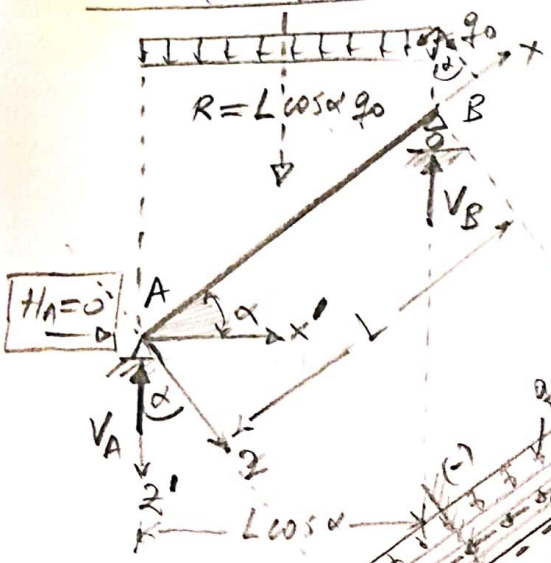
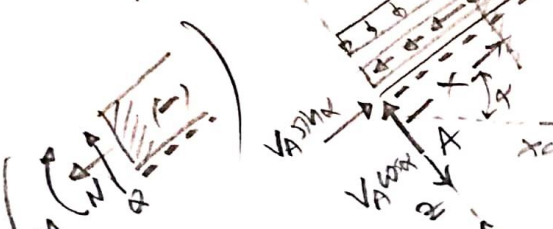
12ο Παράδειγμα – Κατανομή δυνάμεων ($q_0 \frac{N}{m}$: μ ορθόγωνης κατανομής)





- **Αντιδράσεις στηρίξεων (A x'z')**

$\Sigma F_{x'} = 0 \Rightarrow \boxed{H_A = 0}$

$\left(\Sigma M_A = 0 \rightarrow L\cos\alpha\, V_B - \frac{L\cos\alpha}{2} \underbrace{L\cos\alpha}_{R}\, q_0 = 0 \Rightarrow\right.$

$\Rightarrow \boxed{V_B = \frac{L\cos\alpha}{2} q_0}$

$+\Sigma F_{z'} = 0 \Rightarrow V_A = L\cos\alpha\, q_0 - \frac{L\cos\alpha}{2} q_0 \Rightarrow$

$\Rightarrow \boxed{V_A = \frac{L\cos\alpha}{2} q_0} \quad (= V_B)$

- **N, Q, M (A x z)**

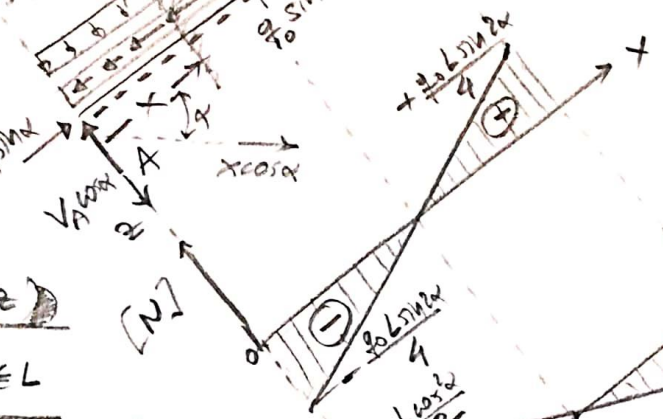
Τμήμα AB, $0 \le x \le L$

$\bullet N(x) = -V_A \sin\alpha + q_0 \sin\alpha \cdot (x\cos\alpha) =$

$= -\frac{L\cos\alpha \sin\alpha}{2} q_0 + q_0 \sin\alpha\cos\alpha \cdot x$

$= \frac{q_0}{2} \sin 2\alpha \cdot x - \frac{q_0 L}{4} \sin 2\alpha$ (ευθεία)

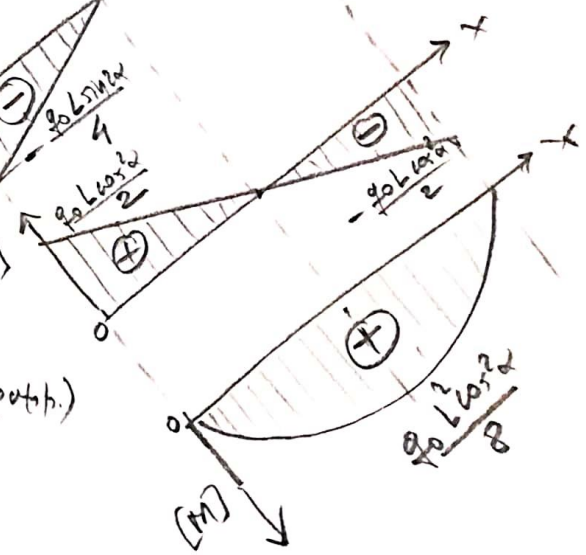
$N(L) = +\frac{q_0 L}{4} \sin 2\alpha$



$\bullet Q(x) = +V_A \cos\alpha - q_0 \cos\alpha \cdot (x\cos\alpha) =$

$= \frac{L\cos^2\alpha}{2} q_0 - q_0 \cos^2\alpha\, x$ (ευθεία)

$Q(L) = -\frac{q_0 L \cos^2\alpha}{2}$



$\bullet M(x) = x V_A \cos\alpha - \frac{x}{2} q_0 \cos\alpha \cdot (x\cos\alpha) = x \frac{L\cos^2\alpha}{2} q_0 - \frac{x^2}{2} q_0 \cos^2\alpha$ (παραβολ.)

$M(0) = 0$, $M(L) = 0$ ✓.

Κοιλότητα $\frac{d^2M}{dx^2} = -q_0 \cos^2\alpha < 0$ άρα.

Μέγ. $\frac{dM}{dx} = -Q(x) = \frac{L\cos^2\alpha}{2} q_0 - q_0 \cos^2\alpha\, x = 0 \rightarrow x = \frac{L}{2}$ άρα M_{max}:

$M_{max} = \frac{L}{2} \frac{L\cos^2\alpha}{2} q_0 - \frac{L^2}{4} \frac{q_0 \cos^2\alpha}{2} = \frac{q_0 L^2 \cos^2\alpha}{8}$