrix form as:

$$\vec{j} = \begin{bmatrix} \vec{J}_{wE} \\ \vec{J}_{NaE} \\ \vec{J}_{CaE} \\ \vec{J}_{ClE} \end{bmatrix}, \qquad \vec{f} = \begin{bmatrix} \varrho_w \vec{\nabla} \mu_{wE} \\ \varrho_{Na} \vec{\nabla} \mu^{ec}_{NaE} \\ \varrho_{Ca} \vec{\nabla} \mu^{ec}_{CaE} \\ \varrho_{Cl} \vec{\nabla} \mu^{ec}_{ClE} \end{bmatrix}$$

$\vec{j}$: vector of fluxes

$\vec{f}$: vector of gradients of the electrochemical potentials, weighted by the mass densities of the elements

We have an isotropic diffusion matrix κ, which relates $\vec{j}$ and $\vec{f}$ via the relation

$$\vec{j} = [\kappa]\vec{f}$$

The vector of unknowns, ~~f t~~ to be found via the FE (finite element method) method, is

$$\underline{X} = \begin{bmatrix} \vec{u} \\ \hline \mu^{ec}_{wE} \\ \mu^{ec}_{NaE} \\ \mu^{ec}_{CaE} \\ \mu^{ec}_{ClE} \\ \hline m^{wI} \\ m^{NaI} \\ m^{CaI} \end{bmatrix}
\begin{array}{l} \leftarrow \text{displacement vector} \\ \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} \text{extrafibrillar electrochemical potentials (all scalars)} \\ \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \text{masses of intrafibrillar components (all scalars)} \end{array}$$

In 3d space, we overall have 3 (components of $\vec{u}$) + 7 scalars = 10 unknowns (scalar unknowns), at each node of the FE mesh.

The semi-discrete, ~~non~~-linear, first order equation to be solved, are written as

$$\mathbb{R} = \mathbb{F}^{surf}(\mathbb{X}) - \mathbb{F}^{int}\left(\mathbb{X}, \frac{d\mathbb{X}}{dt}\right) = 0$$

where

$\mathbb{R}$: is the rediduul to be minimised

$\mathbb{F}^{surf}$: is the vector of surface forces, acting on the boundary of the medium, from the exterior.

$\mathbb{F}^{int}$: is the vector of internal forces developing in the interior of the medium.

Experimental setup of the evaluation of the accuracy of the FE solutions