Aspects of the finite element formulation and illustrative results

In 3d space we have 3 (number of space dimensions) + 7 = 10 field (governing equations), namely:

1) Balance of momentum
2) Balance of mass for the whole medium (related mainly to water transfer)
3) Balance of mass of ions (between EF and IF) phases
4) Equation of mass transfer

The primary unkowns are

1) The displacement vector $\vec{u}$
2) The electrochemical potentials of ions of the extrafibrillar phase μ^{ec}_{kE} (they are scalars)

   k: Kth species (ion)
   E: extrafibrillar compartment
   ec: electrochemical
3) The independent intrafibrillar masses m^{iI}
   i: ith element (species) in the intrafibrillar phase (I). They are also scalars

The unknowns in vector form

$$\underbrace{\vec{X}}_{\text{vector of unkowns}} = \left[\begin{array}{c} \vec{u} \\ \hline \mu^{ec}_{wE} \\ \mu^{ec}_{NaE} \\ \mu^{ec}_{CaE} \\ \mu^{ec}_{ClE} \\ \hline m^{wI} \\ m^{NaI} \\ m^{CaI} \end{array}\right] \begin{array}{l} \rightarrow \text{displacement vector} \\ \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} \rightarrow \text{Extrafibrillar electrochemical Potentials of ions} \\ \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \rightarrow \text{Intrafibrillar masses} \end{array}$$

The final system of equations to be solved the form

$$\vec{R} = \vec{F}^{surf}(\vec{S}, \vec{X}) - \vec{F}^{int}\left(\vec{X}, \frac{d\vec{X}}{dt}\right) = \vec{0} \quad (1)$$

Residual vector (unbalanced)

surface force vector on the boundary of the medium

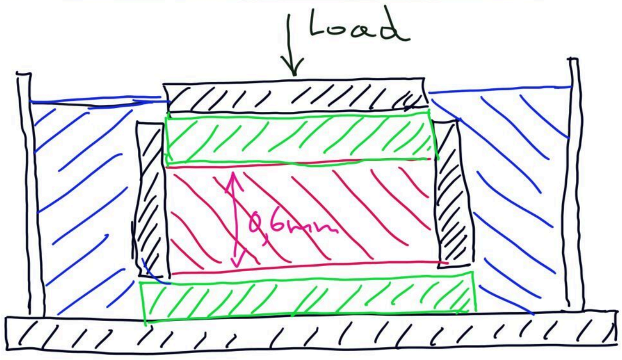
The residual $\vec{R}$ has to be minimised

The system of equations (1) may be nonlineal.

$\vec{S}$: external surface of the medium

Once vector $\vec{X}$ of unkowns has been calulated, all other unknowns can be found (e.g pressures, concentrations of species, volume fractions of species etc.)

Results of experiments modelled by the FE method



Cylindrical specimen of cartilage

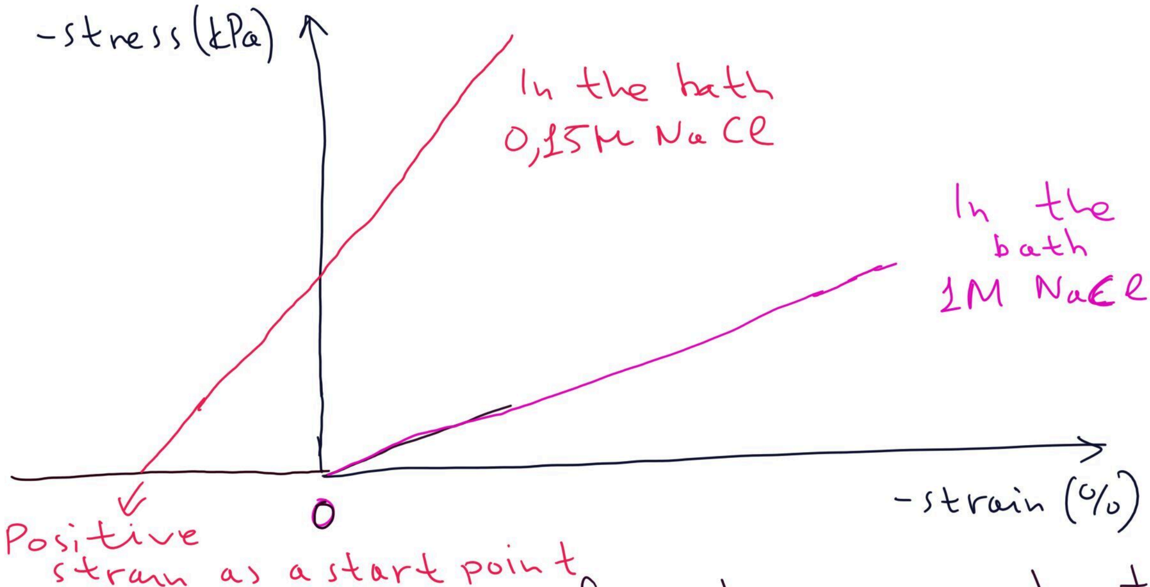
Plates made of porous medium (cylindrical)

Cylindrical plate where a compressive load is applied

Cylindrical tank walls

Bath of varying chemical composition

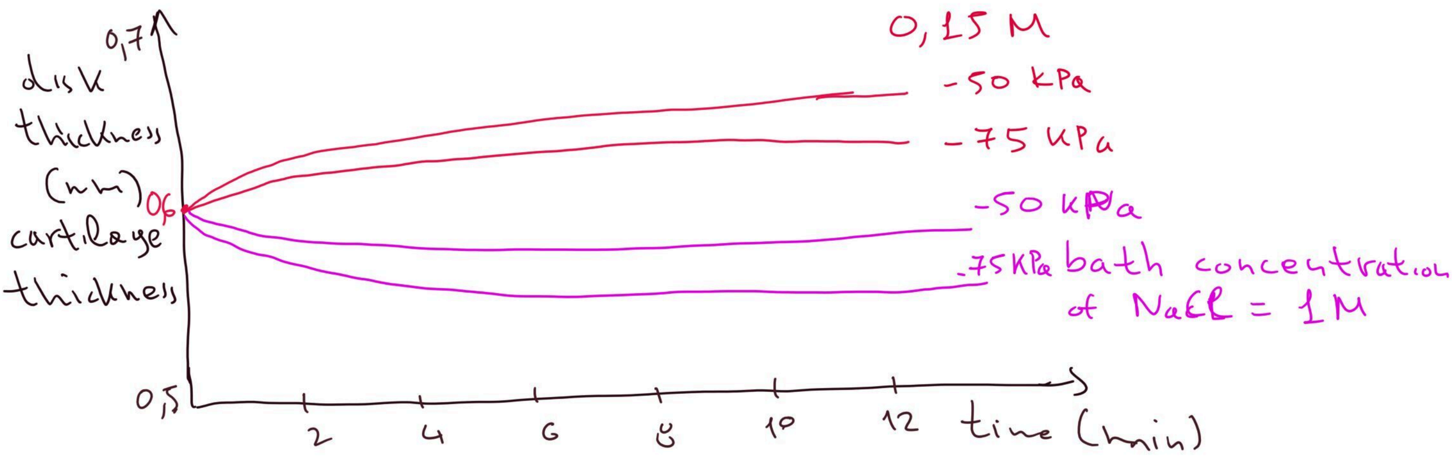
We apply alternate mechanical and chemical loading. Chemical loading is applied by changing the composition of the bath.



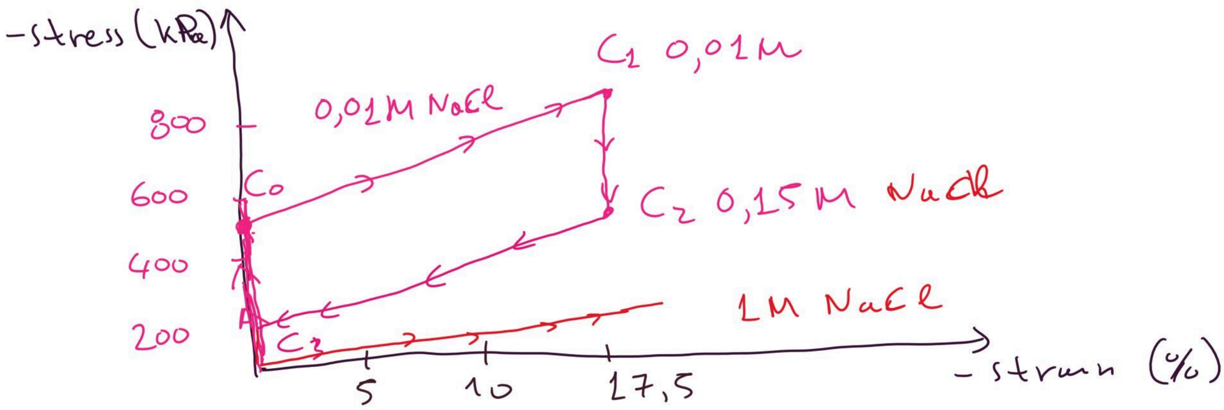
Stress-strain curves for two concentrations of NaCl, namely of 1M ~~...~~ and 0,15M

For low concentration of NaCl, the experiment starts from positive strain, without having applied any stress (any load externally).

Variation of the specimen thickness with time



How the chemical composition change, alters the stress strain curve?



Corresponding stress-time curves