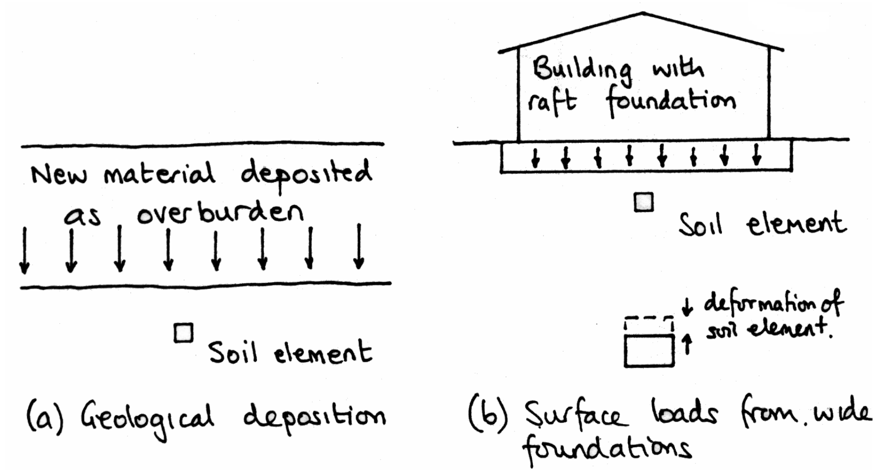
Consolidation

Reminder: What is consolidation?

Consolidation: Time-dependent compression of soil under constant total stress that is accompanied by dissipation of excess pore pressure.

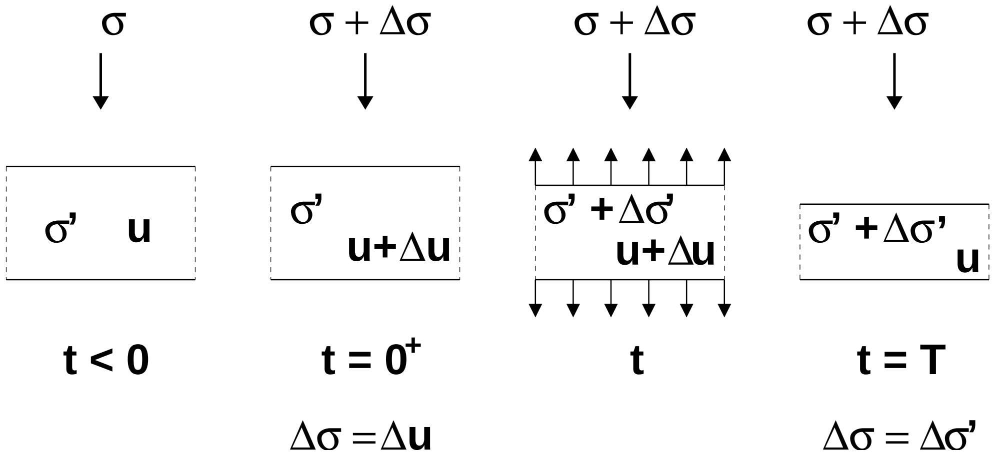
In Part-II we discussed **one dimensional** compression and consolidation.

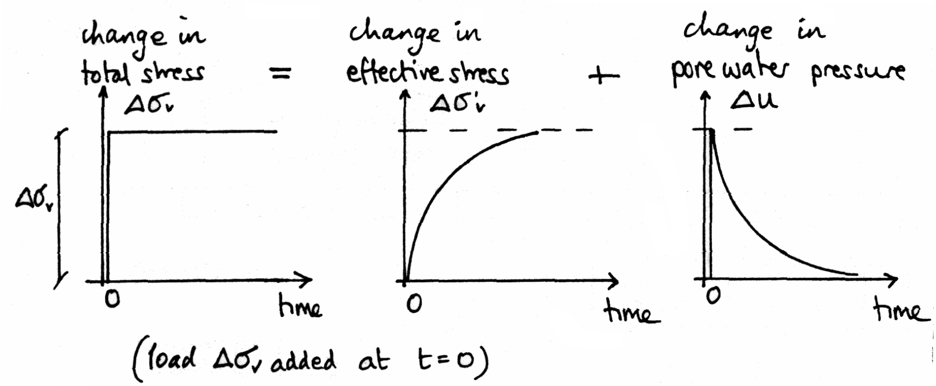
- Vertical but no lateral deformation; vertical strain ϵ_z equal to volumetric ϵ_v.
- Variations only with depth.
- Simplified approach; applicable to e.g. the soil under a shallow foundation.



The process of consolidation is time-dependent because:

- Soil volume can change only if soil particles rearrange, changing the volume of voids.
- If the voids are full of water, flow needs to take place to accommodate compression.
- Flow needs time to occur, as soil permeability is finite.



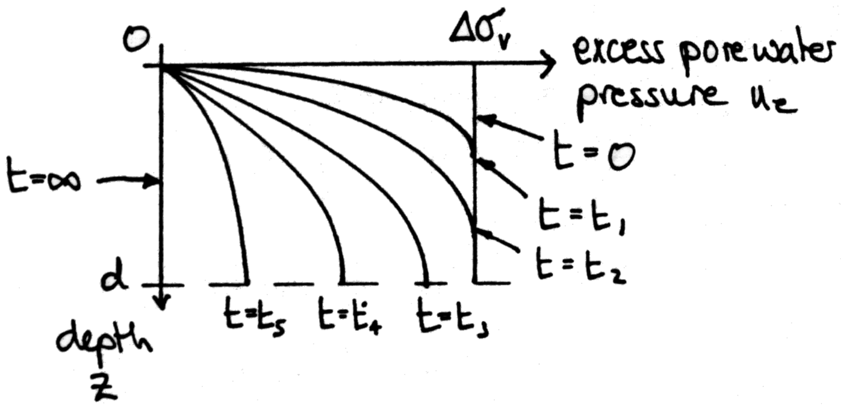


- The applied load increases total stress by $\Delta\sigma$.
- Skeleton can't take more load till particles rearrange; **excess** pore pressures generated.
- Pore pressure increase creates a hydraulic gradient; flow commences.
- Due to flow excess pore pressures gradually diffuse; effective stresses gradually increase.
- Finally, the skeleton takes all the extra load; pore pressures back to their equilibrium value.

Isochrones of excess pore pressure

A line showing the distribution of excess pore pressure at a particular time is called the **isochrone of excess pore pressure** for that time.

- In one-dimensional consolidation the isochrones consisted of excess pore pressure plotted vs depth.



Typical isochrones for a consolidating layer that drains from the top only are shown on the left.

In Part-II we saw how we can determine the isochrones at different times, and how we can use them to calculate settlements over time.

In Part-II we assumed that:

- Consolidating layers are relatively thin, so an initially uniform distribution of uniform excess pore pressure with depth can be assumed.
- There is no lateral, only vertical flow.
- The soil is linear elastic, homogeneous and isotropic.
- Soil stiffness and permeability do not vary during the consolidation process.
- Deformations are small.

In Part-II we dealt with simple problems that were not far from one-dimensional. But:

- What if a problem is truly two- or even three-dimensional?
- What if the layer is not thin?
- What if lateral flow is important?
- What if there is anisotropy, or even multiple materials?
- (Soil is **not** linear elastic and some properties change with effective stress.)

Then it is necessary to solve the general consolidation problem numerically.

Consolidation involves both soil skeleton deformation and fluid flow. Therefore we will use for the equations of equilibrium and of continuity of fluid flow.

We need however to take into account that soil deformation is governed by increments of the effective stress $\boldsymbol{\sigma}'$, rather than total stress $\boldsymbol{\sigma}$.

Equilibrium:

- $\boldsymbol{\Sigma} \cdot \nabla + \mathbf{b} = \mathbf{0}$ or, equivalently, $\mathbf{L}^\top \boldsymbol{\sigma} + \mathbf{b} = \mathbf{0}$
- $\boldsymbol{\sigma}' = \boldsymbol{\sigma} + \mathbf{m}^\top p_w$ is the effective stress, where $\mathbf{m}^\top = \{1\ 1\ 1\ 0\ 0\ 0\}$
- $\boldsymbol{\sigma}' = \mathbf{D}' \cdot \boldsymbol{\epsilon}$
- $\boldsymbol{\epsilon} = \mathbf{L} \cdot \mathbf{u}$

where $\mathbf{L}$ is a differential operator, p_w the pore pressure and $\mathbf{D}'$ the drained elastic moduli. The boundary conditions are:

- $\mathbf{u} = \mathbf{u_0}$ on S_u (the part of S where displacements are prescribed.)
- $\boldsymbol{\Sigma} \cdot \mathbf{n} = \mathbf{t}$ on S_σ (the part of S where stresses are applied.)

Note that the definition of effective stress assumes that positive stress is *tensile* while positive pore pressure is *compressive*.

Equilibrium: weak form

We have already worked out the weak form to be:

$$\int_V \delta\boldsymbol{\epsilon}^\top \boldsymbol{\sigma}\, dV = \int_V \delta\mathbf{u}^\top \mathbf{b}\, dV + \int_S \delta\mathbf{u}^\top \mathbf{t}\, dS$$

where $\delta\mathbf{u}$ a virtual displacement field and $\delta\boldsymbol{\epsilon}$ the corresponding virtual strain. Substituting the definition of effective stress we obtain:

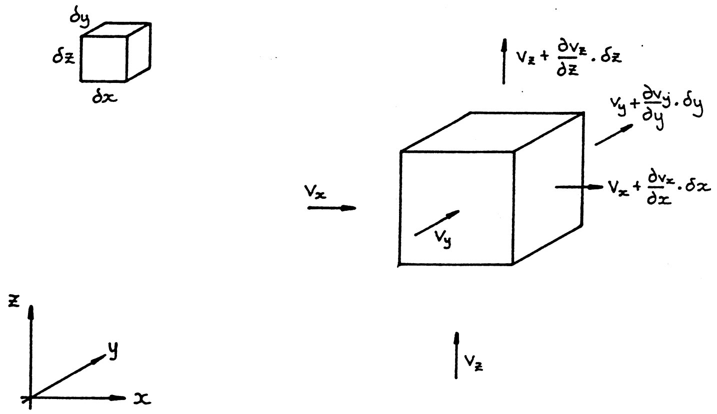
$$\int_V \delta\boldsymbol{\epsilon}^\top \boldsymbol{\sigma'}\, dV - \int_V \delta\boldsymbol{\epsilon}^\top \mathbf{m}\, p_w\, dV = \int_V \delta\mathbf{u}^\top \mathbf{b}\, dV + \int_S \delta\mathbf{u}^\top \mathbf{t}\, dS$$

and finally the weak form of the equations of equilibrium is written as:

$$\boxed{\int_V \delta\boldsymbol{\epsilon}^\top \mathbf{D}'\boldsymbol{\epsilon}\, dV - \int_V \delta\boldsymbol{\epsilon}^\top \mathbf{m}\, p_w\, dV = \int_V \delta\mathbf{u}^\top \mathbf{b}\, dV + \int_S \delta\mathbf{u}^\top \mathbf{t}\, dS}$$

We note that the equation contains *two* unknown fields: the displacement $\mathbf{u}$ (through its derivatives $\boldsymbol{\epsilon}$) and the pore pressure p_w.

Formulation of the coupled problem: Continuity



Flow through inf/mal cube $V_0 = dx \times dy \times dz$

- “negative” faces: v_x, v_y, v_z.
- “positive” faces:$v_x + \dfrac{\partial v_x}{\partial x} dx$, $v_y + \dfrac{\partial v_y}{\partial y} dy$, $v_z + \dfrac{\partial v_z}{\partial z} dz$

In: $q_{in} = v_x \, dydz + v_y \, dxdz + v_z \, dxdy$

Out: $q_{out} = (v_x + \dfrac{\partial v_x}{\partial x} dx) \, dydz + (v_y + \dfrac{\partial v_y}{\partial y} dy) \, dxdz + (v_z + \dfrac{\partial v_z}{\partial z} dz) \, dxdy$

Assume that the reference volume V_0 changes (increases) by dV during a time interval dt.

Also assume conservation of mass and incompressibility. Then:

$$q_{in} = q_{out} + \frac{dV}{dt} \Longrightarrow \left(\nabla^\top \mathbf{v}\right) V_0 = \frac{dV}{dt} \Longrightarrow \nabla^\top \cdot \mathbf{v} = \frac{\partial \epsilon_v}{\partial t} \Longrightarrow \boxed{\nabla^\top \cdot \mathbf{v} = \mathbf{m}^\top \cdot \frac{\partial \boldsymbol{\epsilon}}{\partial t}}$$

where $\mathbf{v}$ the seepage velocity, ϵ_v the volumetric strain and $\epsilon_v < 0$ for compression.

The total head at a point is defined as: $h = s + \dfrac{p_w}{\rho_w g}$, where:

- p_w: the total pore pressure.
- s: the distance from the datum for head measurement.
- ρ_w: the density of the fluid (water.)
- g: the acceleration of gravity.

Darcy's Law: $\mathbf{v} = -\mathbf{K} \cdot \nabla h$, where $\mathbf{K}$ the permeability matrix

Defining the gravity acceleration vector as $\mathbf{g} = -g(\nabla s)$ and substituting the definition of total head and Darcy's law into the equation of continuity we obtain:

$$\frac{1}{\rho_w g} \nabla^\top \left\{ \mathbf{K} \left(\nabla p_w - \rho_w \mathbf{g} \right) \right\} = -\mathbf{m}^\top \cdot \frac{\partial \boldsymbol{\epsilon}}{\partial t}$$

Continuity: weak form

We can determine the weak form of the flow equation by the usual procedure. Assume a virtual total pore pressure field δp_w consistent with prescribed boundary pore pressures. Then:

$$\int_V \left[\frac{1}{\rho_w g}\nabla^\top \{\mathbf{K}\,(\nabla p_w - \rho_w \mathbf{g})\}\right]\delta p_w\, dV = -\int_V \mathbf{m}^\top \cdot \frac{\partial \boldsymbol{\epsilon}}{\partial t}\delta p_w\, dV$$

and carrying out the usual algebraic manipulations:

$$\frac{1}{\rho_w g}\int_V \left(\nabla^\top \delta p_w\right)\mathbf{K}\,(\nabla p_w)\, dV = \frac{1}{g}\int_V \left(\nabla^\top \delta p_w\right)\mathbf{K}\cdot \mathbf{g}\, dV + \int_V \mathbf{m}^\top \cdot \frac{\partial \boldsymbol{\epsilon}}{\partial t}\cdot \delta p_w\, dV - \int_S \delta p_w \cdot \mathbf{v}^\top \cdot \mathbf{n}\, dS$$

We interpolate displacements and pore pressure as:

- $u = \mathbf{N_u} \cdot \hat{\mathbf{u}}$
- $p_w = \mathbf{N_p} \cdot \hat{\mathbf{p}}$

where $\mathbf{N_u}$ and $\mathbf{N_p}$ shape functions and $\hat{\mathbf{u}}$ and $\hat{\mathbf{p}}$ degrees of freedom.

We also define the matrices: $\mathbf{B_u} = \mathbf{L} \cdot \mathbf{N_u}$ and $\mathbf{B_p} = \nabla \mathbf{N_p}$ and remind that $\boldsymbol{\epsilon} = \mathbf{L} \cdot \mathbf{u}$.

The weak forms of the equilibrium and continuity equations can then be discretised as follows:

Discretised form of equilibrium

$$\int_V \delta\boldsymbol{\epsilon}^\top \mathbf{D}'\boldsymbol{\epsilon}\, dV - \int_V \delta\boldsymbol{\epsilon}^\top \mathbf{m}\, p_w\, dV = \int_V \delta\mathbf{u}^\top \mathbf{b}\, dV + \int_S \delta\mathbf{u}^\top \mathbf{t}\, dS \longrightarrow$$

$$\int_V \mathbf{B_u}^\top \mathbf{D}'\mathbf{B_u}\, dV \cdot \hat{\mathbf{u}} - \int_V \mathbf{B_u}^\top \mathbf{m}\, \mathbf{N_p}\, dV \cdot \hat{\mathbf{p}} = \int_V \mathbf{N_u}^\top \mathbf{b}\, dV + \int_S \mathbf{N_u}^\top \mathbf{t}\, dS \longrightarrow$$

$$\boxed{\mathbf{K_{uu}} \cdot \hat{\mathbf{u}} + \mathbf{K_{up}} \cdot \hat{\mathbf{p}} = \mathbf{f_u}}$$

where:

$\mathbf{K_{uu}} = \int_V \mathbf{B_u}^\top \mathbf{D}'\mathbf{B_u}\, dV$, $\mathbf{K_{up}} = -\int_V \mathbf{B_u}^\top \mathbf{m}\, \mathbf{N_p}\, dV$, and

$\mathbf{f_u} = \int_V \mathbf{N_u}^\top \mathbf{b}\, dV + \int_S \mathbf{N_u}^\top \mathbf{t}\, dS$

$$\frac{1}{\rho_w g}\int_V \left(\nabla^\top \delta p_w\right)\mathbf{K}\left(\nabla p_w\right)dV = \frac{1}{g}\int_V \left(\nabla^\top \delta p_w\right)\mathbf{K}\cdot\mathbf{g}\,dV$$

$$+\int_V \delta p_w\,\mathbf{m}^\top\cdot\frac{\partial \boldsymbol{\epsilon}}{\partial t}dV - \int_S \delta p_w\cdot\mathbf{v}^\top\cdot\mathbf{n}\,dS \longrightarrow$$

$$-\int_V \mathbf{N_p}^\top\mathbf{m}^\top\mathbf{B_u}dV\cdot\frac{\partial\hat{\mathbf{u}}}{\partial t} + \frac{1}{\rho_w g}\int_V \mathbf{B_p}^\top\mathbf{K}\mathbf{B_p}dV\cdot\hat{\mathbf{p}} =$$

$$\frac{1}{g}\int_V \mathbf{B_p}^\top\mathbf{K}\cdot\mathbf{g}\,dV - \int_S \mathbf{N_p}^\top\cdot\mathbf{v}^\top\cdot\mathbf{n}\,dS \longrightarrow$$

$$\boxed{\mathbf{K_{pu}} \cdot \dot{\hat{\mathbf{u}}} + \mathbf{K_{pp}} \cdot \hat{\mathbf{p}} = \mathbf{f_p}}$$

where:

$$\mathbf{K_{pp}} = \frac{1}{\rho_w g} \int_V \mathbf{B_p}^\top \mathbf{K} \mathbf{B_p} dV,$$

$\mathbf{K_{pu}} = \mathbf{K_{up}}^\top$, and

$$\mathbf{f_p} = \frac{1}{g} \int_V \mathbf{B_p}^\top \mathbf{K} \cdot \mathbf{g} \, dV - \int_S \mathbf{N_p}^\top \cdot \mathbf{v}^\top \cdot \mathbf{n} \, dS$$

We arrived at two equations that are coupled:

$$\mathbf{K_{uu}} \cdot \hat{\mathbf{u}} + \mathbf{K_{up}} \cdot \hat{\mathbf{p}} = \mathbf{f_u} \qquad \mathbf{K_{pu}} \cdot \dot{\hat{\mathbf{u}}} + \mathbf{K_{pp}} \cdot \hat{\mathbf{p}} = \mathbf{f_p}$$

Nevertheless they cannot be solved directly because the second one contains the *rate* of the displacement $\dot{\hat{\mathbf{u}}}$, rather than the displacement $\hat{\mathbf{u}}$.

We need to discretise in time as well as space!

- We assume that we know the solution $(\hat{\mathbf{u}}_\mathbf{n}, \hat{\mathbf{p}}_\mathbf{n})$ at the end of timestep n, corresponding to time t.
- We want to determine $(\hat{\mathbf{u}}_\mathbf{n+1} = \hat{\mathbf{u}}_\mathbf{n} + \Delta\hat{\mathbf{u}}, \hat{\mathbf{p}}_\mathbf{n+1} = \hat{\mathbf{p}}_\mathbf{n} + \Delta\hat{\mathbf{p}})$ at the end of timestep $n+1$, corresponding to time $t + \Delta t$.
  - Determine $(\Delta\hat{\mathbf{u}}, \Delta\hat{\mathbf{p}})$ for Δt.

Writing the first equation at both t and $t + \Delta t$ and subtracting:

$$\boxed{\mathbf{K_{uu}} \cdot \mathbf{\Delta \hat{u}} + \mathbf{K_{up}} \cdot \mathbf{\Delta \hat{p}} = \mathbf{\Delta f_u}}$$

To discretise the second one, we assume the following approximation in time:

$$\mathbf{\hat{u}_{n+1}} = \mathbf{\hat{u}_n} + \Delta t \left((1 - \theta)\dot{\mathbf{\hat{u}}}_\mathbf{n} + \theta \dot{\mathbf{\hat{u}}}_\mathbf{n+1} \right)$$

i.e. that the average derivative of $\mathbf{\hat{u}}$ over Δt is a weighted sum of the derivatives at the beginning ($\dot{\mathbf{\hat{u}}}_\mathbf{n}$) and the end ($\dot{\mathbf{\hat{u}}}_\mathbf{n+1}$) of the interval.

- $\theta = 0$: (Unstable) explicit scheme $\mathbf{\hat{u}_{n+1}} = \mathbf{\hat{u}_n} + \Delta t \dot{\mathbf{\hat{u}}}_\mathbf{n}$; only uses information available at time t.
- $\theta = 1$: (Stable) backward difference $\mathbf{\hat{u}_{n+1}} = \mathbf{\hat{u}_n} + \Delta t \theta \dot{\mathbf{\hat{u}}}_\mathbf{n+1}$; implicit scheme so the solution must be determined iteratively.
- To ensure stability, $\theta \geq 1/2$. For $\theta = 1/2$: Crank-Nicholson scheme.

Writing the second equation at both t and $t + \Delta t$, premultiplied by $(1 - \theta)$ and θ respectively:

$$\mathbf{K_{pu}} \cdot \theta \dot{\hat{\mathbf{u}}}_{\mathbf{n+1}} + \mathbf{K_{pp}} \cdot \theta \hat{\mathbf{p}}_{\mathbf{n+1}} = \theta \mathbf{f_{p,n+1}}$$

$$\mathbf{K_{pu}} \cdot (1 - \theta)\, \dot{\hat{\mathbf{u}}}_{\mathbf{n}} + \mathbf{K_{pp}} \cdot (1 - \theta)\, \hat{\mathbf{p}}_{\mathbf{n}} = (1 - \theta)\, \mathbf{f_{p,n}}$$

Adding up yields:

$$\mathbf{K_{pu}} \cdot \frac{\boldsymbol{\Delta} \hat{\mathbf{u}}}{\Delta t} + \mathbf{K_{pp}} \cdot \left(\theta \hat{\mathbf{p}}_{\mathbf{n+1}} + (1 - \theta)\, \hat{\mathbf{p}}_{\mathbf{n}}\right) = \theta \mathbf{f_{p,n+1}} + (1 - \theta)\, \mathbf{f_{p,n}} \longrightarrow$$

$$\mathbf{K_{pu}} \cdot \boldsymbol{\Delta} \hat{\mathbf{u}} + \theta \Delta t \mathbf{K_{pp}} \cdot \Delta \hat{\mathbf{p}} = \left(\theta \mathbf{f_{p,n+1}} + (1 - \theta)\, \mathbf{f_{p,n}} - \mathbf{K_{pp}} \cdot \hat{\mathbf{p}}_{\mathbf{n}}\right) \Delta t$$

In the following we consider the case $\theta = 1$ that is implemented in ABAQUS:

$$\boxed{\mathbf{K_{pu}} \cdot \boldsymbol{\Delta} \hat{\mathbf{u}} + \Delta t \mathbf{K_{pp}} \cdot \Delta \hat{\mathbf{p}} = \left(\mathbf{f_{p,n+1}} - \mathbf{K_{pp}} \cdot \hat{\mathbf{p}}_{\mathbf{n}}\right) \Delta t}$$

Finally the system of equations to solve becomes:

$$\begin{bmatrix} \mathbf{K_{uu}} & \mathbf{K_{up}} \\ \mathbf{K_{up}}^\top & \Delta t \mathbf{K_{pp}} \end{bmatrix} \cdot \begin{Bmatrix} \Delta \hat{\mathbf{u}} \\ \Delta \hat{\mathbf{p}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{\Delta f_u} \\ \left(\mathbf{f_{p,n+1}} - \mathbf{K_{pp}} \cdot \hat{\mathbf{p}}_\mathbf{n}\right) \Delta t \end{Bmatrix}$$

$\mathbf{u}$ and p_w are interpolated independently. However:

- p_w is driven by changes in volumetric strain, i.e. derivatives of $\mathbf{u}$.
- At the undrained limit $\mathbf{K} = \mathbf{0} \longrightarrow \mathbf{K_{pp}} = \mathbf{0}$ a solution exists only if $\mathbf{u}$ has more degrees of freedom than p_w: $n_u > n_p$

For these reasons typical elements that perform well interpolate the pore pressure with a polynomial one order lower than the displacement.

- E.g. biquadratic quadrilateral for $\mathbf{u}$ and bilinear quadrilateral for p_w

- Use a transient *SOILS analysis step.
- Use elements that interpolate the displacement and the pore pressure.
  - Typically biquadratic displacement, bilinear pore pressure.
- Supply displacement and pore pressure boundary conditions.
- Prescribe boundary loads and seepage conditions.
  - May vary over time using *AMPLITUDE
  - Complex conditions such as drainage-only can also be used.
- Prescribe initial conditions of void ratio, pore pressure and effective stress.
  - *INITIAL CONDITIONS, TYPE=RATIO
  - *INITIAL CONDITIONS, TYPE=PORE PRESSURE
  - *INITIAL CONDITIONS, TYPE=STRESS or *INITIAL CONDITIONS, TYPE=STRESS, GEOSTATIC
  - In all three cases you will need to hand-edit the keywords in.
- Run an initial *GEOSTATIC step before *SOILS to obtain equilibrium.

In a transient analysis you need to define some timestepping parameters:

- Initial, minimum and maximum timestep.
- Total time period to be modelled. (Alternatively require the analysis to run till steady-state is reached.)
- Maximum pore pressure change during the increment: defines the tolerance with which the solution is obtained.

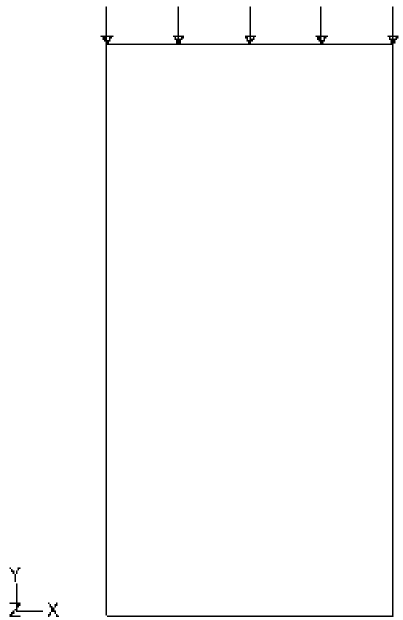
There are two different ways of deciding timestep size.

- Timesteps of fixed size.
  - Generally not a good idea unless done for specific reasons and you already know that the step you prescribe is small enough.
- Timesteps automatically adjusted by ABAQUS.
  - Should be preferred. Allows automatic adjustment of timestep size to optimise accuracy vs. computational time.

Last but not least: remember to be consistent with units.

Assume a $2m$-thick layer of soil consolidating in one-dimensional compression under a load of $100kPa$. The load is applied instantaneously at $t = 0$ and remains constant thereafter. The soil has permeability $k = 10^{-7} m/sec$ and is linear elastic with $E = 10MPa$ and $\nu = 0.3$.

- Determine the settlement vs time curve of the layer.
- Plot profiles of excess pore pressure vs depth.



Setting up the analysis:

- Geometry and material properties are trivial to enter.
- So is the mesh and the supports.
- Parameters requiring further thought:
  - Application of load? Drainage/pore pressure?

If the load is applied instantaneously, uniform excess pore pressure should develop throughout the sample. If we want to reproduce this response and "set up" excess pore pressure before we allow it to dissipate, we can:

- Run a first analysis step where the load is applied but drainage is prohibited. Then the pore pressure should rise to match the applied load.
- Run a second analysis step where we change one of the boundary conditions to allow drainage. As excess pore pressure is present, consolidation will start.

- Create a first step of the *Geostatic type.
  - Create supports. Apply vertical load. Solve for equilibrium; this sets up excess pore pressures.
- Create a second step of the *Soils (transient) type.
  - This step inherits all loads and supports.
  - Introduce an additional boundary condition: set pore pressure to be zero at the top.
  - Running the step will solve the consolidation problem.

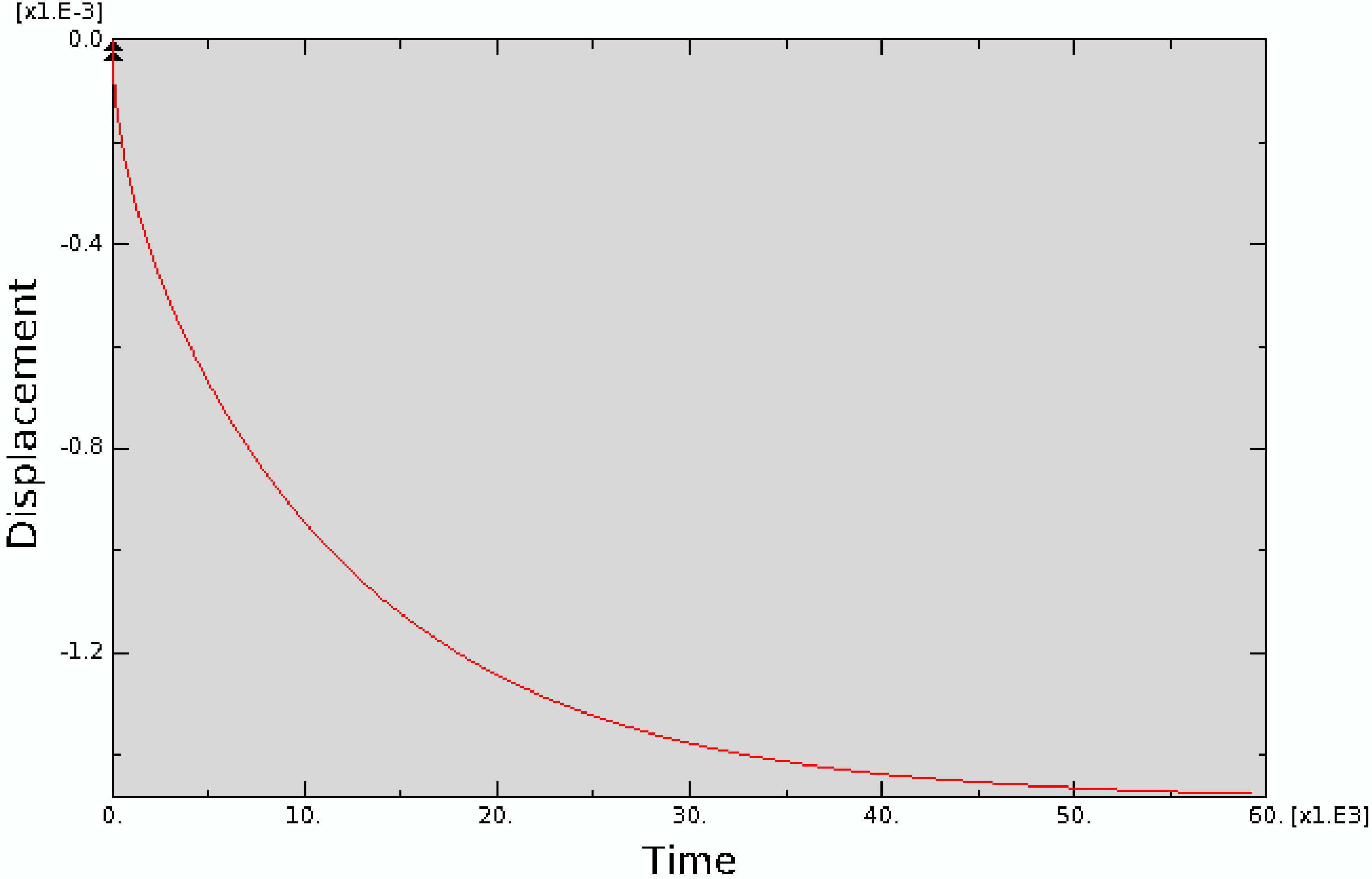
In the following we will see some details of the model:

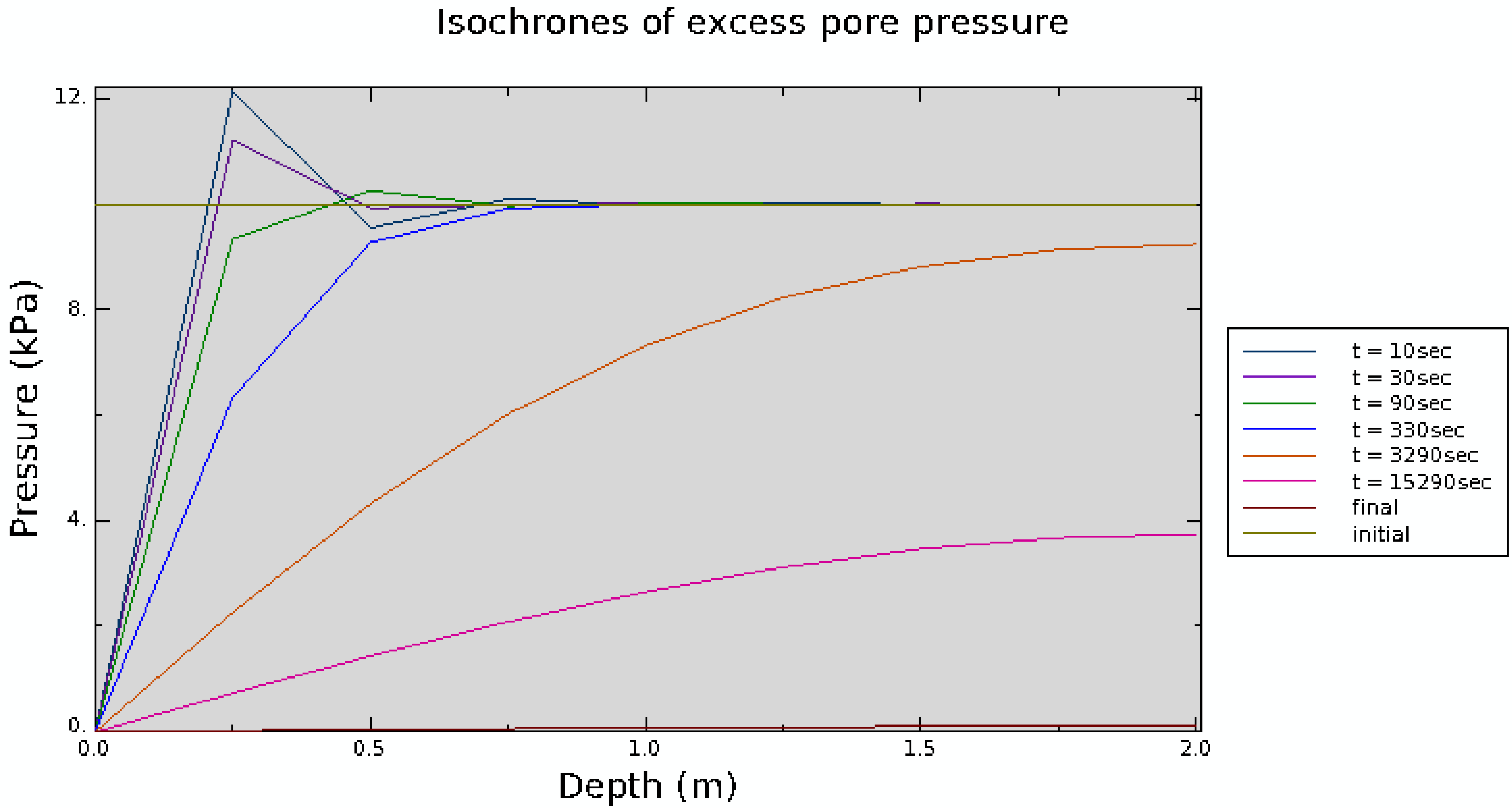
- Boundary conditions and loads have been assigned to different steps.
- Initial conditions on the void ratio have been hand-edited in the input file.
- Time incrementation parameters have been prescribed.
- Assumptions on when steady state conditions are reached were made.

The backward-difference scheme used for time integration is *unconditionally stable*, i.e. will always converge. However it is not always accurate!

- Too large timesteps: may lead to inaccuracies, although not cumulative.
- Too small timesteps: may lead to spurious pore pressure oscillations.
  - Problematic if soil plasticity models are used (see next few lectures!)
- Generally $\Delta t \geq \dfrac{\gamma_w (\Delta l)^2}{6 \cdot E \cdot k}$, where Δl a representative element length.

PS: The model is called “Column” and is in the “Consolidation.cae” database. The file will be available on Blackboard for you to download and play with.


[x1.E-3]
0.0
-0.4
-0.8
-1.2
Displacement
0.
10.
20.
30.
40.
50.
60.
[x1.E3]
Time
settlement


Isochrones of excess pore pressure
Pressure (kPa)
Depth (m)
12.
8.
4.
0.
0.0
0.5
1.0
1.5
2.0
t = 10sec
t = 30sec
t = 90sec
t = 330sec
t = 3290sec
t = 15290sec
final
initial

The oscillations could be due to the initial timestep being too small:

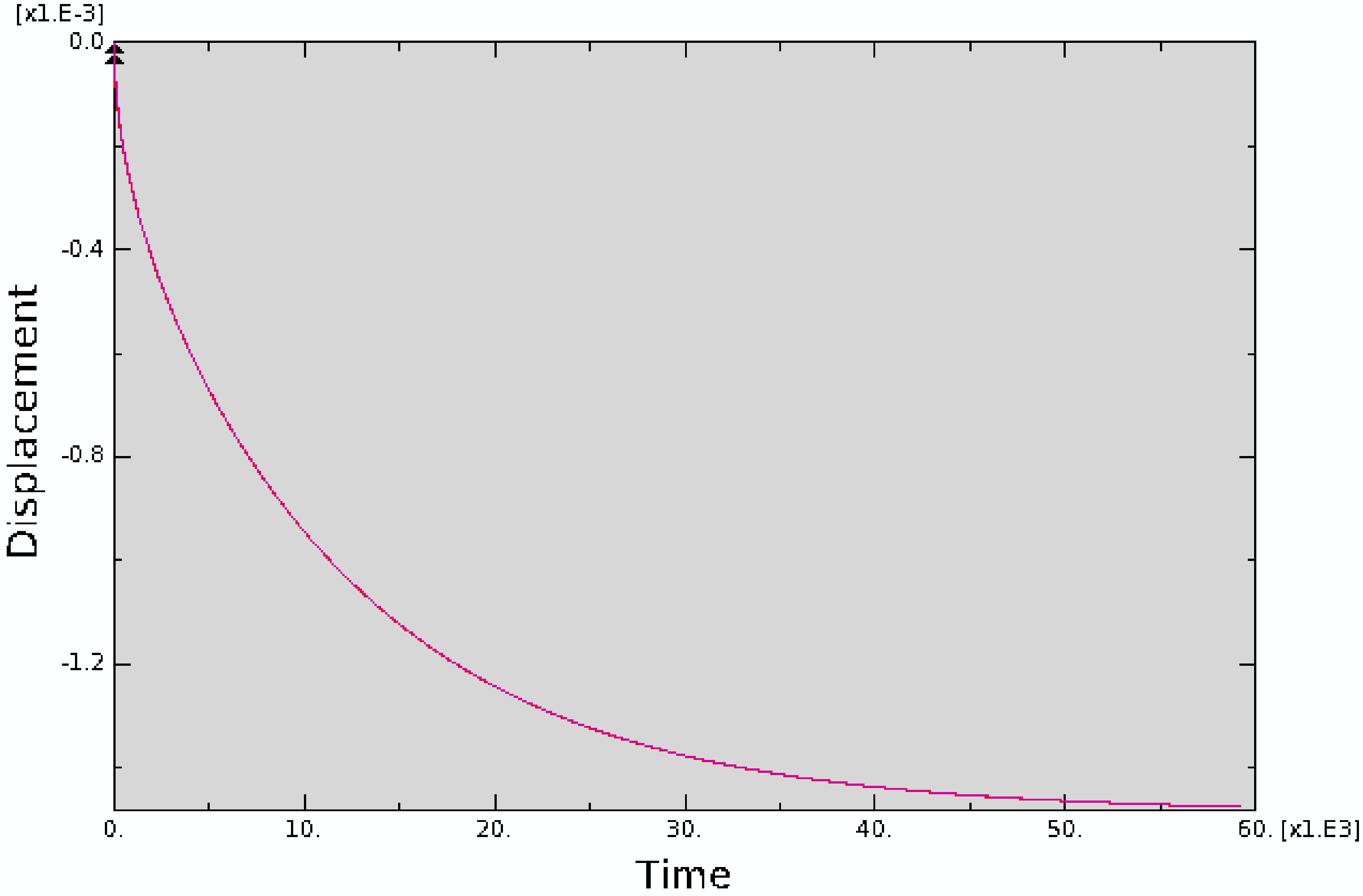
$$\Delta t \geq \frac{\gamma_w (\Delta l)^2}{6 \cdot E \cdot k} \longrightarrow \Delta t \geq \frac{10 \cdot 0.25^2}{6 \cdot 10000 \cdot 10^{-7}} \longrightarrow \Delta t \geq 104sec$$

while we used $\Delta t = 10sec$.

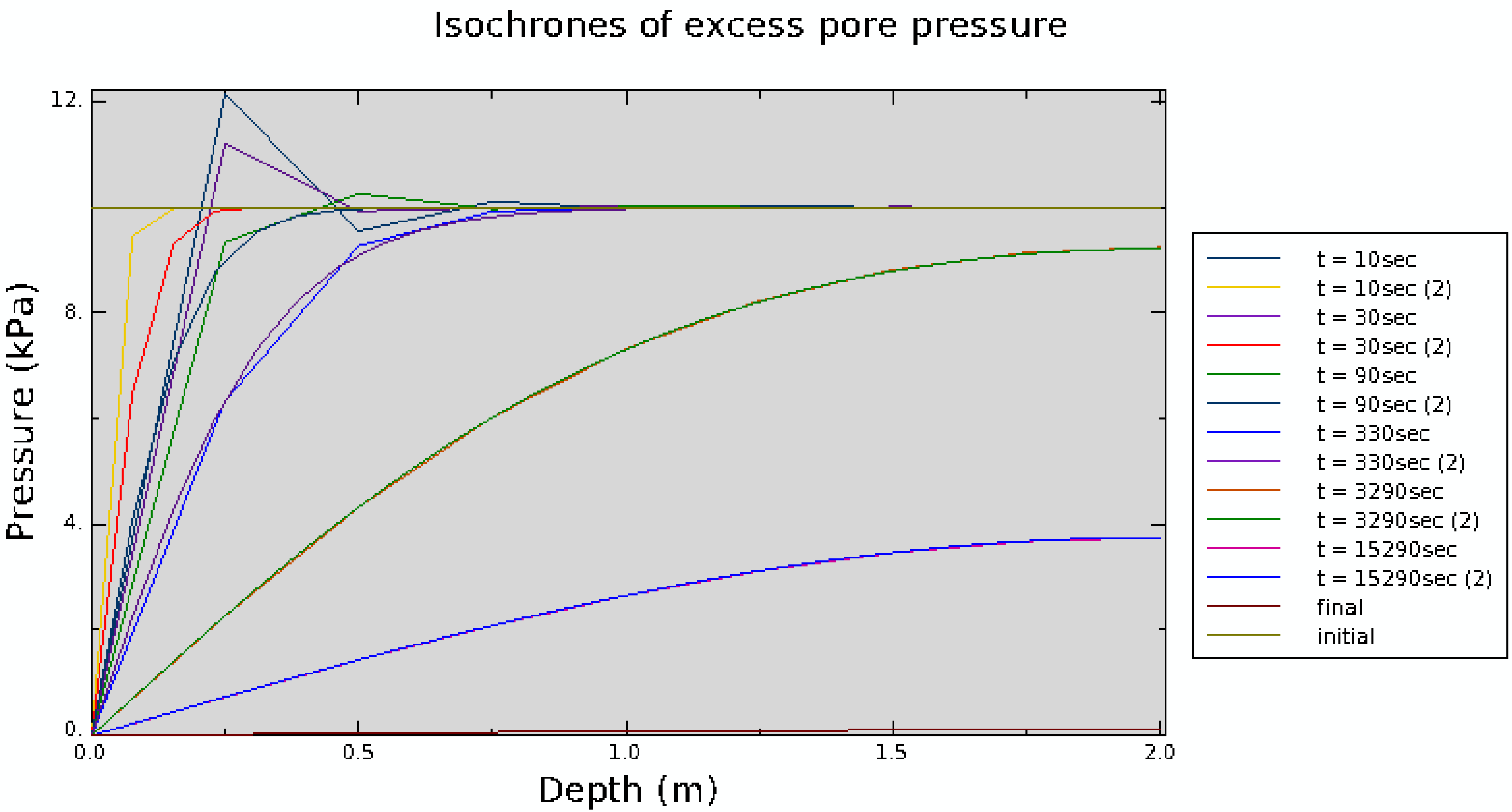
To eliminate the oscillations we need to either increase the initial timestep or refine the mesh.

Let us assume that we indeed want to use $\Delta t = 10sec$ for the initial parts of the consolidation process. Then from the above $\Delta l \leq 7.75 \cdot 10^{-2}m$, i.e. we need at least 26 elements along the depth of $2m$.

Results from a model with refined mesh 4×26 (named “Column_2”) are presented in the following.



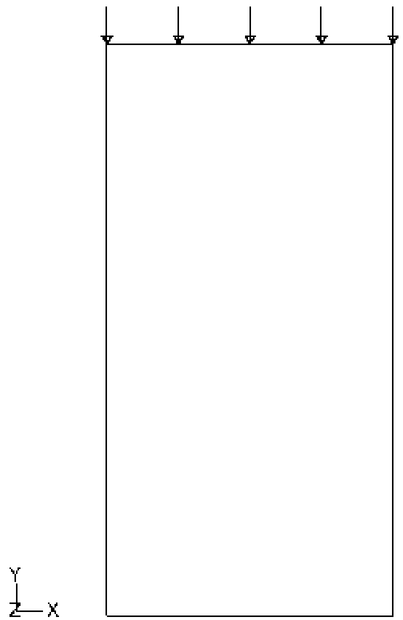
No change in the predicted settlement vs. time.



Initial isochrones no longer exhibit oscillations.

Assume a $2m$-thick layer of soil consolidating in one-dimensional compression under a load of $100kPa$. The load is applied **over the first $\mathbf{2000}$sec** and remains constant thereafter. The soil has permeability $k = 10^{-7}m/sec$ and is linear elastic with $E = 10MPa$ and $\nu = 0.3$.

- Determine the settlement vs time curve of the layer.
- Plot profiles of excess pore pressure vs depth.



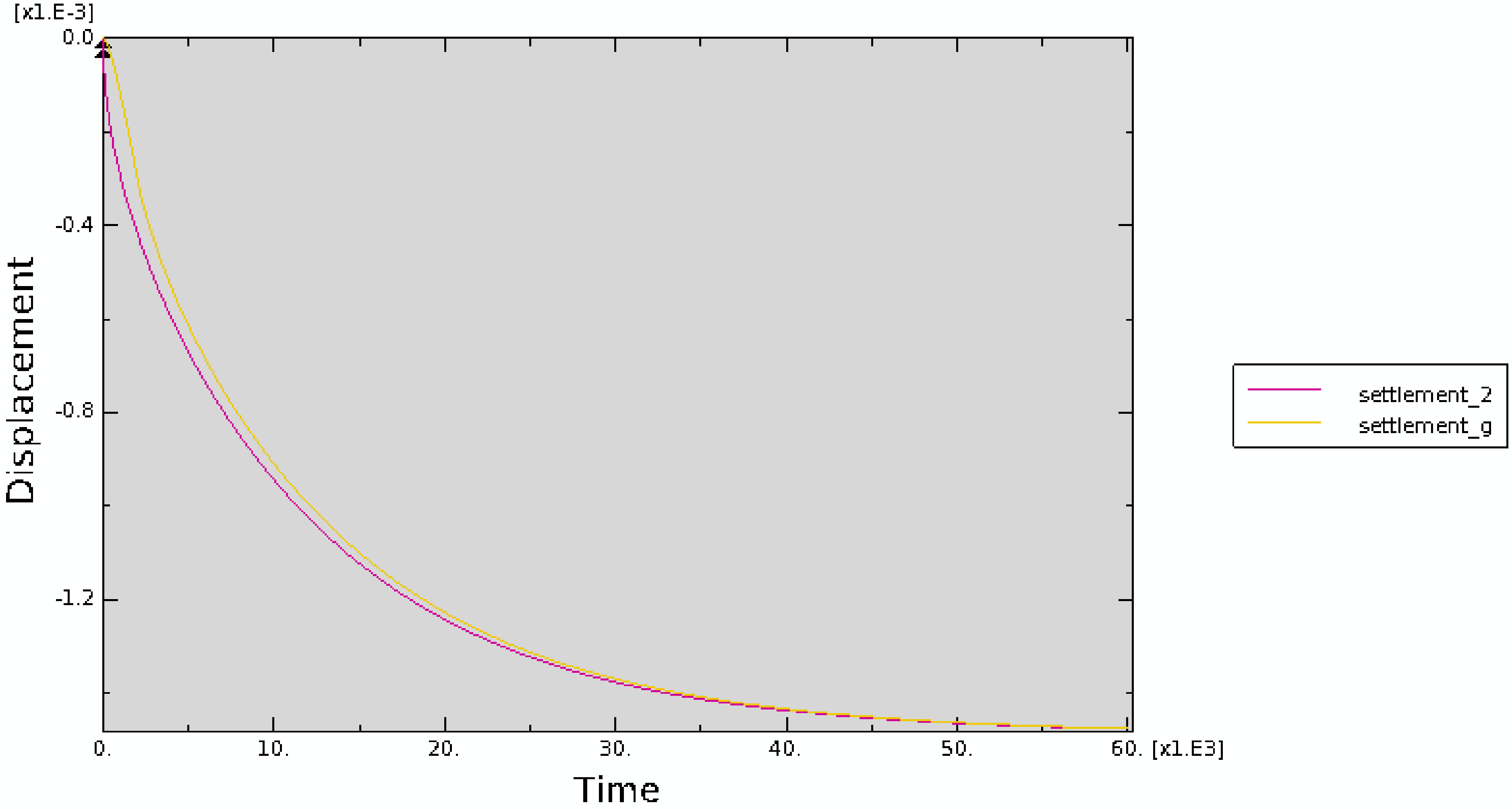
Setting up the analysis:

- Geometry and material properties are trivial to enter.
- So is the mesh and the supports.
- Parameters requiring further thought:
  - Application of load? Drainage/pore pressure?

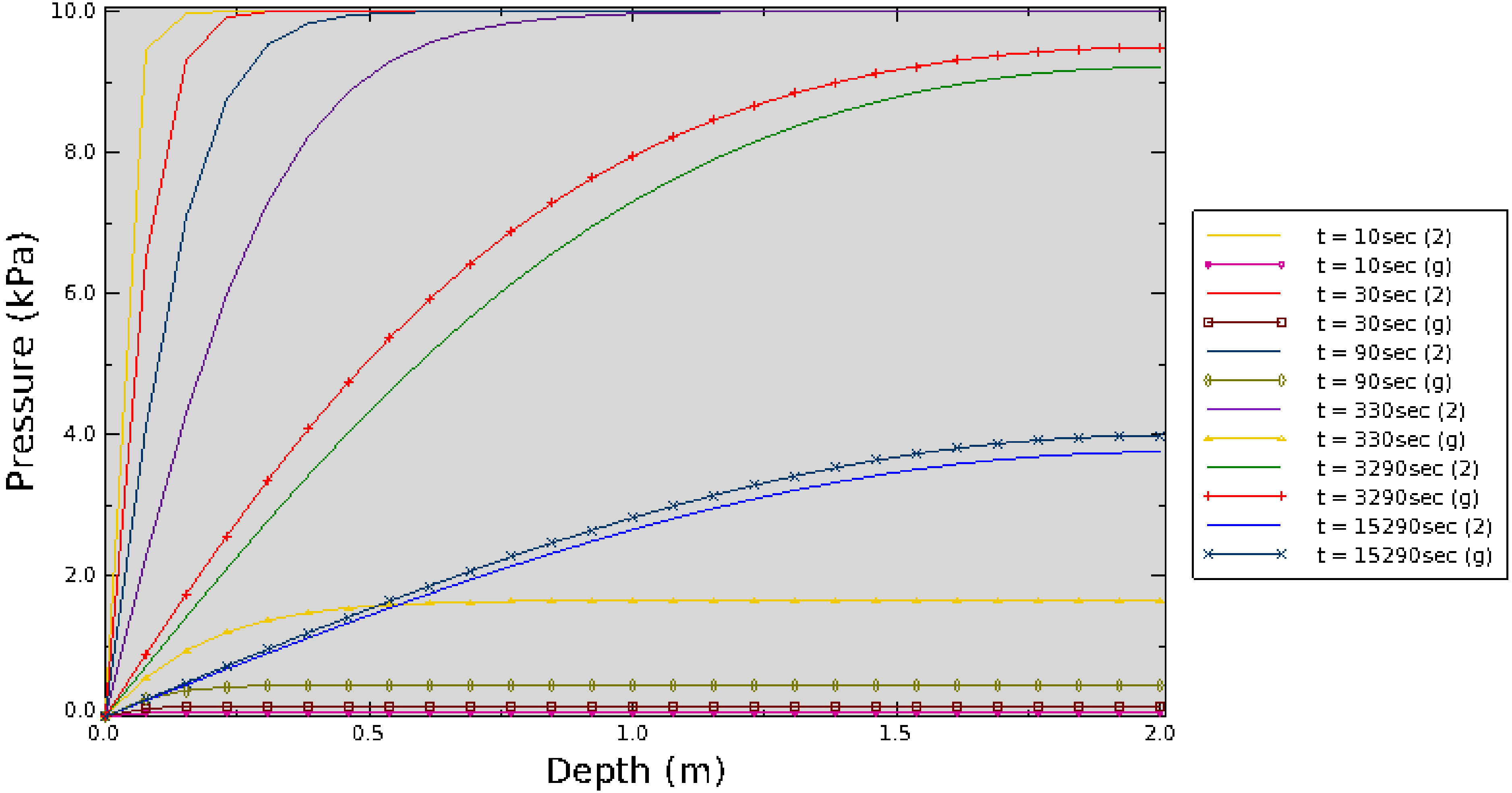
The load is **not** applied instantaneously, so drainage (and consolidation) take place under a gradually increasing external load.

- No need for a first analysis step to set up uniform excess pore pressure.
- Drainage boundary condition should be active from the outset.
- Load should be applied with an *AMPLITUDE to tell ABAQUS that it should be ramped up over $100 sec$.

We use the same fine mesh as in “Column_2”. Results from the new model, named “Column_gradual”, are presented in the following.



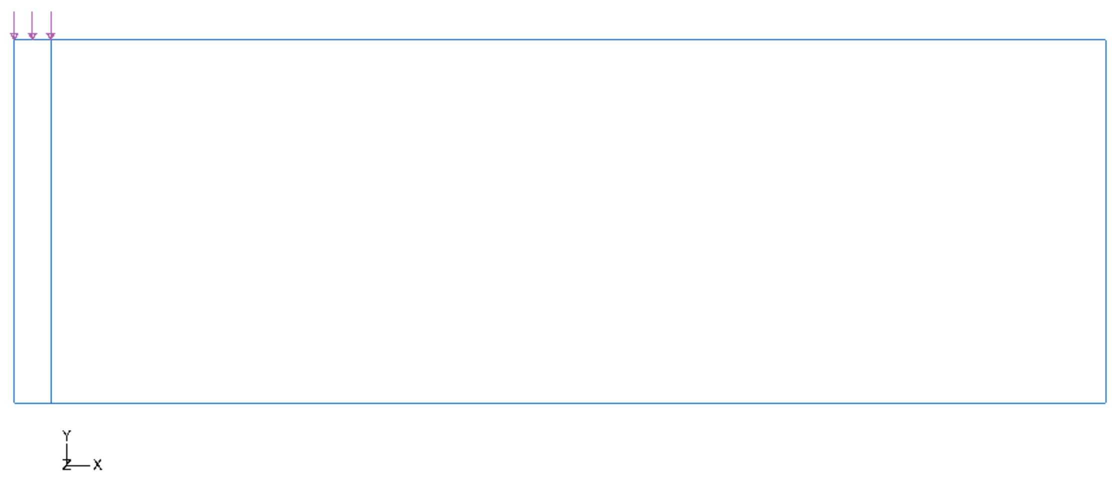
Settlement vs. time predicted more gradual.



Isochrones show slow ramping up of pore pressure, attainment of a maximum and dissipation.

Assume a $2m$-wide footing on a $10m$-thick layer of soil consolidating under a load of $100kPa$. The soil is underlain by impermeable bedrock. The load is applied over the first $5days$ and remains constant thereafter. The soil has permeability $k = 10^{-9}m/sec = 8.64 \cdot 10^{-5}m/day$ and is linear elastic with $E = 10MPa$ and $\nu = 0.3$. Pore pressures are initially hydrostatic.

- Determine the settlement vs time curve of the layer.
- Plot contours of excess pore pressure and vertical effective stress vs depth.



Setting up the analysis (model “Footing” in the “Consolidation.cae” database):

- Geometry, material properties, mesh and supports are trivial to enter.
- Parameters requiring further thought:
  - Application of load? Drainage/pore pressure?
  - Initial condition of pore pressure?

An initial distribution of hydrostatic pore pressure must be set up. Thereafter drainage and consolidation take place under a gradually increasing load.

- Introduce a hydrostatic pore pressure by hand-editing *INITIAL CONDITIONS in the input file.
- Apply load using an *AMPLITUDE: ramp up over $5 days$.
- A boundary condition should ensure that pore pressures remain hydrostatic “far enough” from the footing.
  - Layer will be draining from the top as well as laterally.

The analysis can be set up in two steps as follows:

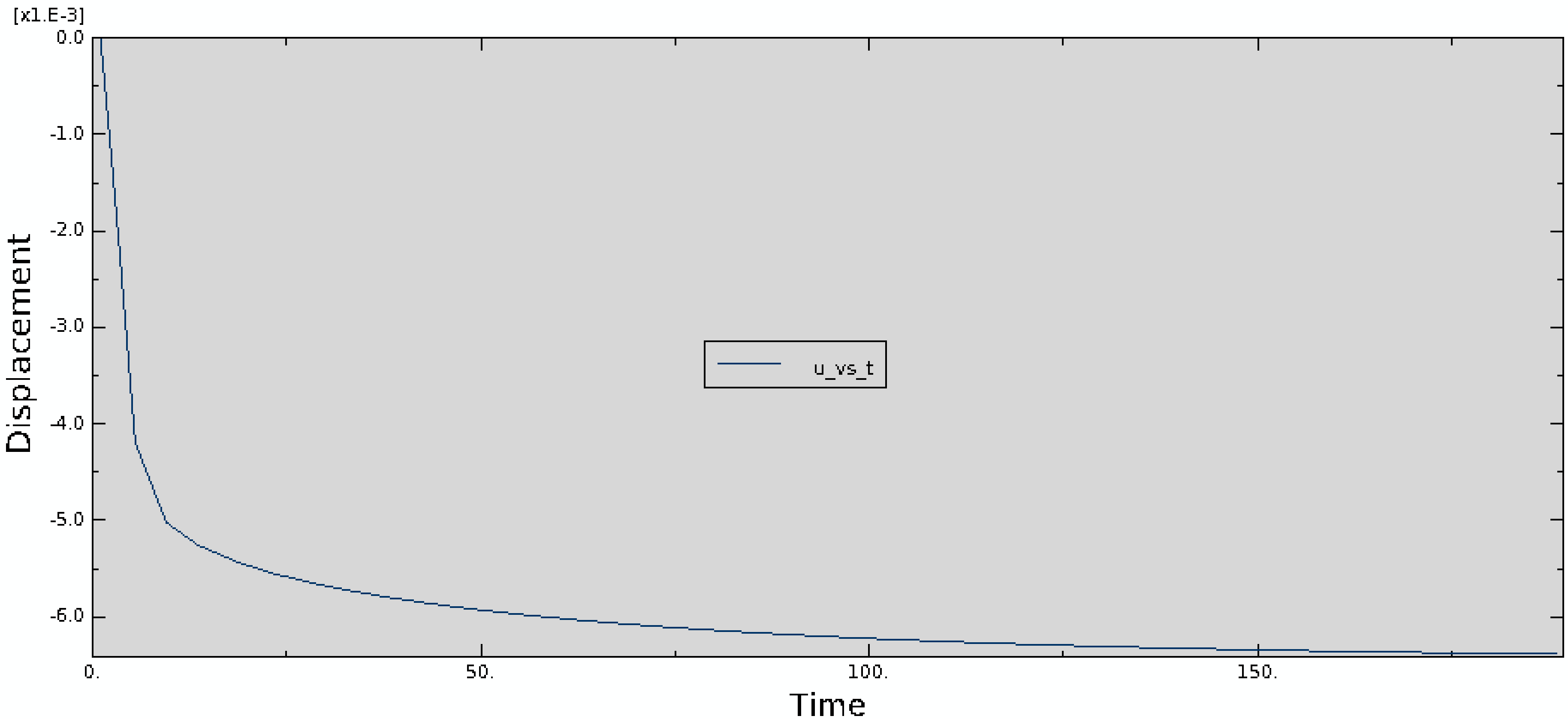
- Introduce supports.
- Introduce a hydrostatic pore pressure by hand-editing *INITIAL CONDITIONS in the input file.
- Run a *Geostatic step.

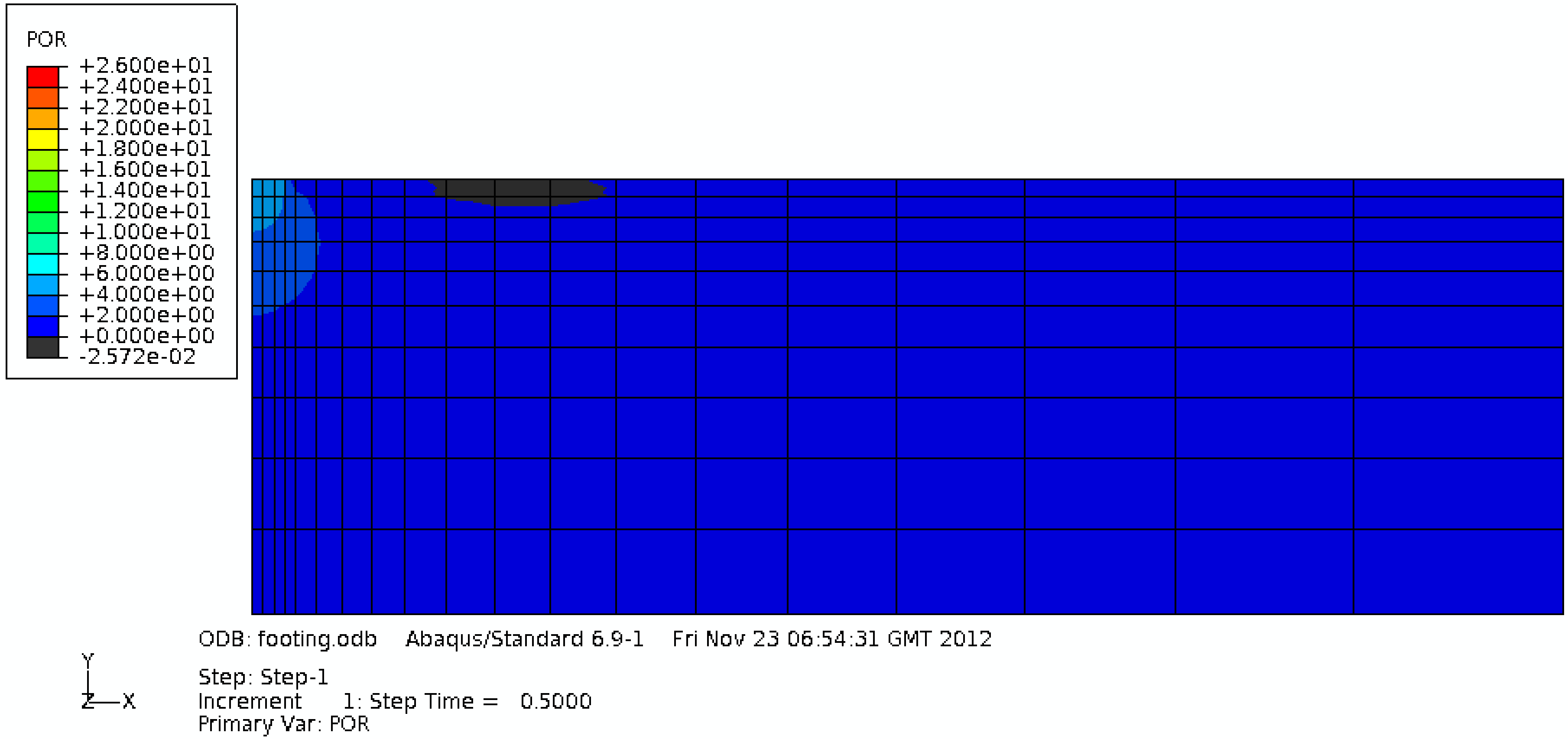
At the end of this step deformation should be zero, as no loads have been prescribed, however initial pore pressures will have the right (hydrostatic) value.

Then:

- Introduce the load.
- Fix far-field pore pressure to its current (hydrostatic) value.
- Fix top-surface pore pressure to its current (zero) value.
- Run a *Soils step.

It is this step that carries out the consolidation analysis.


[x1.E-3]
0.0
-1.0
-2.0
-3.0
-4.0
-5.0
-6.0
Displacement
u_vs_t
0.
50.
100.
150.
Time


POR
+2.600e+01
+2.400e+01
+2.200e+01
+2.000e+01
+1.800e+01
+1.600e+01
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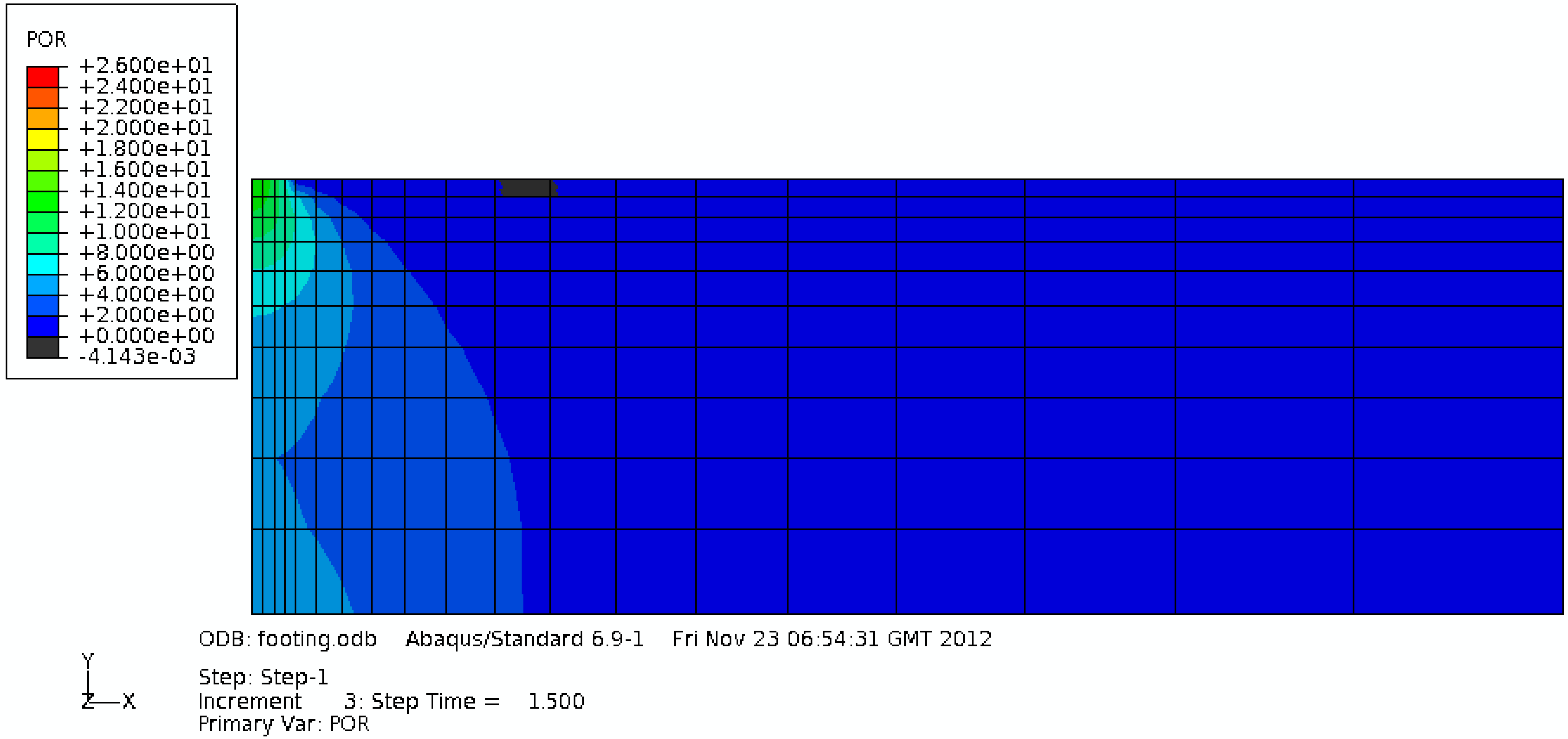
+1.200e+01
+1.000e+01
+8.000e+00
+6.000e+00
+4.000e+00
+2.000e+00
+0.000e+00
-2.572e-02
ODB: footing.odb Abaqus/Standard 6.9-1 Fri Nov 23 06:54:31 GMT 2012
Y
Z X
Step: Step-1
Increment 1: Step Time = 0.5000
Primary Var: POR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+2.400e+01
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+1.800e+01
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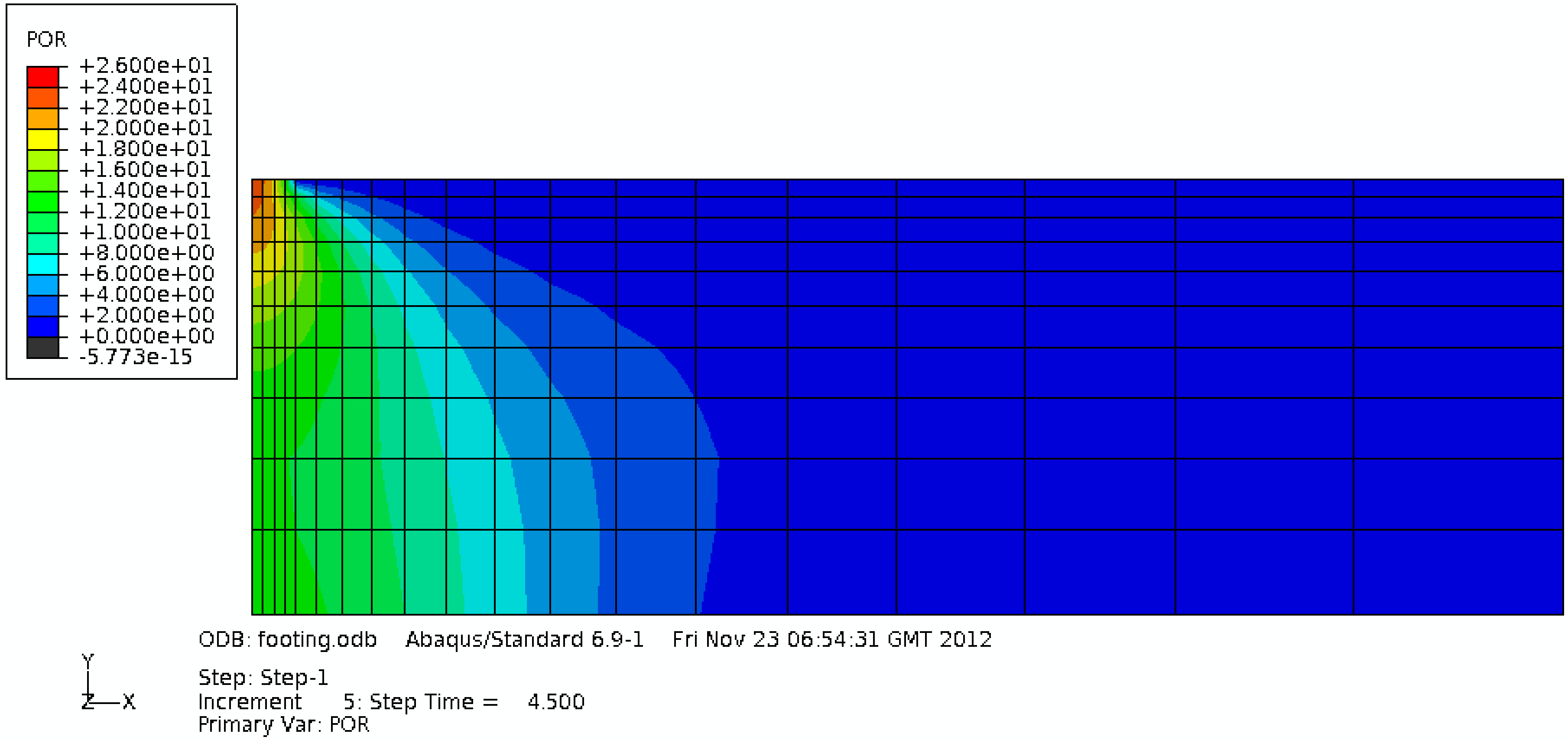
+1.200e+01
+1.000e+01
+8.000e+00
+6.000e+00
+4.000e+00
+2.000e+00
+0.000e+00
-4.143e-03
ODB: footing.odb Abaqus/Standard 6.9-1 Fri Nov 23 06:54:31 GMT 2012
Step: Step-1
Increment 3: Step Time = 1.500
Primary Var: POR
Y
Z
X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+2.400e+01
+2.200e+01
+2.000e+01
+1.800e+01
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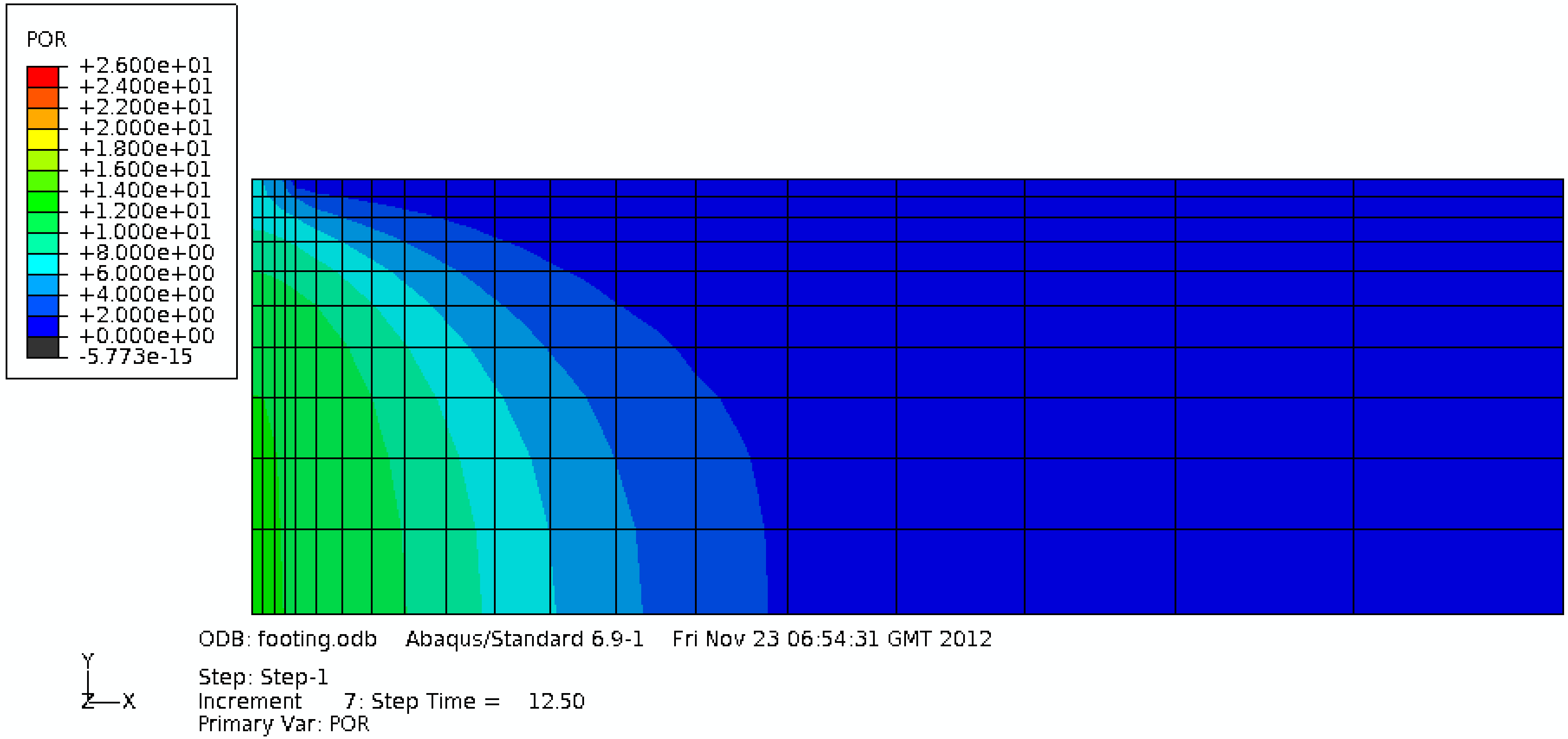
+1.200e+01
+1.000e+01
+8.000e+00
+6.000e+00
+4.000e+00
+2.000e+00
+0.000e+00
-5.773e-15
ODB: footing.odb Abaqus/Standard 6.9-1 Fri Nov 23 06:54:31 GMT 2012
Y
Z X
Step: Step-1
Increment 5: Step Time = 4.500
Primary Var: POR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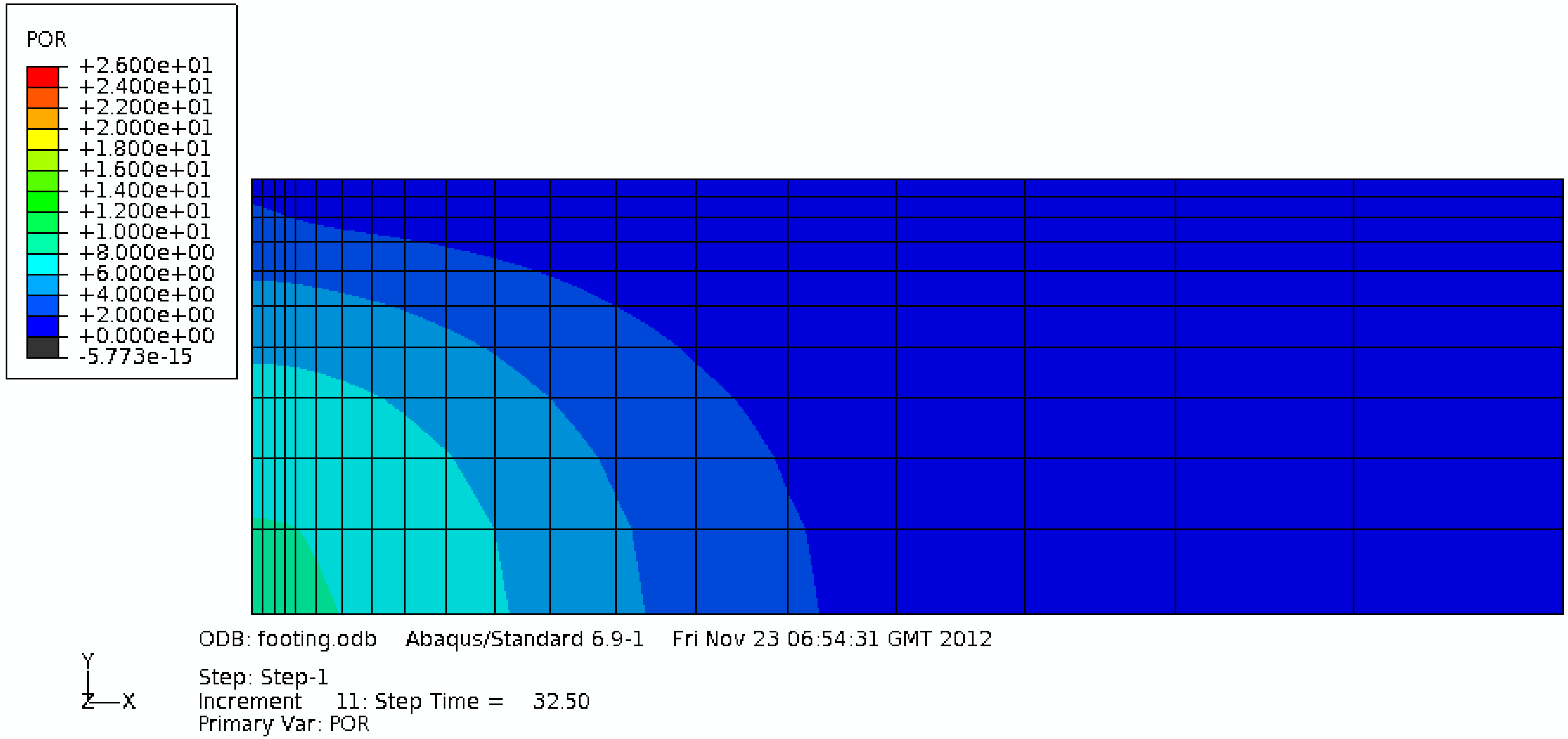
+1.200e+01
+1.000e+01
+8.000e+00
+6.000e+00
+4.000e+00
+2.000e+00
+0.000e+00
-5.773e-15
ODB: footing.odb Abaqus/Standard 6.9-1 Fri Nov 23 06:54:31 GMT 2012
Y
Z X
Step: Step-1
Increment 7: Step Time = 12.50
Primary Var: POR



POR
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+2.400e+01
+2.200e+01
+2.000e+01
+1.800e+01
+1.600e+01
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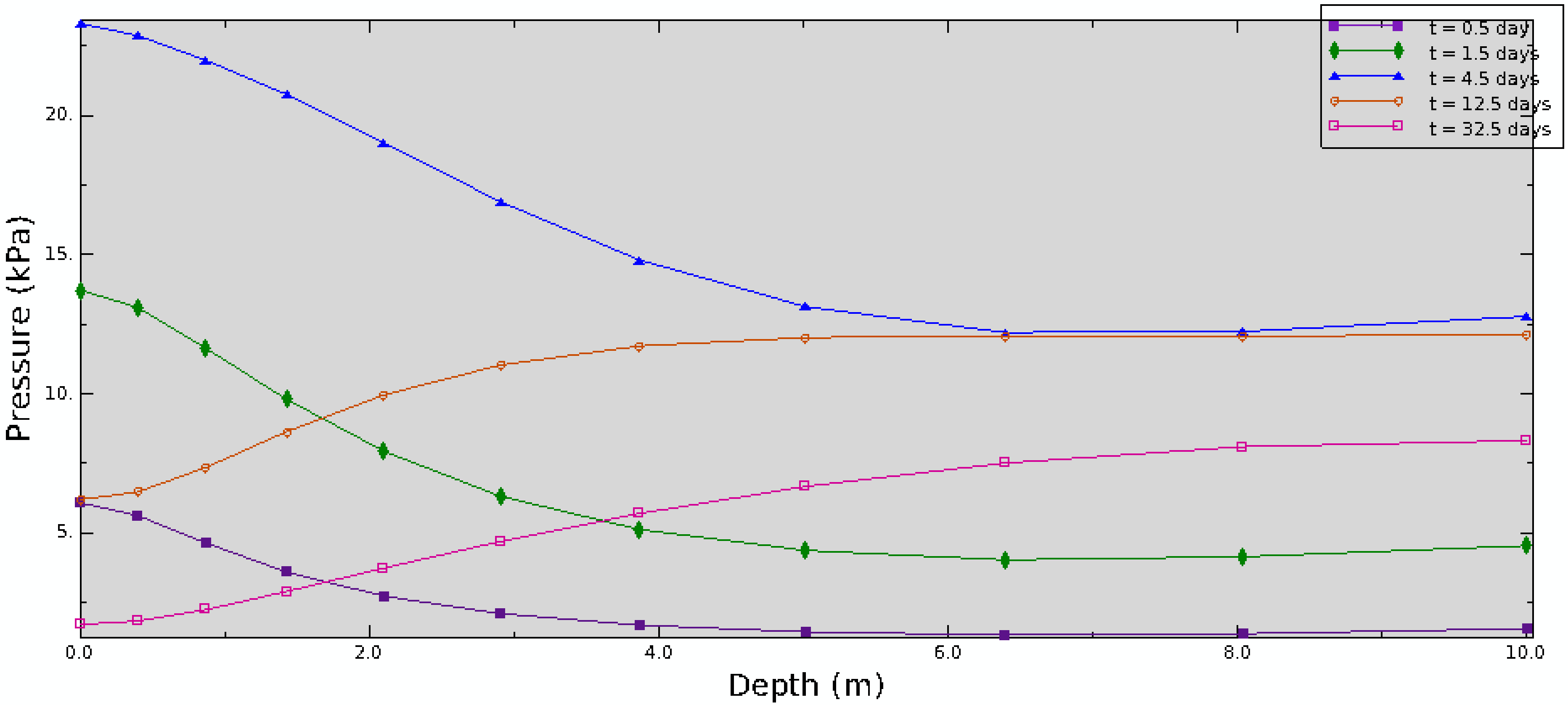
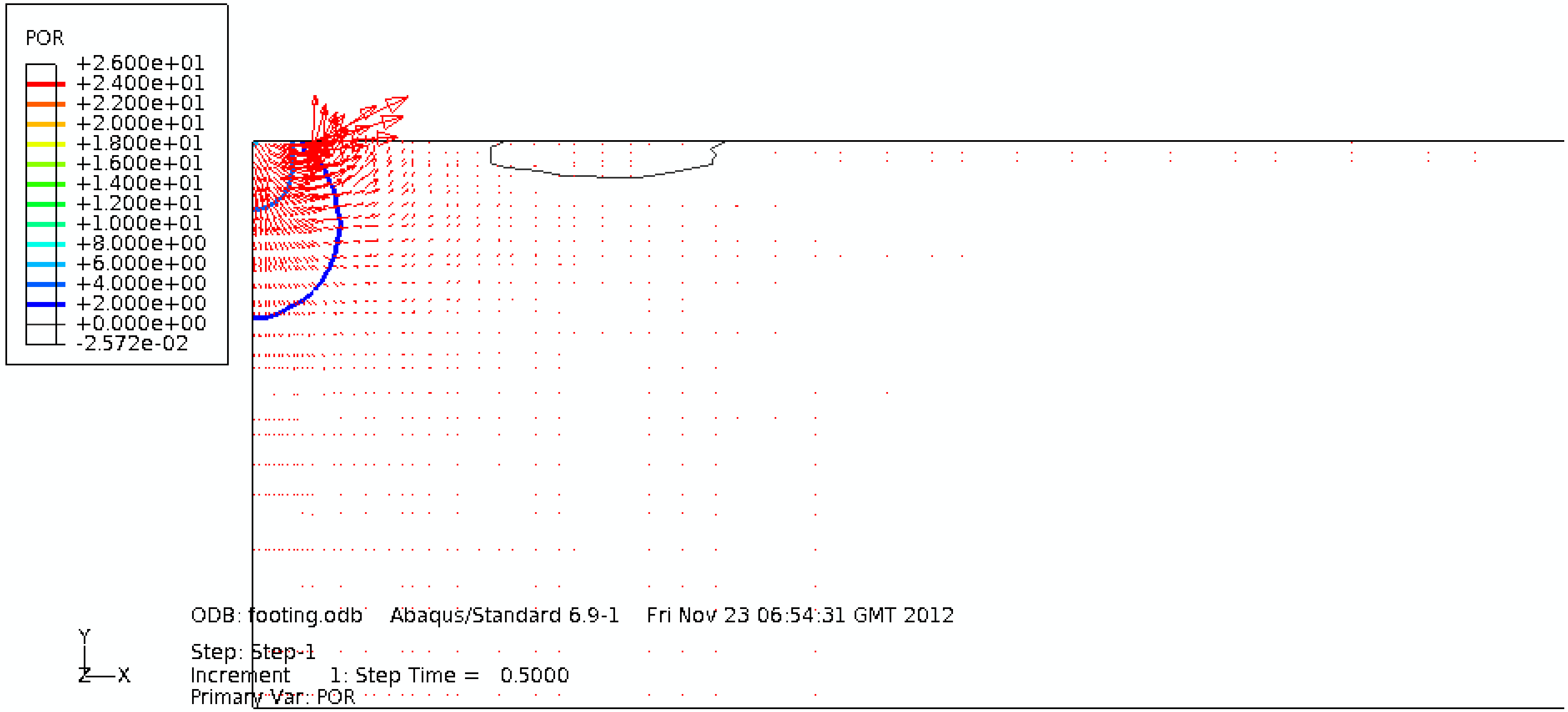
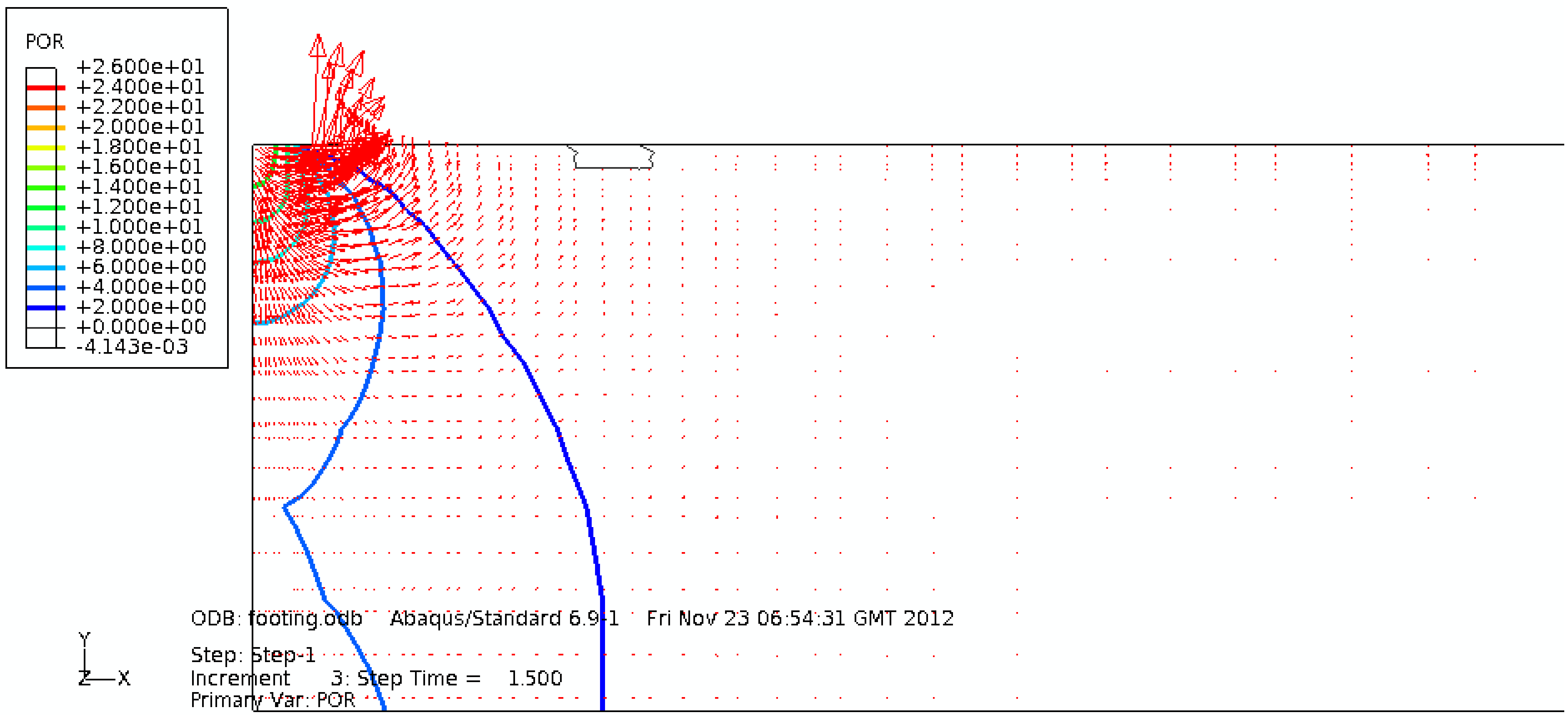
+1.200e+01
+1.000e+01
+8.000e+00
+6.000e+00
+4.000e+00
+2.000e+00
+0.000e+00
-5.773e-15
ODB: footing.odb Abaqus/Standard 6.9-1 Fri Nov 23 06:54:31 GMT 2012
Y
Z X
Step: Step-1
Increment 11: Step Time = 32.50
Primary Var: POR



t = 0.5 day
t = 1.5 days
t = 4.5 days
t = 12.5 days
t = 32.5 days
Pressure (kPa)
20.
15.
10.
5.
0.0
2.0
4.0
6.0
8.0
10.0
Depth (m)



POR
+2.600e+01
+2.400e+01
+2.200e+01
+2.000e+01
+1.800e+01
+1.600e+01
+1.400e+01
+1.200e+01
+1.000e+01
+8.000e+00
+6.000e+00
+4.000e+00
+2.000e+00
+0.000e+00
-2.572e-02
ODB: footing.odb Abaqus/Standard 6.9-1 Fri Nov 23 06:54:31 GMT 2012
Step: Step-1
Increment 1: Step Time = 0.5000
Primary Var: POR
Y
Z
X


POR
+2.600e+01
+2.400e+01
+2.200e+01
+2.000e+01
+1.800e+01
+1.600e+01
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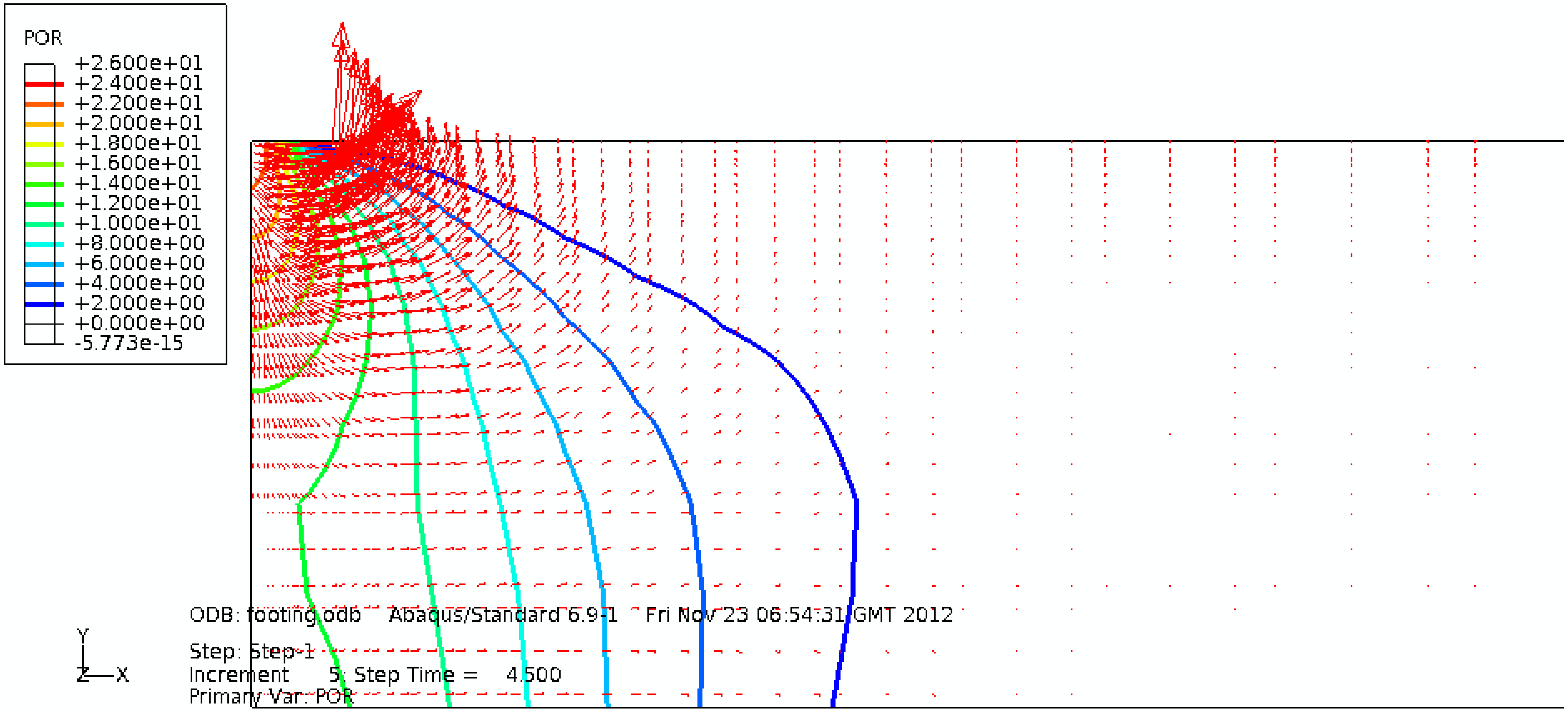
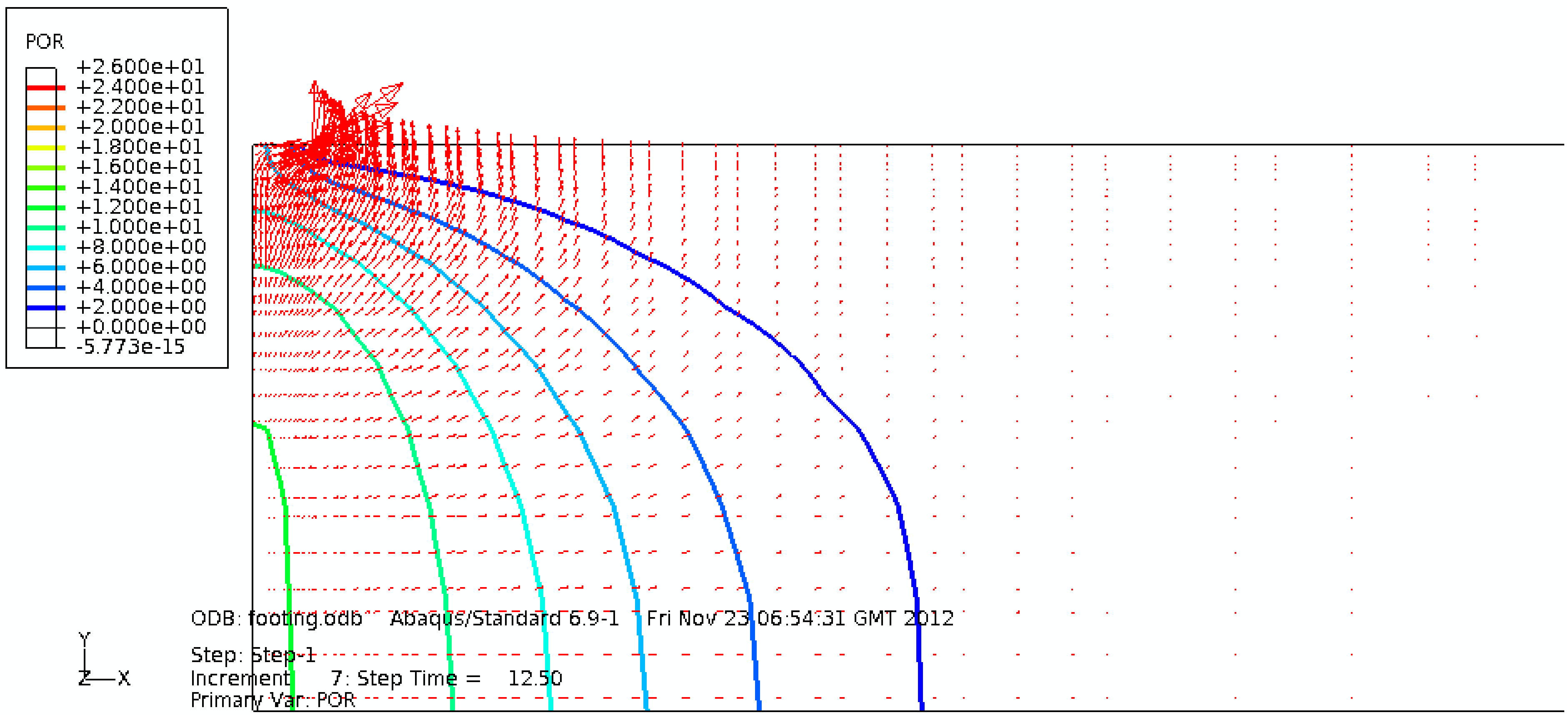
+1.200e+01
+1.000e+01
+8.000e+00
+6.000e+00
+4.000e+00
+2.000e+00
+0.000e+00
-4.143e-03
ODB: footing.odb Abaqus/Standard 6.9-1 Fri Nov 23 06:54:31 GMT 2012
Step: Step-1
Increment 3: Step Time = 1.500
Primary Var: POR
Y
Z
X



POR
+2.600e+01
+2.400e+01
+2.200e+01
+2.000e+01
+1.800e+01
+1.600e+01
+1.400e+01
+1.200e+01
+1.000e+01
+8.000e+00
+6.000e+00
+4.000e+00
+2.000e+00
+0.000e+00
-5.773e-15
ODB: footing.odb Abaqus/Standard 6.9-1 Fri Nov 23 06:54:31 GMT 2012
Step: Step-1
Increment 5: Step Time = 4.500
Primary Var: POR
Y
Z X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+1.600e+01
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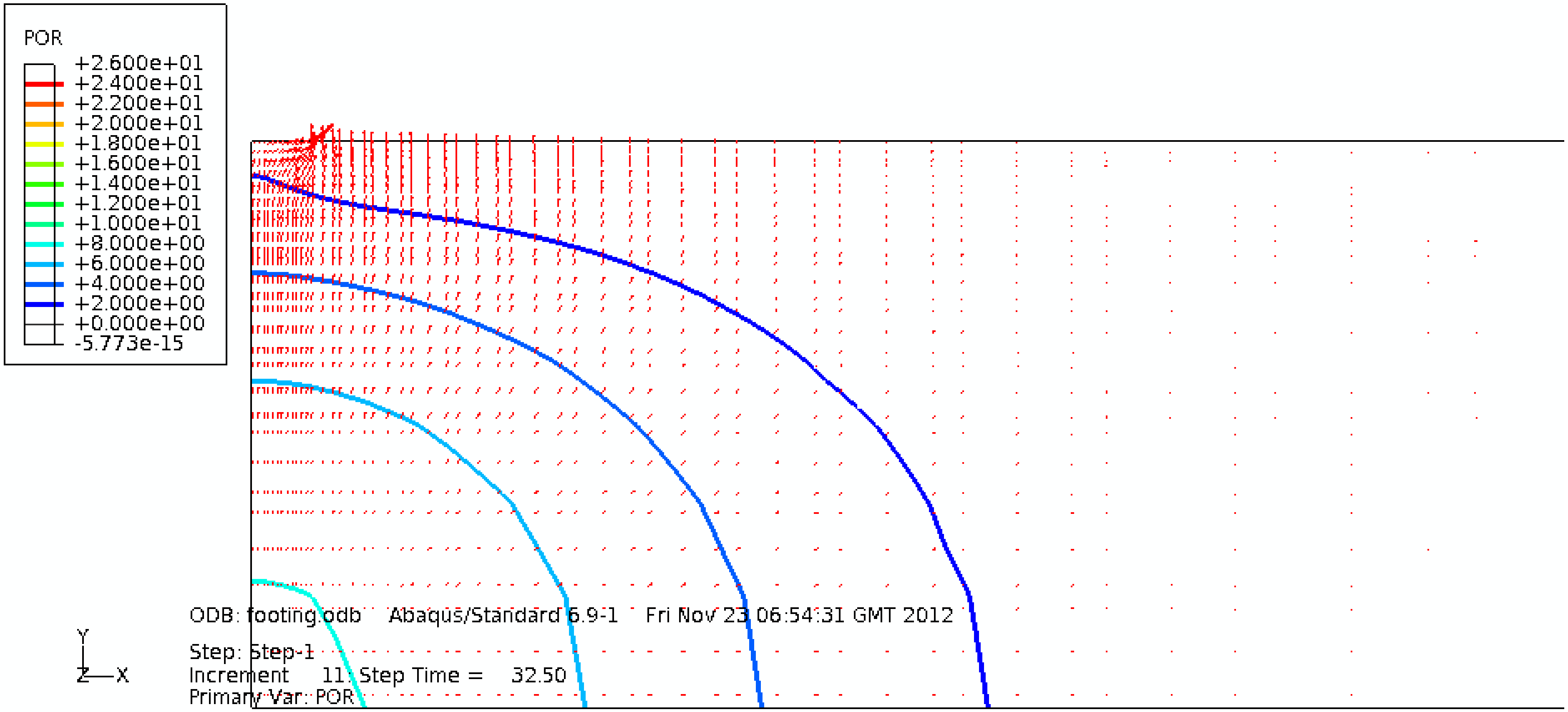
+1.200e+01
+1.000e+01
+8.000e+00
+6.000e+00
+4.000e+00
+2.000e+00
+0.000e+00
-5.773e-15
ODB: footing.odb Abaqus/Standard 6.9-1 Fri Nov 23 06:54:31 GMT 2012
Step: Step-1
Increment 7: Step Time = 12.50
Primary Var: POR
Y
Z
X


POR
+2.600e+01
+2.400e+01
+2.200e+01
+2.000e+01
+1.800e+01
+1.600e+01
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+1.200e+01
+1.000e+01
+8.000e+00
+6.000e+00
+4.000e+00
+2.000e+00
+0.000e+00
-5.773e-15
ODB: footing.odb Abaqus/Standard 6.9-1 Fri Nov 23 06:54:31 GMT 2012
Step: Step-1
Increment 11: Step Time = 32.50
Primary Var: POR
Y
Z
X