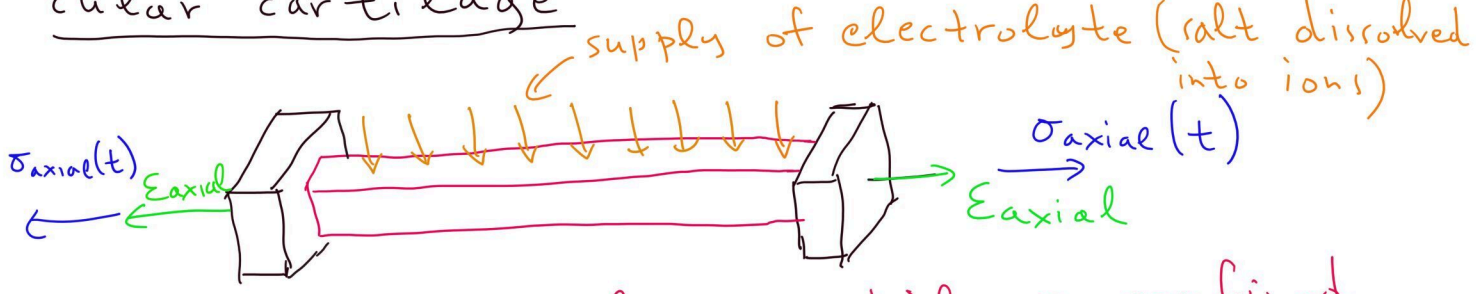
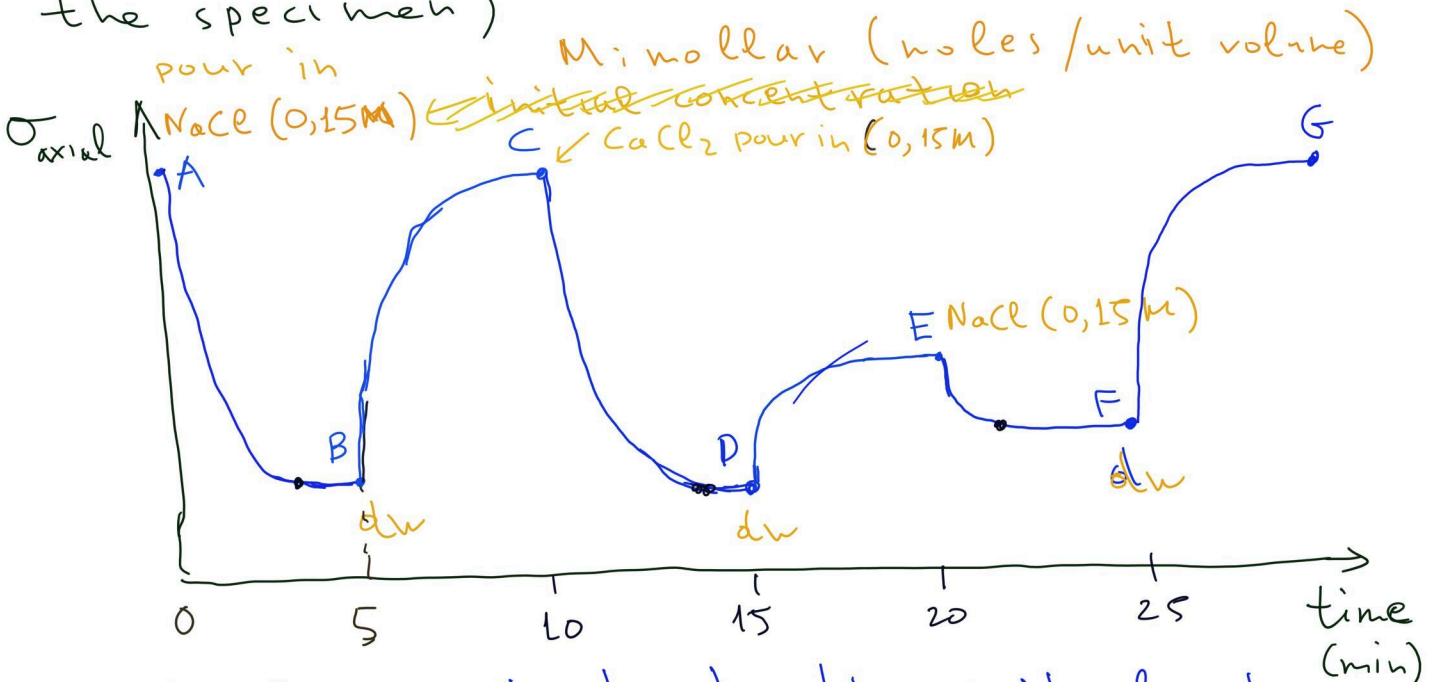
Uniaxial traction experiment of bovine articular cartilage



Strip of articular cartilage confined between two blocks. Applied initially an axial tensile strain of 5%, which is kept constant with time t. The strip is submerged in a bath, whose composition of dissolved species can change.

Graph depicting the change of the axial stress when the chemical composition changes (no external forces are applied on the specimen)



Point G corresponds to the initial stress at point A. The stress level depends on the kind of the electrolyte that is added.

The chemo-poroelastic field and constitutive equations

Balance of mass of the porous medium: $\operatorname{div}\left(n^s \vec{v}_s + n^w \vec{v}_w\right) = 0 \quad (1)$

$\vec{v}_s$: velocity of solid phase
$\vec{v}_w$: velocity of fluid phase
n^s: volume fraction of the solid phase in the porous medium
n^w: volume fraction of the fluid phase in the porous medium

s: solid phase
w: fluid phase
c: chemical substance

Balance of mass of the chemical substance:

$$\frac{\partial C}{\partial t} + \operatorname{div} \vec{J} = 0 \quad (2)$$

$C = C(x,t)$: concentration of the chemical in moles / m^3.
$\vec{J} = C(\vec{v}_c - \vec{v}_w)$: chemical flux
$\vec{v}_c$: absolute velocity of the chemical
Also from Fick's law of diffusion we have

$$\vec{J} = -D \vec{\nabla} C \quad (3)$$

where D is the coefficient of molecular diffusion

Balance of momentum of the solid phase:

$$\operatorname{div} \underset{\sim}{\sigma}_s + k_{sd}(\vec{v}_w - \vec{v}_s) - p_w \vec{\nabla} n^w = 0 \quad (4)$$

Balance of momentum of the fluid phase:

$$\operatorname{div} \underset{\sim}{\sigma}_w - k_{sd}(\vec{v}_w - \vec{v}_s) + p_w \vec{\nabla} n^w = 0 \quad (5)$$

$\underset{\sim}{\sigma} = \underset{\sim}{\sigma}_s + \underset{\sim}{\sigma}_w$: total stress equals the partial stresses in the solid and fluid phases

The partial fluid stress is

$$\underset{\sim}{\sigma}^{w} = -n^{w} p_w \underset{\sim}{I} \quad (6)$$

p_w: intrinsic fluid pressure (preexistent in the porous medium)

$\underset{\sim}{I}$: 2nd order identity tensor

The diffusion equation of the chemical, if we take into account Fick's law, takes the form

$$\frac{\partial C}{\partial t} - D \frac{\partial^2 C}{\partial t^2} = 0 \quad (7)$$

(7): constitutive equation for the diffusion of the chemical

The costitutive equation of the porous medium, is given via

$$\underset{\sim}{\sigma}' = \lambda \operatorname{tr} \underset{\sim}{\epsilon}' \underset{\sim}{I} + 2\mu \underset{\sim}{\epsilon}' \quad (8)$$

where

$\underset{\sim}{\sigma}' = \underset{\sim}{\sigma} + p_w \underset{\sim}{I}$: the effective stress of the po

$\underset{\sim}{\varepsilon}' = \underset{\sim}{\varepsilon} + \alpha_c C \underset{\sim}{I}$: the effective strain

$\underset{\sim}{\varepsilon} = \underset{\sim}{\varepsilon}_s + \underset{\sim}{\varepsilon}_w$: total strain

α_c: coefficient of chemical contraction

λ, μ: Lame constants for the drained solid phase.

Equations (4) and (5) lead to

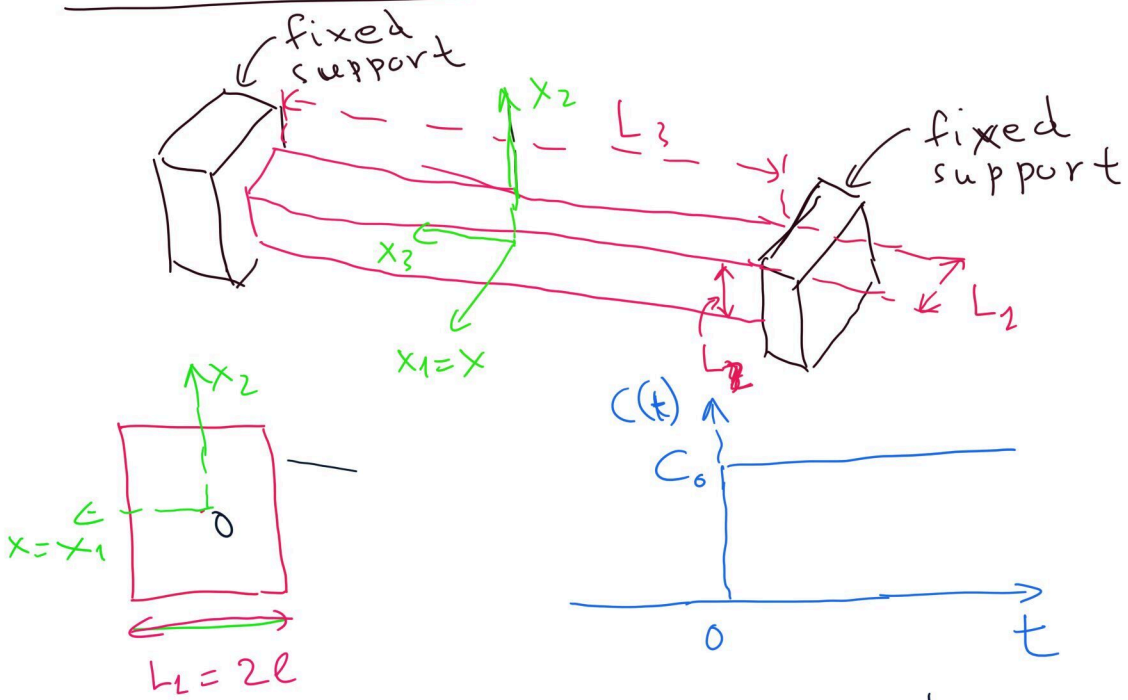
$$\operatorname{div} \underset{\sim}{\sigma} = 0 \quad (9)$$

Also for the flow of water within the porous medium we have Darcy's law

$$n^{w}(\vec{v}_w - \vec{v}_s) = -k_H \vec{\nabla} p_{p_L} \quad (10)$$

K_H: hydraulic permeability of the porous medium

Modelling of the uniaxial deformation of a rectangular specimen of soft tissue due to chemical loading.



We will formulate the equations ~~for the origin O (the middle po~~ for the points $0 \leq x \leq l$, at the horizontal plane in the middle ~~horizontal~~ section, along the length of the specimen. We will have an one-dimensional, ~~in~~ time dependent problem. Independent variables will be (x, t).

A tensile axial stress σ_{33}, along the axial direction is expected to develop after adding salt in the bath where the cartilage strip is submerged in.