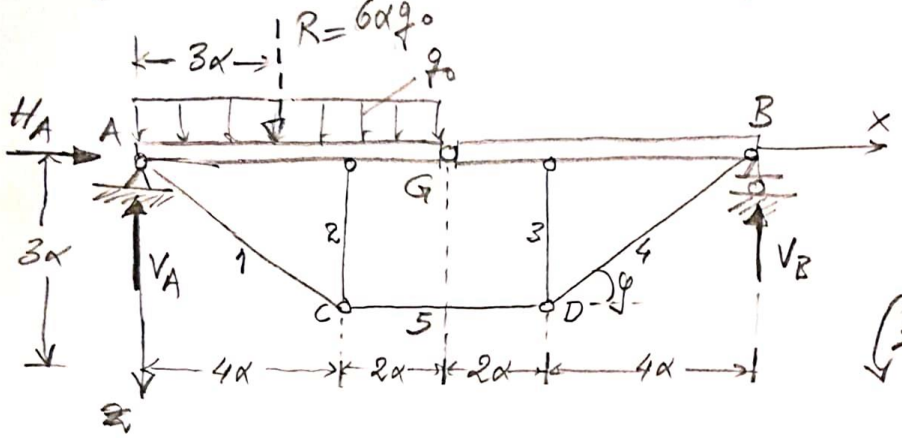
Μικτοί φορείς (N, Q, M)

[Παράδειγμα 2, σελ. 237–240, Γεωργιάδης–Ζήσης]



(Γ ο π α π ι ώ σ τ α – Σ τ ρ ο φ ώ σ τ α ε, έ ω ς κ α ι έ χ ω)

Αντιδράσεις στηρίξεων

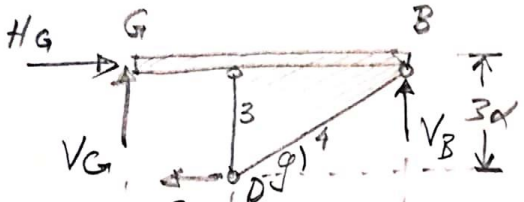
$\curvearrowleft \Sigma M_A = 0 \Rightarrow 12\alpha V_B - 3\alpha \cdot 6\alpha q_0 = 0$

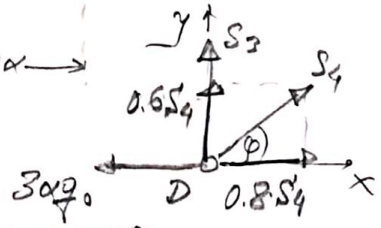
$\Rightarrow \boxed{V_B = \frac{3 \cdot 6\alpha q_0}{3 \cdot 4} = \frac{3}{2}\alpha q_0 \text{ kN}}$

$\Sigma F_x = 0 \Rightarrow \boxed{H_A = 0 \text{ kN}}$, $\Sigma F_y = 0 \Rightarrow \boxed{V_A = 6\alpha q_0 - \frac{3}{2}\alpha q_0 = \frac{9\alpha q_0}{2} \text{ kN}}$

Κινηματικοί S_1, S_2, S_3, S_4, S_5

$\curvearrowleft \Sigma M_G = 0 \Rightarrow 3\alpha S_5 = 6\alpha V_B \Rightarrow \boxed{S_5 = 3\alpha q_0 \text{ kN}}$





$\varphi = \tan^{-1}\frac{3\alpha}{4\alpha} = 36.87°$

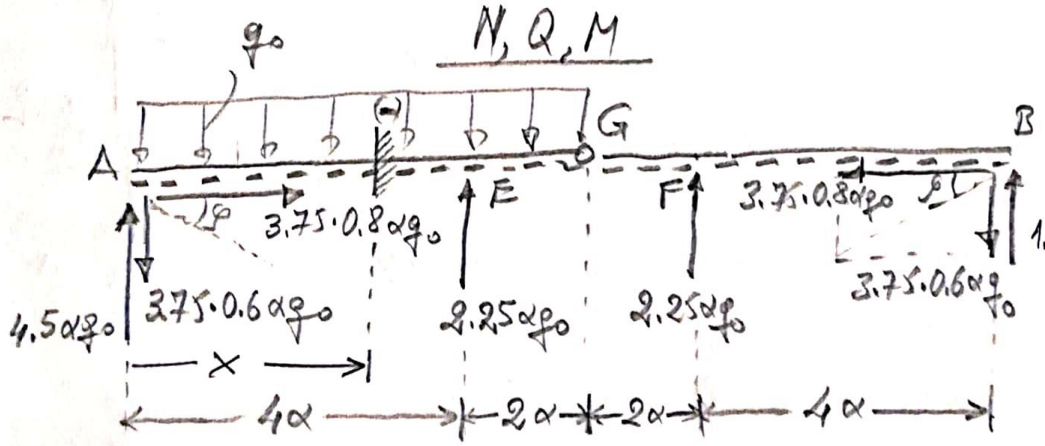
$\sin\varphi = 0.60$, $\cos\varphi = 0.80$

Κόμβος D

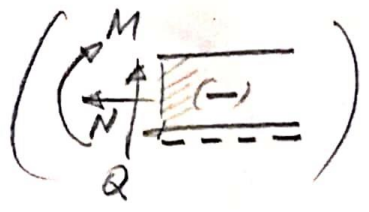
$\Sigma F_x = 0 \rightarrow \boxed{S_4 = 3.75\alpha q_0}$, $\Sigma F_y = 0 \Rightarrow \boxed{S_3 = -0.6 S_4 = -2.25\alpha q_0 \text{ kN}}$ Θλιπτική

Και στις υπόλοιπες: $\boxed{S_1 = 3.75\alpha q_0 \text{ kN}}$, $\boxed{S_2 = -2.25\alpha q_0 \text{ kN}}$ Θλιπτική

N, Q, M



(3.75·0.8 = 3.0
3.75·0.6 = 2.25)



Τμήμα AE, $0 \le x \le 4\alpha$

$N(x) = -3\alpha q_0$ kN, σταθ. διαφορετικά

$Q(x) = 4.5\alpha q_0 - 2.25\alpha q_0 - q_0 x = 2.25\alpha q_0 - q_0 x$ kN, γραμμικό

$Q(0) = +2.25\alpha q_0$ kN, $Q(4\alpha) \overset{\text{αρ.}}{=} -1.75\alpha q_0$ kN, για $x = 2.25\alpha$, $Q^{AE}(x) = 0$ kN σημείο M^{AE}_{max}.

$M(x) = x \cdot 4.5\alpha q_0 - x \cdot 2.25\alpha q_0 - \frac{x}{2} \cdot x q_0 = 2.25\alpha q_0 x - \frac{q_0}{2} x^2$ kNm, παραβολικό

$M(0) = 0$, $M(4\alpha) = \alpha^2 q_0$, $M^{AE}_{max} = M(2.25\alpha) = 2.53\alpha^2 q_0$ kNm

$M'(0) = Q(0) = 2.25\alpha q_0$ kN (κλίση), $M'(4\alpha) = Q(4\alpha) = -1.75\alpha q_0$ kN (κλίση)

$M''(x) = -q(x) = -q_0 \frac{\text{kN}}{\text{m}}$ άρα, αρνητικό καμπυλόγραμμο.

Τμήμα EG, $4\alpha \le x \le 6\alpha$ (όχι πλέον Γ, αλλά πάλι q_0)

$N(x) = -3\alpha q_0$ kN, παντού

$Q(x) = Q(4\alpha)^{\text{αρ}} + 2.25\alpha q_0 - q_0(x - 4\alpha) = -1.75\alpha q_0 + 2.25\alpha q_0 + 4\alpha q_0 - q_0 x =$

$= 4.5\alpha q_0 - q_0 x$ kN, γραμμικό

$Q(4\alpha)^{\delta} = 0.5\alpha q_0$ kN, $Q(6\alpha) = -1.5\alpha q_0$, για $x = 4.5\alpha$, $Q^{EG}(x) = 0$ kN σημείο M^{EG}_{max}.

$\boxed{M(x)} = x \cdot 4.5\alpha q_0 - x \cdot 2.25\alpha q_0 + (x - 4\alpha) 2.25\alpha q_0 - \frac{x}{2} q_0 x =$

$= 2.25\alpha q_0 \cdot x + 2.25\alpha q_0 \cdot x - 9\alpha^2 q_0 - \frac{q_0}{2} x^2 =$

$= -9\alpha^2 q_0 + 4.5\alpha q_0 \cdot x - \frac{q_0}{2} x^2$ kNm, παραβολικό

(*) 1η $M(x) = M(4\alpha) + Q(4\alpha)^{\delta}(x - 4\alpha) - \frac{x - 4\alpha}{2} q_0 (x - 4\alpha) =$

$= \alpha^2 q_0 + 0.5\alpha q_0 (x - 4\alpha) - \frac{q_0}{2}(x^2 + 16\alpha^2 - 8\alpha x) =$

$= \alpha^2 q_0 - 2\alpha^2 q_0 + 0.5\alpha q_0 x - \frac{q_0}{2} x^2 - 8\alpha^2 q_0 + 4\alpha q_0 x =$

$= -9\alpha^2 q_0 + 4.5\alpha q_0 x - \frac{q_0}{2} x^2$ kNm ✓

(*,*) 1η $M(x) = M(4\alpha) + Q(4\alpha)^{\text{αρ}}(x - 4\alpha) + (x - 4\alpha) 2.25\alpha q_0 - \frac{x - 4\alpha}{2} q_0 (x - 4\alpha) =$ στο E

$= \alpha^2 q_0 - 1.75\alpha q_0 (x - 4\alpha) + (x - 4\alpha) 2.25\alpha q_0 - \frac{q_0}{2}(x^2 + 16\alpha^2 - 8\alpha q_0) =$

$= \alpha^2 q_0 + 0.5\alpha q_0 (x - 4\alpha) - \frac{q_0}{2}(x^2 + 16\alpha^2 - 8\alpha q_0) = \ldots$ kNm ✓

$M(4\alpha) = \alpha^2 q_0$ kNm ✓, $M(6\alpha) = 0$ kNm ✓ (πρόβλεψη)

$M^{EG}_{max} = M(4.5\alpha) = 1.125\alpha^2 q_0$ kNm

$M'(4\alpha) = Q(4\alpha)^{\delta} = 0.5\alpha q_0$ kN (κλίση), $M'(6\alpha) = Q(6\alpha) = -1.5\alpha q_0$ kN (κλίση)

$M''(x) = -q_0$ kN/m < 0, κρηπιδωτή καμπυλότητα

Τμήμα GF, $6\alpha \leq x \leq 8\alpha$

$N(x) = -3.0\alpha q_0$ kN, παδιρό

$Q(x) = Q(6\alpha) = -1.5\alpha q_0$ kN, παδιρό

$M(x) = M(6\alpha) + (x-6\alpha)Q(6\alpha) = 0 - (x-6\alpha)1.5\alpha q_0 = \underbrace{9\alpha q_0 - 1.5\alpha q_0 x}_{\text{γραμμικό}}$ kNm

$M(6\alpha) = 0$ kNm ✓, $M(8\alpha) = -3\alpha^2 q_0$ kNm

Τμήμα FB, $8\alpha \leq x \leq 12\alpha$

$N(x) = -3.0\alpha q_0$ kN, παδιρό

$Q(x) = Q(8\alpha)^{\alpha\rho} + 2.25\alpha q_0 = -1.5\alpha q_0 + 2.25\alpha q_0 = \underbrace{+0.75\alpha q_0}_{\text{παδιρό}}$ kN ✓

Αναζητώ νέα πρόβλεψη στη στενή διαστολή:

$M(x) = (12\alpha - x)\underbrace{(1.5 - 2.25)}_{-0.75}\alpha q_0 = \underbrace{-9\alpha^2 q_0 + 0.75\alpha q_0 x}_{\text{γραμμικό}}$ kNm,

$M(12\alpha) = 0$ kNm ✓, $M(8\alpha) = -3\alpha^2 q_0$ kNm ✓

(29.0)

Διαγράμματα