Planar distribution of fibers

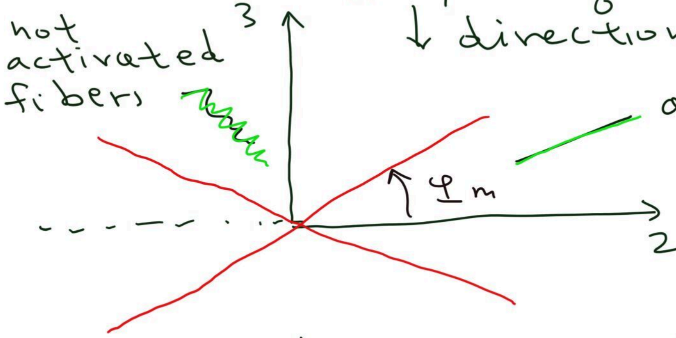
In the plane containing the axes 2 and 3 (axis 1 out of plane)

$$\underrightarrow{m}_c = \begin{bmatrix} 0 \\ \cos\varphi \\ \sin\varphi \end{bmatrix}$$

Doing similar averaging as before, we come up with a relation between the Lame constants and the meterial Property of a single fiber.

$$\lambda^c = \mu^c = \frac{K^c}{8}$$

↓ direction of contraction (along axis 3)

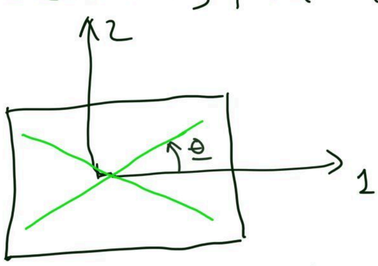


Inside the cones $-\varphi_m \leq \varphi \leq \varphi_m$ we have activated fibers.

The transition boundary from activation to non activation is found from the relation

$$\underset{\sim}{\epsilon} : \underset{\sim}{M}_c \geq 0 \Rightarrow \cot\varphi = \frac{m_{c_2}^2}{m_{c_3}^2} \geq \cot\varphi_m = \frac{-\varepsilon_{33}}{\varepsilon_{22}}$$

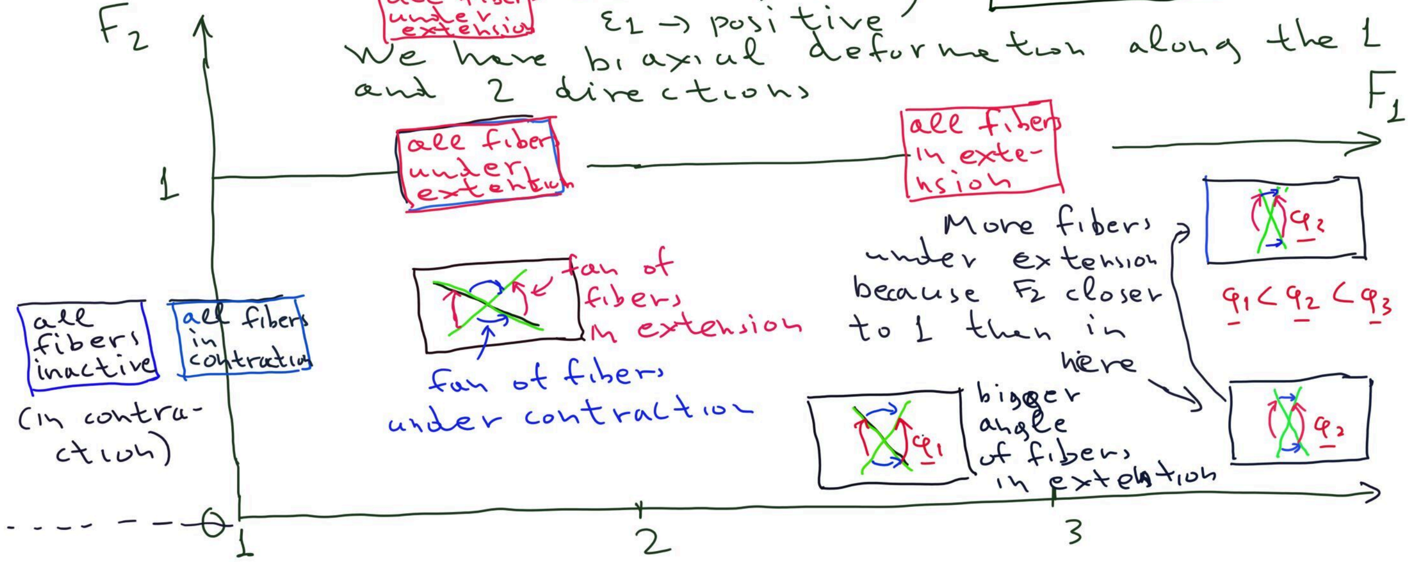
In case of large deformation (strains) a similar problem is the following, in the plane 1–2



$0 \leq F_2 = 1 + \varepsilon_2 \leq 1$: contraction along the 2 direction

$F_1 = 1 + \varepsilon_1 \geq 1$ ($\varepsilon_2 \to$ negative, $\varepsilon_1 \to$ positive)

We have biaxial deformation along the 1 and 2 directions



Relevant calculations for the graph

$\underline{\vec{m}}\ (\cos\underline{\theta}, \sin\underline{\theta}, 0)$: unit normal along each fiber

$$\ell^2 = \underline{\vec{m}} \cdot \underset{\sim}{C} \cdot \underline{\vec{m}} = F_1^2 + \sin^2\underline{\theta}\,(F_2^2 - F_1^2)$$: deformed length of ~~a~~ fiber in the $\underline{\vec{m}}$ direction

$\underset{\sim}{C}$: Right Cauchy Green deformation tensor

$$\underset{\sim}{C} = \underset{\sim}{F}^T \cdot \underset{\sim}{F}$$

$\underset{\sim}{F}$: deformation gradient tensor

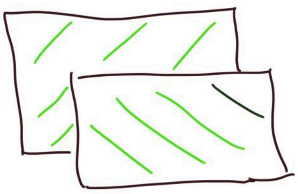
$$\underset{\sim}{F} = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix}$$

The fibers that are in extension have a length > 1 and this leads to the result

$$\ell > 1 \Rightarrow \sin^2\theta_{critical} = \frac{(F_1^2 - 1)}{(F_1^2 - F_2^2)}$$

regarding the angles which seperate active and inactive fibers.

In the stroma part of the cornea we have e.g. two families of fibers



Cornea cosists of a matrix meterial (water with proteins and chemicals), reinforced with fibers

The strain energy density ~~can~~ have an additive form

$$W_{stroma} = W_{matrix}(\underline{\vec{x}}) + \int_0^{\pi} W_{fibril}(\underline{\vec{x}}, \underline{\theta}) D(\underline{\vec{x}}, \underline{\theta})\, d\underline{\theta}$$

$D(\underline{\vec{x}}, \underline{\theta})$: probability density function of the distribution of the fibers.

The fabric tensor $\underset{\sim}{G}$ is a second order symmetric and traceless tensor, which is used ~~the~~ to describe anisotropic, mechanical or transport properties

$$\text{microstructural properties} \xrightarrow{\underset{\sim}{G}} \text{macroscopic level}$$

The probability density function may take the form

$D(\vec{m}) = D_0 + \vec{m} \cdot \underset{\sim}{G} \cdot \vec{m}$

In the plane the above relation may take the form

$$D(\vec{m}) = \frac{1}{2\pi}\left[1 + \hat{G}\cos(\theta - \hat{\theta})\right]$$

$\hat{G}$: eigenvalue of $\underset{\sim}{G}$

$\hat{\theta}$: particular ange related to anisotropy. Also the eigevector of $\underset{\sim}{G}$.

For an isotropic distribution, $\hat{G} = 0$.

The range of the eigenvalues of $\underset{\sim}{G}$, ~~are~~ is

$$-1 < \hat{G} < 1$$

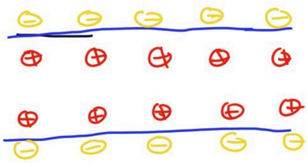
The eigenvectors may be defined by a single angle $\hat{\theta}$

The extreme values of the quantity $2\pi D(\vec{n})$ namely the $1 \pm \hat{G} \in [0,2]$

Couplings (electro-chemo-mechanical) in soft tissues

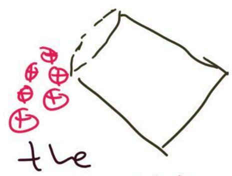
Consentrations of dissolved chemical species (e.g. ions) and associated electrical charges play an important role in the mechanical behavior of soft tissues.

Proteoglycans: Consisting of negatively charges particles ⊖

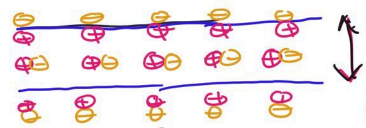


Ions: positevely charged particles (Na^+, Ca^+) or negatively charged particles

Repulsion of negatively charged GAG's (the components of PG's (proteoglycans) Electronutrality in ensured.



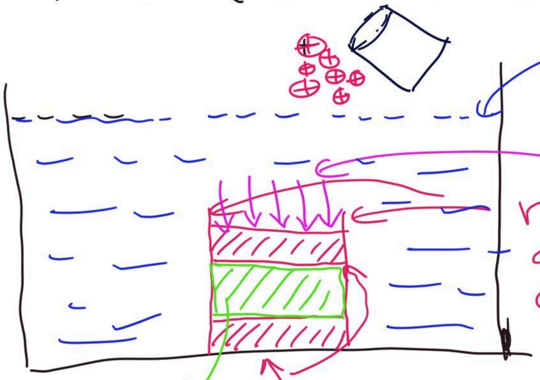
Pour in the mixture positively charged ions



The ability to sustain compressive forces is reduced

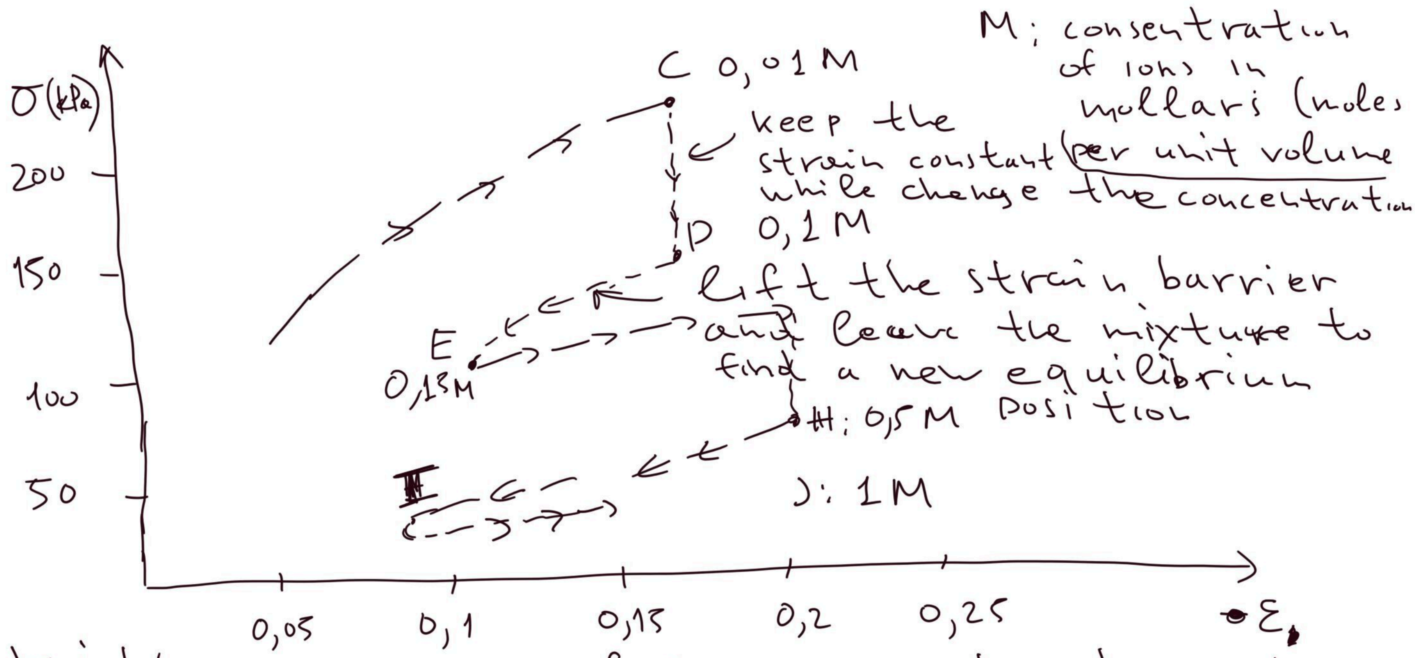
Repulsive forcec due to ⊖, are reduced due to the presnce of the ⊕

Experiment of confined compression of articular cartilage



The composition of the bath is changed by pouring in positively charged ions (e.g NaCl)

Stress-strain curve from the experiment



with increase of ion concentration, the stiffness ~~be~~ decreases. Also under constant strain, the stress decreases. The mixture becomes more flexible (deformable) with the increase in ion concentration.