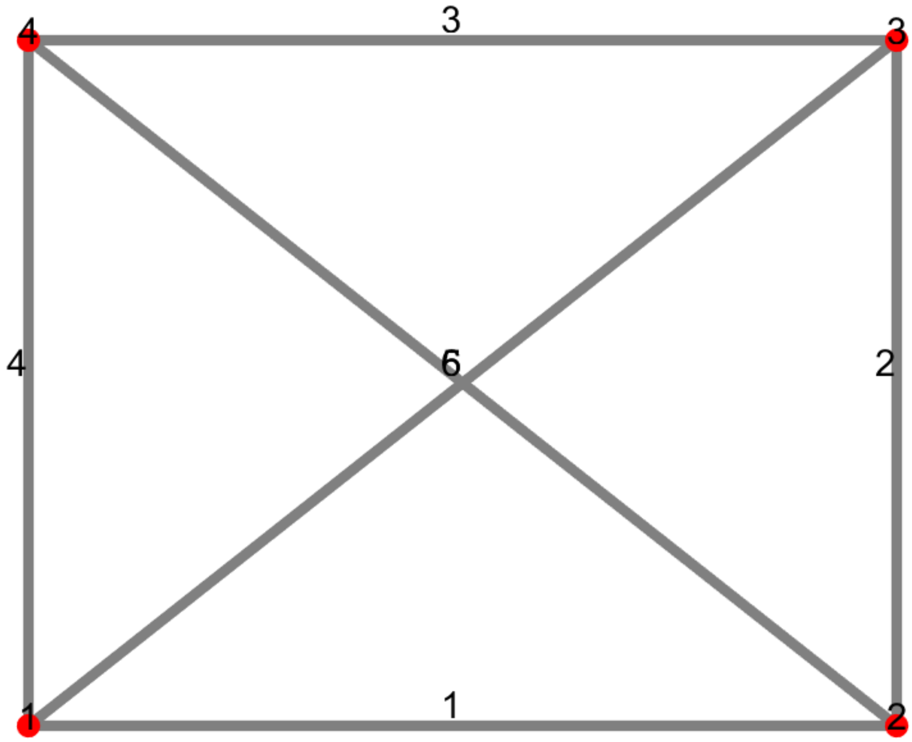

Figure 1: Our Structure to be solved

1 Exercise1a.tex

Our target is to solve the structure shown in Fig. 1 using our Matrix Structural Analysis Methods - Yeeeh!

The element orientation angles are

Element: 1

$\phi^1 = 0.0^o$

Element: 2

$\phi^2 = 90.0^o$

Element: 3

$\phi^3 = 180.0^o$

Element: 4

$\phi^4 = 90.0^o$

Element: 5

$\phi^5 = 36.87^o$

Element: 6

$\phi^6 = 143.13^o$

The element transformation matrices are

$$[\Lambda^i_{PT}] = \begin{bmatrix} \cos\phi^i & \sin\phi^i & 0 & 0 \\ -\sin\phi^i & \cos\phi^i & 0 & 0 \\ 0 & 0 & \cos\phi^i & \sin\phi^i \\ 0 & 0 & -\sin\phi^i & \cos\phi^i \end{bmatrix}$$

Element: 1

$$[\Lambda^1_{PT}] = \begin{pmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1.0 \end{pmatrix} \tag{1}$$

Element: 2

$$[\Lambda^2_{PT}] = \begin{pmatrix} 0 & 1.0 & 0 & 0 \\ -1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 \\ 0 & 0 & -1.0 & 0 \end{pmatrix} \tag{2}$$

Element: 3

$$[\Lambda^3_{PT}] = \begin{pmatrix} -1.0 & 0 & 0 & 0 \\ 0 & -1.0 & 0 & 0 \\ 0 & 0 & -1.0 & 0 \\ 0 & 0 & 0 & -1.0 \end{pmatrix} \tag{3}$$

Element: 4

$$[\Lambda^4_{PT}] = \begin{pmatrix} 0 & 1.0 & 0 & 0 \\ -1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 \\ 0 & 0 & -1.0 & 0 \end{pmatrix} \tag{4}$$

Element: 5

$$[\Lambda^5_{PT}] = \begin{pmatrix} 0.8 & 0.6 & 0 & 0 \\ -0.6 & 0.8 & 0 & 0 \\ 0 & 0 & 0.8 & 0.6 \\ 0 & 0 & -0.6 & 0.8 \end{pmatrix} \tag{5}$$

Element: 6

$$[\Lambda^6_{PT}] = \begin{pmatrix} -0.8 & 0.6 & 0 & 0 \\ -0.6 & -0.8 & 0 & 0 \\ 0 & 0 & -0.8 & 0.6 \\ 0 & 0 & -0.6 & -0.8 \end{pmatrix} \tag{6}$$

The element stiffness matrices at their local coordinate system are

$$[k^i] = \frac{EA^i}{L^i} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

Element: 1

$$\left[k^1\right] = \begin{pmatrix} 525000.0 & 0 & -525000.0 & 0 \\ 0 & 0 & 0 & 0 \\ -525000.0 & 0 & 525000.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{8}$$

Element: 2

$$\left[k^2\right] = \begin{pmatrix} 700000.0 & 0 & -700000.0 & 0 \\ 0 & 0 & 0 & 0 \\ -700000.0 & 0 & 700000.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{9}$$

Element: 3

$$\left[k^3\right] = \begin{pmatrix} 525000.0 & 0 & -525000.0 & 0 \\ 0 & 0 & 0 & 0 \\ -525000.0 & 0 & 525000.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{10}$$

Element: 4

$$\left[k^4\right] = \begin{pmatrix} 700000.0 & 0 & -700000.0 & 0 \\ 0 & 0 & 0 & 0 \\ -700000.0 & 0 & 700000.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{11}$$

Element: 5

$$\left[k^5\right] = \begin{pmatrix} 420000.0 & 0 & -420000.0 & 0 \\ 0 & 0 & 0 & 0 \\ -420000.0 & 0 & 420000.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{12}$$

Element: 6

$$\left[k^6\right] = \begin{pmatrix} 420000.0 & 0 & -420000.0 & 0 \\ 0 & 0 & 0 & 0 \\ -420000.0 & 0 & 420000.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{13}$$

The element stiffness matrices at the global coordinate system are

$$\left[\bar{k}^i\right] = \left[\Lambda^i_{PT}\right]^T \left[k^i\right] \left[\Lambda^i_{PT}\right] \tag{14}$$

Element: 1

$$\left[\bar{k}^1\right] = \begin{pmatrix} 525000.0 & 0 & -525000.0 & 0 \\ 0 & 0 & 0 & 0 \\ -525000.0 & 0 & 525000.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{15}$$

Element: 2

$$\left[\bar{k}^2\right] = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 700000.0 & 0 & -700000.0 \\ 0 & 0 & 0 & 0 \\ 0 & -700000.0 & 0 & 700000.0 \end{pmatrix} \tag{16}$$

Element: 3

$$\left[\bar{k}^3\right] = \begin{pmatrix} 525000.0 & 0 & -525000.0 & 0 \\ 0 & 0 & 0 & 0 \\ -525000.0 & 0 & 525000.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{17}$$

Element: 4

$$\left[\bar{k}^4\right] = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 700000.0 & 0 & -700000.0 \\ 0 & 0 & 0 & 0 \\ 0 & -700000.0 & 0 & 700000.0 \end{pmatrix} \tag{18}$$

Element: 5

$$\left[\bar{k}^5\right] = \begin{pmatrix} 268800.0 & 201600.0 & -268800.0 & -201600.0 \\ 201600.0 & 151200.0 & -201600.0 & -151200.0 \\ -268800.0 & -201600.0 & 268800.0 & 201600.0 \\ -201600.0 & -151200.0 & 201600.0 & 151200.0 \end{pmatrix} \tag{19}$$

Element: 6

$$\left[\bar{k}^6\right] = \begin{pmatrix} 268800.0 & -201600.0 & -268800.0 & 201600.0 \\ -201600.0 & 151200.0 & 201600.0 & -151200.0 \\ -268800.0 & 201600.0 & 268800.0 & -201600.0 \\ 201600.0 & -151200.0 & -201600.0 & 151200.0 \end{pmatrix} \tag{20}$$

The stiffness matrix of the entire structure is

$$
[\bar{K}] = \begin{pmatrix}
793800.0 & 201600.0 & -525000.0 & 0 & -268800.0 & -201600.0 & 0 & 0 \\
201600.0 & 851200.0 & 0 & 0 & -201600.0 & -151200.0 & 0 & -700000.0 \\
-525000.0 & 0 & 793800.0 & -201600.0 & 0 & 0 & -268800.0 & 201600.0 \\
0 & 0 & -201600.0 & 851200.0 & 0 & -700000.0 & 201600.0 & -151200.0 \\
-268800.0 & -201600.0 & 0 & 0 & 793800.0 & 201600.0 & -525000.0 & 0 \\
-201600.0 & -151200.0 & 0 & -700000.0 & 201600.0 & 851200.0 & 0 & 0 \\
0 & 0 & -268800.0 & 201600.0 & -525000.0 & 0 & 793800.0 & -201600.0 \\
0 & -700000.0 & 201600.0 & -151200.0 & 0 & 0 & -201600.0 & 851200.0
\end{pmatrix} \tag{21}
$$

The constraint permuation matrix V is

$$[V] = \begin{pmatrix} 0 & 0 & 1.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \\ 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{22}$$

And the permuted stiffness matrix for the entire structure becomes

$$\left[\bar{K}\right]_m = [V]\left[\bar{K}\right][V]^T \tag{23}$$

$$\left[\bar{K}\right]_m = \begin{pmatrix} 793800.0 & 0 & 0 & -268800.0 & 201600.0 & -525000.0 & 0 & -201600.0 \\ 0 & 793800.0 & 201600.0 & -525000.0 & 0 & -268800.0 & -201600.0 & 0 \\ 0 & 201600.0 & 851200.0 & 0 & 0 & -201600.0 & -151200.0 & -700000.0 \\ -268800.0 & -525000.0 & 0 & 793800.0 & -201600.0 & 0 & 0 & 201600.0 \\ 201600.0 & 0 & 0 & -201600.0 & 851200.0 & 0 & -700000.0 & -151200.0 \\ -525000.0 & -268800.0 & -201600.0 & 0 & 0 & 793800.0 & 201600.0 & 0 \\ 0 & -201600.0 & -151200.0 & 0 & -700000.0 & 201600.0 & 851200.0 & 0 \\ -201600.0 & 0 & -700000.0 & 201600.0 & -151200.0 & 0 & 0 & 851200.0 \end{pmatrix} \tag{24}$$

The vector of external nodal loads is

$$\{\bar{P}_{ext}\} = \begin{pmatrix} 0 \\ 0 \\ 100.0 \\ 0 \\ 100.0 \\ 0 \\ 100.0 \\ -100.0 \end{pmatrix} \tag{25}$$

which, when permuted becomes

$$\{\bar{P}_{m,ext}\} = \begin{pmatrix} 100.0 \\ 100.0 \\ 0 \\ 100.0 \\ -100.0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{26}$$

Solving the system of linear equations, the nodal displacements are eventually

$$\{\bar{\Delta}\} = \begin{pmatrix} 0 \\ 0 \\ 0.00034039 \\ 0 \\ 0.00054872 \\ -0.00012996 \\ 0.00058929 \\ -0.000058532 \end{pmatrix} \tag{27}$$

The deformed configuration of the structure is shown Fig. 2 - We did it!

Finaly, the element load vectors at their local coordinate system are Element: 1

$$\{F^1\} = \begin{pmatrix} -178.7 \\ 0 \\ 178.7 \\ 0 \end{pmatrix} \tag{28}$$

Element: 2

$$\{F^2\} = \begin{pmatrix} 90.972 \\ 0 \\ -90.972 \\ 0 \end{pmatrix} \tag{29}$$

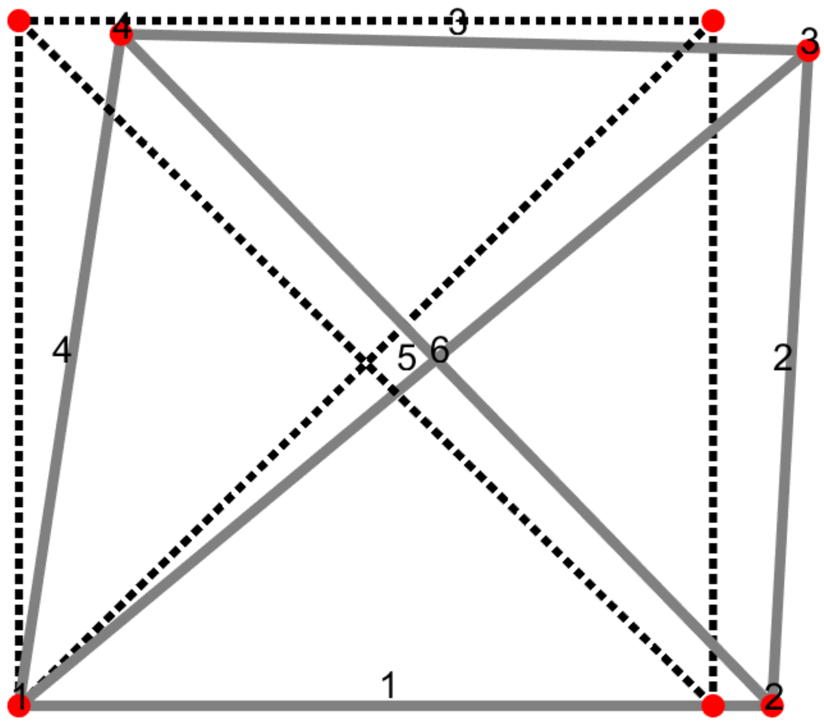

Figure 2: Deformed Configuration

Element: 3

$$\{F^3\} = \begin{pmatrix} 21.296 \\ 0 \\ -21.296 \\ 0 \end{pmatrix} \tag{30}$$

Element: 4

$$\{F^4\} = \begin{pmatrix} 40.972 \\ 0 \\ -40.972 \\ 0 \end{pmatrix} \tag{31}$$

Element: 5

$$\{F^5\} = \begin{pmatrix} -151.62 \\ 0 \\ 151.62 \\ 0 \end{pmatrix} \tag{32}$$

Element: 6

$$\{F^6\} = \begin{pmatrix} 98.38 \\ 0 \\ -98.38 \\ 0 \end{pmatrix} \tag{33}$$