

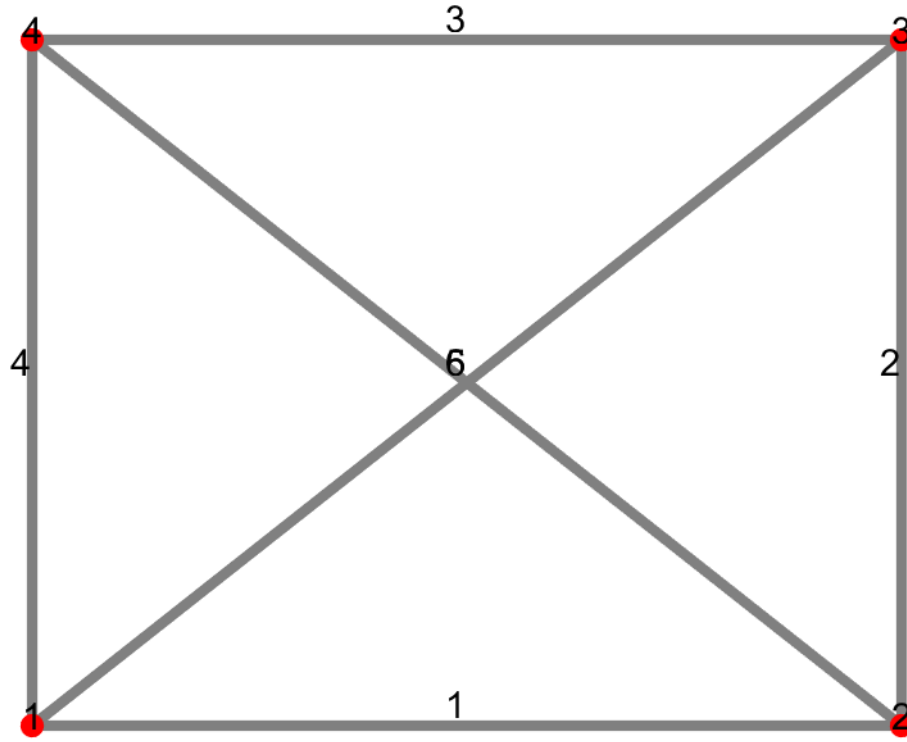


Figure 1: Our Structure to be solved

1 Exercise1b.tex

Our target is to solve the structure shown in Fig. 1 using our Matrix Structural Analysis Methods - Yeeeh!

The element orientation angles are

Element: 1

$$\phi^1 = 0.0^\circ$$

Element: 2

$$\phi^2 = 90.0^\circ$$

Element: 3

$$\phi^3 = 180.0^\circ$$

Element: 4

$$\phi^4 = 90.0^\circ$$

Element: 5

$$\phi^5 = 36.87^\circ$$

Element: 6

$$\phi^6 = 143.13^\circ$$

The element transformation matrices are

$$[\Lambda_{PT}^i] = \begin{bmatrix} \cos \phi^i & \sin \phi^i & 0 & 0 \\ -\sin \phi^i & \cos \phi^i & 0 & 0 \\ 0 & 0 & \cos \phi^i & \sin \phi^i \\ 0 & 0 & -\sin \phi^i & \cos \phi^i \end{bmatrix}$$

Element: 1

$$[\Lambda_{PT}^1] = \begin{pmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1.0 \end{pmatrix} \quad (1)$$

Element: 2

$$[\Lambda_{PT}^2] = \begin{pmatrix} 0 & 1.0 & 0 & 0 \\ -1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 \\ 0 & 0 & -1.0 & 0 \end{pmatrix} \quad (2)$$

Element: 3

$$[\Lambda_{PT}^3] = \begin{pmatrix} -1.0 & 0 & 0 & 0 \\ 0 & -1.0 & 0 & 0 \\ 0 & 0 & -1.0 & 0 \\ 0 & 0 & 0 & -1.0 \end{pmatrix} \quad (3)$$

Element: 4

$$[\Lambda_{PT}^4] = \begin{pmatrix} 0 & 1.0 & 0 & 0 \\ -1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 \\ 0 & 0 & -1.0 & 0 \end{pmatrix} \quad (4)$$

Element: 5

$$[\Lambda_{PT}^5] = \begin{pmatrix} 0.8 & 0.6 & 0 & 0 \\ -0.6 & 0.8 & 0 & 0 \\ 0 & 0 & 0.8 & 0.6 \\ 0 & 0 & -0.6 & 0.8 \end{pmatrix} \quad (5)$$

Element: 6

$$[\Lambda_{PT}^6] = \begin{pmatrix} -0.8 & 0.6 & 0 & 0 \\ -0.6 & -0.8 & 0 & 0 \\ 0 & 0 & -0.8 & 0.6 \\ 0 & 0 & -0.6 & -0.8 \end{pmatrix} \quad (6)$$

The element stiffness matrices at their local coordinate system are

$$[k^i] = \frac{EA^i}{L^i} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (7)$$

Element: 1

$$[k^1] = \begin{pmatrix} 525000.0 & 0 & -525000.0 & 0 \\ 0 & 0 & 0 & 0 \\ -525000.0 & 0 & 525000.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (8)$$

Element: 2

$$[k^2] = \begin{pmatrix} 700000.0 & 0 & -700000.0 & 0 \\ 0 & 0 & 0 & 0 \\ -700000.0 & 0 & 700000.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (9)$$

Element: 3

$$[k^3] = \begin{pmatrix} 525000.0 & 0 & -525000.0 & 0 \\ 0 & 0 & 0 & 0 \\ -525000.0 & 0 & 525000.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (10)$$

Element: 4

$$[k^4] = \begin{pmatrix} 700000.0 & 0 & -700000.0 & 0 \\ 0 & 0 & 0 & 0 \\ -700000.0 & 0 & 700000.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (11)$$

Element: 5

$$[k^5] = \begin{pmatrix} 420000.0 & 0 & -420000.0 & 0 \\ 0 & 0 & 0 & 0 \\ -420000.0 & 0 & 420000.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (12)$$

Element: 6

$$[k^6] = \begin{pmatrix} 420000.0 & 0 & -420000.0 & 0 \\ 0 & 0 & 0 & 0 \\ -420000.0 & 0 & 420000.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (13)$$

The element stiffness matrices at the global coordinate system are

$$[\bar{k}^i] = [\Lambda_{PT}^i]^T [k^i] [\Lambda_{PT}^i] \quad (14)$$

Element: 1

$$[\bar{k}^1] = \begin{pmatrix} 525000.0 & 0 & -525000.0 & 0 \\ 0 & 0 & 0 & 0 \\ -525000.0 & 0 & 525000.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (15)$$

Element: 2

$$[\bar{k}^2] = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 700000.0 & 0 & -700000.0 \\ 0 & 0 & 0 & 0 \\ 0 & -700000.0 & 0 & 700000.0 \end{pmatrix} \quad (16)$$

Element: 3

$$[\bar{k}^3] = \begin{pmatrix} 525000.0 & 0 & -525000.0 & 0 \\ 0 & 0 & 0 & 0 \\ -525000.0 & 0 & 525000.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (17)$$

Element: 4

$$[\bar{k}^4] = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 700000.0 & 0 & -700000.0 \\ 0 & 0 & 0 & 0 \\ 0 & -700000.0 & 0 & 700000.0 \end{pmatrix} \quad (18)$$

Element: 5

$$[\bar{k}^5] = \begin{pmatrix} 268800.0 & 201600.0 & -268800.0 & -201600.0 \\ 201600.0 & 151200.0 & -201600.0 & -151200.0 \\ -268800.0 & -201600.0 & 268800.0 & 201600.0 \\ -201600.0 & -151200.0 & 201600.0 & 151200.0 \end{pmatrix} \quad (19)$$

Element: 6

$$[\bar{k}^6] = \begin{pmatrix} 268800.0 & -201600.0 & -268800.0 & 201600.0 \\ -201600.0 & 151200.0 & 201600.0 & -151200.0 \\ -268800.0 & 201600.0 & 268800.0 & -201600.0 \\ 201600.0 & -151200.0 & -201600.0 & 151200.0 \end{pmatrix} \quad (20)$$

The stiffness matrix of the entire structure is

$$[\bar{K}] = \begin{pmatrix} 793800.0 & 201600.0 & -525000.0 & 0 & -268800.0 & -201600.0 & 0 & 0 \\ 201600.0 & 851200.0 & 0 & 0 & -201600.0 & -151200.0 & 0 & -700000.0 \\ -525000.0 & 0 & 793800.0 & -201600.0 & 0 & 0 & -268800.0 & 201600.0 \\ 0 & 0 & -201600.0 & 851200.0 & 0 & -700000.0 & 201600.0 & -151200.0 \\ -268800.0 & -201600.0 & 0 & 0 & 793800.0 & 201600.0 & -525000.0 & 0 \\ -201600.0 & -151200.0 & 0 & -700000.0 & 201600.0 & 851200.0 & 0 & 0 \\ 0 & 0 & -268800.0 & 201600.0 & -525000.0 & 0 & 793800.0 & -201600.0 \\ 0 & -700000.0 & 201600.0 & -151200.0 & 0 & 0 & -201600.0 & 851200.0 \end{pmatrix} \quad (21)$$

The transformation matrix for the rotated joint constraints is

$$[R_m] = \begin{pmatrix} 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.70711 & 0.70711 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.70711 & 0.70711 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \end{pmatrix} \quad (22)$$

The constraint permutation matrix V is

$$[V] = \begin{pmatrix} 0 & 0 & 1.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \\ 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (23)$$

And the rotated/ permuted stiffness matrix for the entire structure becomes

$$[\bar{K}]_m = [V]^T [R_m]^T [\bar{K}] [R_m] [V] \quad (24)$$

$$[\bar{K}]_m = \begin{pmatrix} 793800.0 & 0 & 0 & -268800.0 & 201600.0 & 201600.0 & -525000.0 & 0 & -201600.0 \\ 0 & 793800.0 & 201600.0 & -525000.0 & 0 & 0 & -268800.0 & -201600.0 & 0 \\ 0 & 201600.0 & 851200.0 & 0 & 0 & 0 & -201600.0 & -151200.0 & -700000.0 \\ -268800.0 & -525000.0 & 0 & 793800.0 & -201600.0 & 0 & 0 & 0 & 201600.0 \\ 201600.0 & 0 & 0 & -201600.0 & 851200.0 & 0 & -700000.0 & -151200.0 & 0 \\ -525000.0 & -268800.0 & -201600.0 & 0 & 0 & 793800.0 & 201600.0 & 0 & 0 \\ 0 & -201600.0 & -151200.0 & 0 & -700000.0 & 201600.0 & 851200.0 & 0 & 0 \\ -201600.0 & 0 & -700000.0 & 201600.0 & -151200.0 & 0 & 0 & 851200.0 & 0 \end{pmatrix} \quad (25)$$

The vector of external nodal loads is

$$\{\bar{P}_{ext}\} = \begin{pmatrix} 0 \\ 0 \\ 100.0 \\ 0 \\ 100.0 \\ 0 \\ 100.0 \\ -100.0 \end{pmatrix} \quad (26)$$

which, when permuted becomes

$$\{\bar{P}_{m,ext}\} = \begin{pmatrix} 100.0 \\ 100.0 \\ 0 \\ 100.0 \\ -100.0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (27)$$

Solving the system of linear equations, the nodal displacements are eventually

$$\{\bar{\Delta}\} = \begin{pmatrix} 0 \\ 0 \\ 0.00023223 \\ 0 \\ 0.00035279 \\ 0.000051485 \\ 0.00036272 \\ -0.000041297 \end{pmatrix} \quad (28)$$

Rotating the nodal displacements back to the global coordinate system results in

$$\{\bar{\Delta}\} = \begin{pmatrix} 0 \\ 0 \\ 0.00016421 \\ 0.00016421 \\ 0.00035279 \\ 0.000051485 \\ 0.00036272 \\ -0.000041297 \end{pmatrix} \quad (29)$$

The deformed configuration of the structure is shown Fig. 2 - We did it!

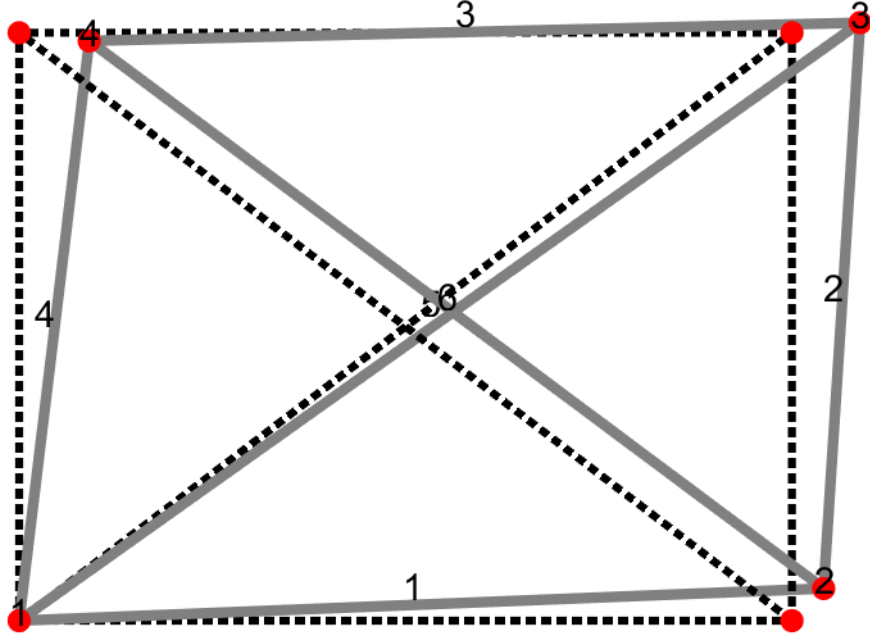


Figure 2: Deformed Configuration

Finally, the element load vectors at their local coordinate system are Element: 1

$$\{F^1\} = \begin{pmatrix} -86.211 \\ 0 \\ 86.211 \\ 0 \end{pmatrix} \quad (30)$$

Element: 2

$$\{F^2\} = \begin{pmatrix} 78.908 \\ 0 \\ -78.908 \\ 0 \end{pmatrix} \quad (31)$$

Element: 3

$$\{F^3\} = \begin{pmatrix} 5.2106 \\ 0 \\ -5.2106 \\ 0 \end{pmatrix} \quad (32)$$

Element: 4

$$\{F^4\} = \begin{pmatrix} 28.908 \\ 0 \\ -28.908 \\ 0 \end{pmatrix} \quad (33)$$

Element: 5

$$\{F^5\} = \begin{pmatrix} -131.51 \\ 0 \\ 131.51 \\ 0 \end{pmatrix} \quad (34)$$

Element: 6

$$\{F^6\} = \begin{pmatrix} 118.49 \\ 0 \\ -118.49 \\ 0 \end{pmatrix} \quad (35)$$