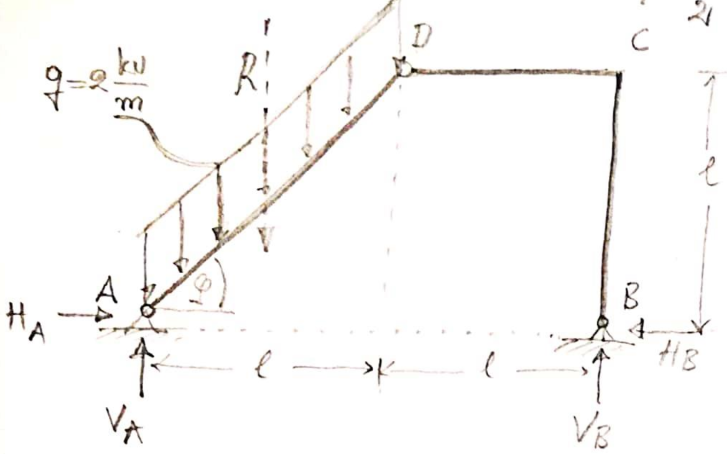
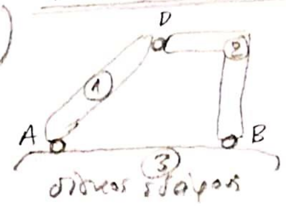
Άσκηση (πλαισιωτό, τριαρθρωτό τόξο)

(q ανά μέτρο μήκος ράβδου: $R = q\frac{l}{\cos\varphi} = \frac{2l}{\frac{\sqrt{2}}{2}} = \frac{4\sqrt{2}}{2}l$)



Ισοστατικότητα, στερεότητα



Τριαρθρωτό τόξο, $n = 3$ δίσκοι, $u = 3\cdot 2 = 6$ δεσμικές ράβδοι.

Ισχύει: $u = 3n - 3$, και A, B, D όχι επί ευθείας, άρα στερεός φορέας.

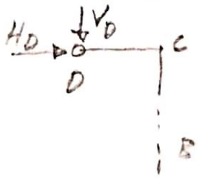
Αντιδράσεις στηρίξεων

(ABCD): $\Sigma F_x = 0 \Rightarrow H_A = H_B$ (1)

$\Sigma M_A = 0 \Rightarrow 2l V_B - \frac{l}{2}R = 0 \Rightarrow V_B = \frac{R}{4} = \frac{1}{4}\cdot\frac{4\sqrt{2}}{2}l = \frac{\sqrt{2}}{2}l\ kN$ (2)

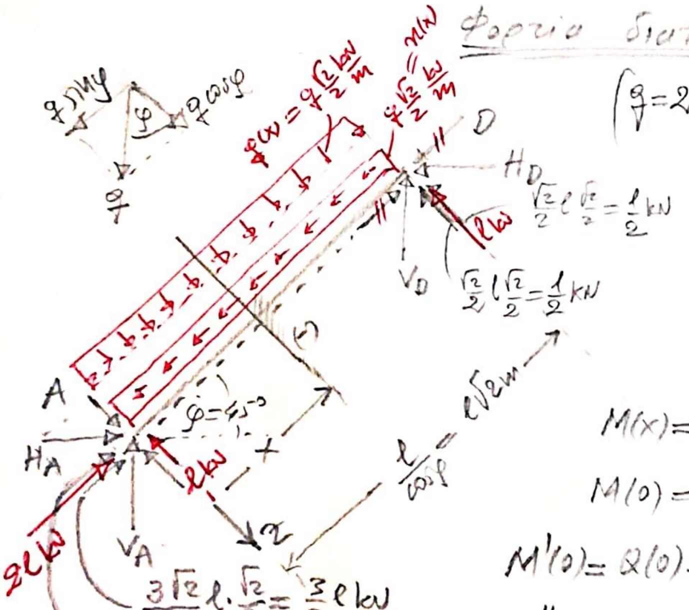
$\Sigma F_z = 0 \Rightarrow V_A = R - V_B = 4\frac{l}{\sqrt{2}} - \frac{\sqrt{2}}{2}l = \frac{3\sqrt{2}}{2}l\ kN$ (3)

(BCD): $\Sigma M_D = 0 \Rightarrow l V_B - l H_A = 0 \Rightarrow H_A \overset{(1)}{=} H_B \overset{(2)}{=} \frac{\sqrt{2}}{2}l\ kN$ (4)



$\Sigma F_x = 0 \Rightarrow H_D = H_B \overset{(4)}{=} \frac{\sqrt{2}}{2}l\ kN$, $\Sigma F_z = 0 \Rightarrow V_D = V_B \overset{(2)}{=} \frac{\sqrt{2}}{2}l\ kN$

Φορτία διατομής N, Q, M



$\left(q = 2\frac{kN}{m}\right)$

• Κεκλιμένο τμήμα AD, $0 \le x \le \frac{l}{\cos\varphi} = l\sqrt{2}$

$N(x) = -2l + q\frac{\sqrt{2}}{2}x = -2l + \sqrt{2}x\ kN$, γραμμ.

$N(0) = -2l$, $N(l\sqrt{2}) = -2l + \sqrt{2}\sqrt{2}l = 0\ kN$ ✓

$Q(x) = l - q\frac{\sqrt{2}}{2}x = l - \sqrt{2}x\ kN$, γραμμ.

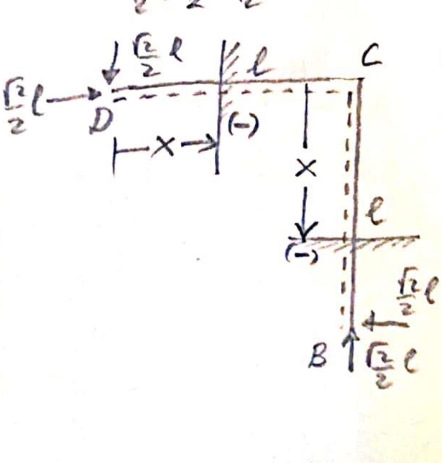
$Q(0) = l\ kN$, $Q(l\sqrt{2}) = l - \sqrt{2}\sqrt{2}l = -l\ kN$ ✓

$M(x) = xl - \frac{x}{2}q\frac{\sqrt{2}}{2}x = xl - \frac{\sqrt{2}}{2}x^2\ kNm$, παραβολικό.

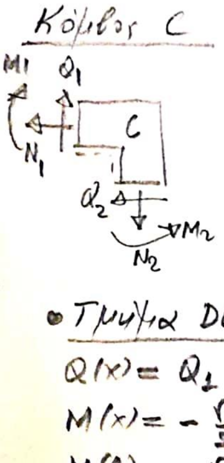
$M(0) = 0\ kNm$, $M(l\sqrt{2}) = l^2\sqrt{2} - \frac{\sqrt{2}}{2}l^2 2 = 0\ kNm$ ✓ (άρθρ. D)

$M'(0) = Q(0) = l\ kN$ (κλίση), $M'(l\sqrt{2}) = Q(l\sqrt{2}) = -l\ kN$ (κλίση)

$M''(x) = -q(x) = -q\frac{\sqrt{2}}{2} = -\sqrt{2}\frac{kN}{m}$ (αρν. καμπυλότητα), $M^{AD}_{max} = M\left(\frac{l\sqrt{2}}{2}\right) = l^2\frac{\sqrt{2}}{4}\ kNm$.



Κόμβος C



$N_1 = -\frac{\sqrt{2}}{2}l\ kN$, $Q_1 = -\frac{\sqrt{2}}{2}l\ kN$, $M_1 = -l\frac{\sqrt{2}}{2}l = -\frac{\sqrt{2}}{2}l^2\ kNm$.

Από ισορροπία:

$N_2 = Q_1 = -\frac{\sqrt{2}}{2}l\ kN$ ✓, $Q_2 = -N_1 = +\frac{\sqrt{2}}{2}l\ kN$ ✓, $M_2 = M_1 = -\frac{\sqrt{2}}{2}l^2\ kNm$ ✓

• Τμήμα DC, $0 \le x \le l$ $N(x) = N_1 = -\frac{\sqrt{2}}{2}l\ kN$, σταθερό διάγραμμα $\left(\frac{dN}{dx} = 0\right)$.

$Q(x) = Q_1 = -\frac{\sqrt{2}}{2}l\ kN$, σταθερό διάγραμμα $\left(\frac{dQ}{dx} = 0\right)$.

$M(x) = -\frac{\sqrt{2}}{2}lx\ kNm$, γραμμικό, $M(0) = 0\ kNm$ ✓ (άρθρ. D),

$M(l) = -\frac{\sqrt{2}}{2}l^2\ kNm = M_1$ ✓

Τμήμα CB, $0 \le x \le \ell$

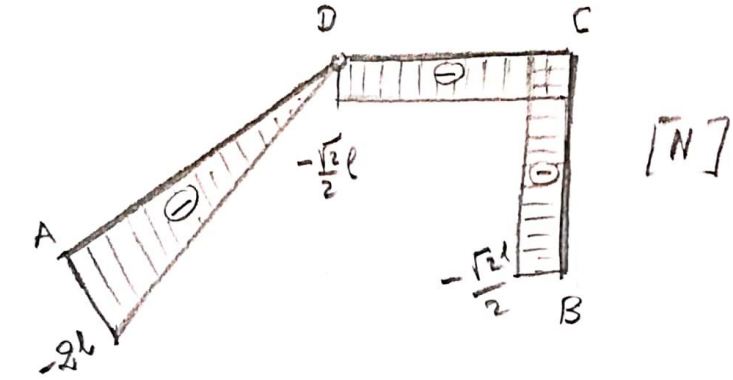
$N(x) = N_2 = -\frac{\sqrt{2}}{2}\ell$ kN, παντού διαφορετικό ($\frac{dN}{dx} = n(x) = 0$)

$Q(x) = Q_2 = \frac{\sqrt{2}}{2}\ell$ kN, ... ($\frac{dQ}{dx} = q(x) = 0$)

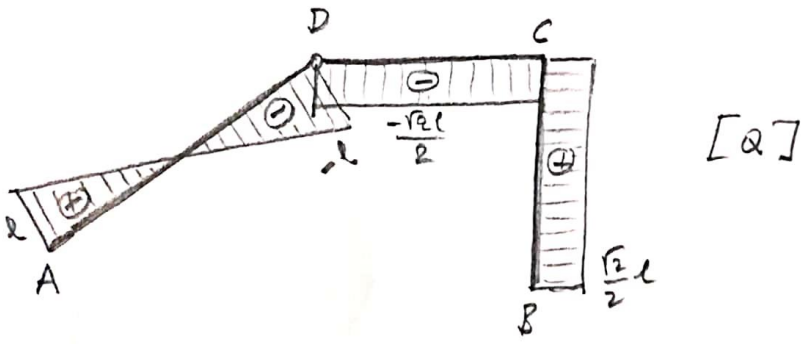
$M(x) = M_2 + x \cdot Q_2 = -\frac{\sqrt{2}}{2}\ell^2 + \frac{\sqrt{2}}{2}\ell \cdot x$ kNm, γραμμική διαφορετική.

$M(0) = -\frac{\sqrt{2}}{2}\ell^2$ kNm ✓, $M(\ell) = 0$ kNm ✓

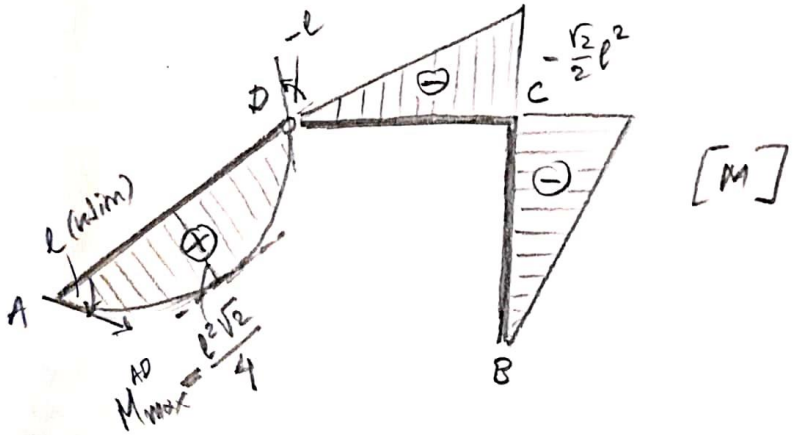
Διαγράμματα N, Q, M.



[N]



[Q]



[M]