Initial and boundary conditions of the 1d problem along the $x_1 \equiv x$ direction.

x_1 is perpendicular to x_3

For the solid phase displacement u_s, s:solid, along the x direction, we have the initial condition

$$u_s(x,t) = 0 \text{ at } t=0 \quad \text{or} \quad u_s(x,0) = 0 \quad (1)$$

for all $-\ell \leq x \leq +\ell$ where ℓ is the half of the thickness of the strip.

For u_s we have the boundary condition

$$u_s(x=0,t) = 0 \quad (2)$$

due to the symmetry in the geometry and the uniformity of the chemical "loading".

For the normal stress (effective stress) along the the x direction, we have the boundary condition

$$\sigma'_{xx} = \sigma'_{11} = \sigma'_{xx}(x = \pm\ell, t) = 0 \Rightarrow$$

$$\Rightarrow \underbrace{H\frac{\partial u_s}{\partial x}}_{\text{pure solid contribution (drained)}} + \underbrace{3K\alpha_c C}_{\text{fluid-solid contribution}} = 0 \quad (3)$$

where

$H = \lambda + 2\mu$, λ, μ : Lamé's constants

$K = \lambda + \frac{2}{3}\mu$

C: concentration of the salt in the bath

α_c: a diffusion coefficient

(3) indicates that we have a partially coupled chemomechanical problem. It's not fully coupled problem since H, K (solid properties) don't depend on α_c (fluid property) and α_c does not depend on H and K.

For the pore pressure, we have the boundary condition

$$P_w(x = \pm\ell, t) = 0 \quad (4)$$

Darcy's law for this problem takes the form

$$K_H \frac{\partial p_w}{\partial x} = \frac{\partial u_s}{\partial t} \quad (5)$$

K_H: hydraulic permeability of the porous medium

The main field equation (single one) takes the form

$$\frac{\partial^2 u_s}{\partial x^2} - \frac{1}{H K_H} \frac{\partial u_s}{\partial t} + \frac{3K}{H} \alpha_c \frac{\partial C}{\partial x} = 0 \quad (6)$$

A separate equation for the pore pressure spatial and temporal evolution is obtained

$$p_w(x,t) = f(H, K, \alpha_c, C(x,t), u_s) \quad (7)$$

A solution for $u_s(x,t)$ is obtained from (6), in terms of $C = C(x,t)$ as well.

The diffusion of the salt in the medium, is uncoupled from the deformation

Diffusion equation: $$\frac{\partial C}{\partial t} - D \frac{\partial^2 C}{\partial x^2} = 0 \quad (8)$$

which has to be solved separately

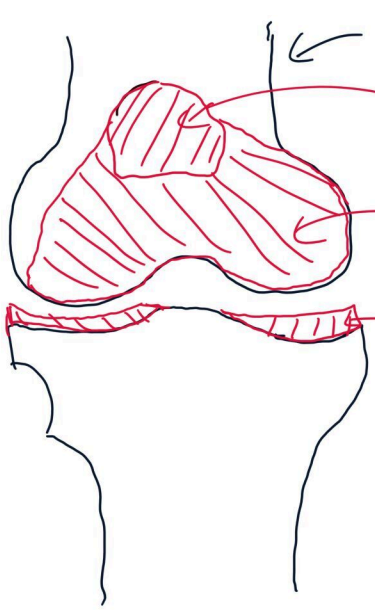
Initial condition for the diffusion problem: $C(x, t=0) = 0$ (9)

Boundary condition: $C(x = \pm l, t) = C_0 \mathcal{H}(t)$

$\mathcal{H}(t)$: Heavyside step function

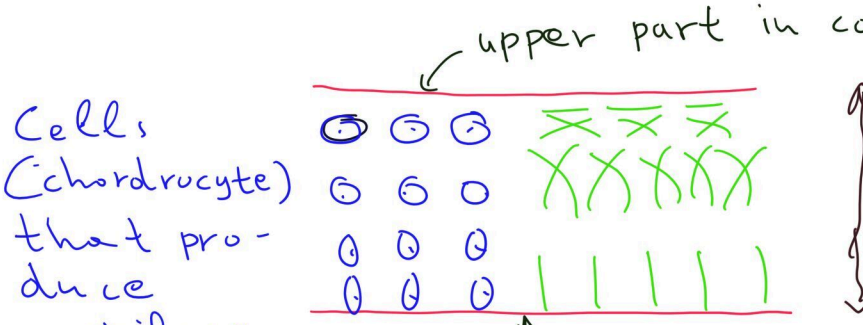
Once $C(x,t)$ is known from the solution of (8), $u_s(x,t)$ and $p_w(x,t)$ can be found from (6) and (7).

Coupled modelling (mechanical + chemical + electrical) of articular cartilage



← Frontal view of the knee joint.

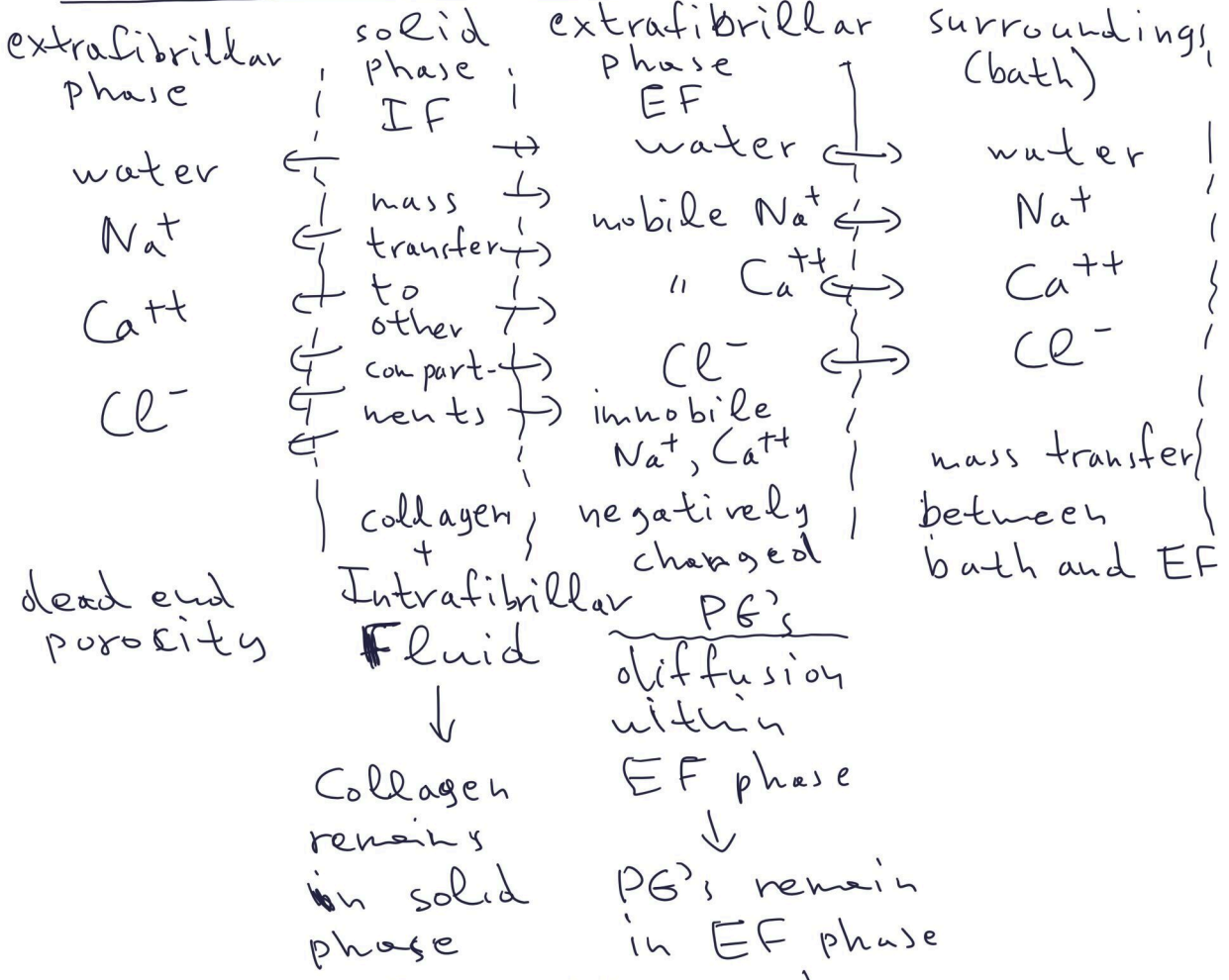
Composition of the cartilage



Collagen fibers :
- Almost horizontal in the upper part, in order to resist to friction due to rotation between the bones.
- Vertical in the lower part in order to anchor in subchondral bone
- At an angle in the middle of the height

Constituents of cartilage are: Collagen, elastin, water, proteoglycans (PG's), ions, e.t.c.

A triphasic model of cartilage



EF: extrafibrillar phase

IF: intrafibrillar phase

Ions cannot move directly from the IF to the bath. They have to move to the EF first.

In living ~~tissue~~ articular cartilage the ~~bath represents a~~ synovial fluid that fills the cartilage space, represents the bath in the experiments.

Measures of the chemical composition

Initial volume of the porous medium: V_0

Current volume of the porous medium (deformed volume): $V = V(t) = V_0 \det \underset{\sim}{F}$

A volume concentration of a species k in the K (capital) phase, per unit deformed

volume, is denoted by

$$n^{kK} = \frac{V_{kK}}{V}, \quad n^K = \sum_{k \in K} n^{kK}, \quad \sum_{K=S,I,E} n^K = 1$$

V_{kK}: volume of species k in the phase K (capital)

S: solid phase ($K = S$)

I: intrafibrillar phase ($K = I$)

E: extrafibrillar phase ($K = E$)

Accordingly we define apparent densitie

$$\rho^{kK} = \frac{M_{kK}}{V} = n^{kK} \rho^k$$

with M_{kK} the mass of constituent k in the phase K, ρ^k the apparent density of costituent k, in the porous medium.

The molar fraction ~~of constituent~~ (concentration) k is written as

$$x_{kK} = \frac{N_{kK}}{N_K}, \quad \sum_{k \in K} x_{kK} = 1, \quad K = I, E, S$$

N: number of moles

The molar concentration per unit volume

$$c_{kK} = \frac{N_{kK}}{V_K}$$