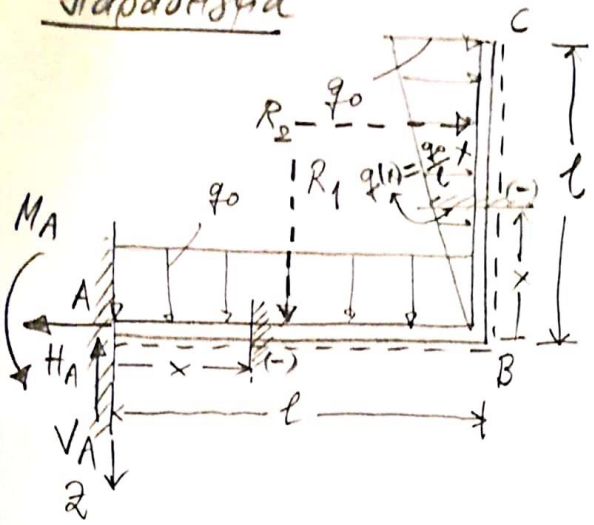
Παράδειγμα



Αντιδράσεις στηρίξεων

$\Sigma F_x = 0 \Rightarrow H_A = R_2 = \frac{q_0\ell}{2}$ kN

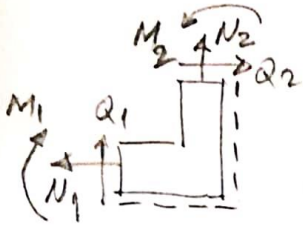
$\Sigma F_z = 0 \Rightarrow V_A = R_1 = q_0\ell$ kN

$\Sigma M_A = 0 \Rightarrow +M_A - \frac{\ell}{2}R_1 - \frac{2\ell}{3}R_2 = 0 \Rightarrow$

$\Rightarrow M_A = \frac{\ell}{2}\left(q_0\ell + \frac{2q_0\ell}{3}\right) = \frac{5q_0\ell^2}{6}$ kNm

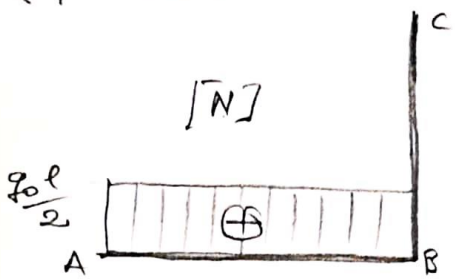
Διατομή Β

$N_2 = 0$ kN, $Q_2 = \frac{q_0\ell}{2}$ kN, $M_2 = -\frac{2\ell}{3}\cdot\frac{q_0\ell}{2} = -\frac{q_0\ell^2}{3}$ kNm



Από ισορροπία στη διατομή Β:

$N_1 = Q_2 = \frac{q_0\ell}{2}$ kN, $Q_1 = -N_2 = 0$ kN, $M_1 = M_2 = -\frac{q_0\ell^2}{3}$ kNm

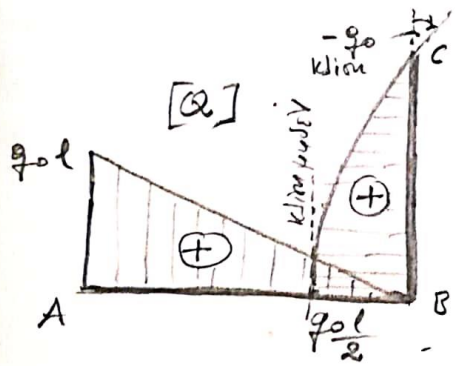


Αξονικός δυνάμης N(x)

AB, $0 \le x \le \ell$: $N(x) = H_A = \frac{q_0\ell}{2}$ kN $(= N_1 \checkmark)$

και $\frac{dN}{dx} = -n(x) = 0 \Rightarrow$ N(x) σταθερό διάγραμμα

BC, $0 \le x \le \ell$: $N(x) = 0$ kN $(= N_2 \checkmark)$, μηδενικό



Τέμνουσες δυνάμης Q(x)

AB, $0 \le x \le \ell$: $Q(x) = V_A - q_0 x = q_0\ell - q_0 x$ kN, γραμμή.

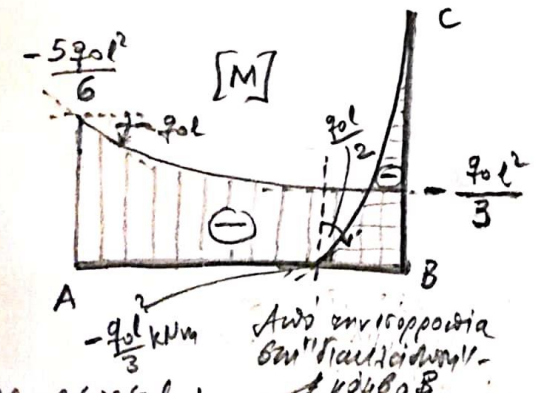
$Q(0) = q_0\ell$ kN ✓, $Q(\ell) = 0$ kN $= Q_1 \checkmark$ $(Q'(x) = -q_0 \checkmark)$

BC, $0 \le x \le \ell$: $Q_2 - \frac{q_0}{\ell}\frac{x^2}{2} = \frac{q_0\ell}{2} - \frac{q_0}{\ell}\frac{x^2}{2}$ kN, παραβολή.

$Q(0) = \frac{q_0\ell}{2}$ kN $= Q_2 \checkmark$, $Q(\ell) = 0$ kN ✓

$Q'(x) = -q(x) = -\frac{q_0}{\ell}x \frac{\text{kN}}{\text{m}} \checkmark$, $Q'(0) = 0 \frac{\text{kN}}{\text{m}}$ (κλίση), $Q'(\ell) = -q_0 \frac{\text{kN}}{\text{m}}$ (κλίση)

$Q''(x) = -\frac{q_0}{\ell}\frac{\text{kN}}{\text{m}^2}$ (αρνητική καμπυλότητα)



Ροπές κάμψης M(x)

AB, $0 \le x \le \ell$: $M(x) = M(0) + \int Q(x)\,dx =$

$= -\frac{5q_0\ell^2}{6} + \int (q_0\ell - q_0 x)\,dx = -\frac{5q_0\ell^2}{6} + q_0\ell x - q_0\frac{x^2}{2}$ kNm, παραβολή

$M(0) = -\frac{5q_0\ell^2}{6}$ kNm ✓, $M(\ell) = -\frac{q_0\ell^2}{3}$ kNm $= M_1 \checkmark$,

$M'(0) = Q(0) = q_0\ell$ kN (κλίση), $M'(\ell) = Q(\ell) = 0$ kN κλίση,

$M''(x) = -q(x) = -q_0 \frac{\text{kN}}{\text{m}}$ (αρνητική καμπυλότητα)

BC, $0 \le x \le \ell$:

$M(x) = M_2 + x \cdot Q_2 - \frac{x}{3}\frac{q_0}{\ell}x \cdot x \cdot \frac{1}{2} = -\frac{q_0\ell^2}{3} - \frac{q_0 x^3}{6\ell} + \frac{q_0\ell}{2}x$ kNm, μηδενικό σ' $M(x) = M_2 + \int Q(x)\,dx \Rightarrow$

$\Rightarrow M(x) = -\frac{q_0\ell^2}{3} + \int\left(\frac{q_0\ell}{2} - \frac{q_0}{2\ell}x^2\right)dx = -\frac{q_0\ell^2}{3} + \frac{q_0\ell}{2}x - \frac{q_0 x^3}{6\ell}$ kNm ✓, $M(0) = -\frac{q_0\ell^2}{3}$ kNm $= M_2 \checkmark$, $M(\ell) = 0$ kNm ✓

$M'(0) = Q(0) = \frac{q_0\ell}{2}$ kN (κλίση), $M'(\ell) = Q(\ell) = 0$ kN (κλίση), $M''(\ell) = -q(x) = -\frac{q_0}{\ell}x \frac{\text{kN}}{\text{m}}$ (αρνητική καμπυλότητα)