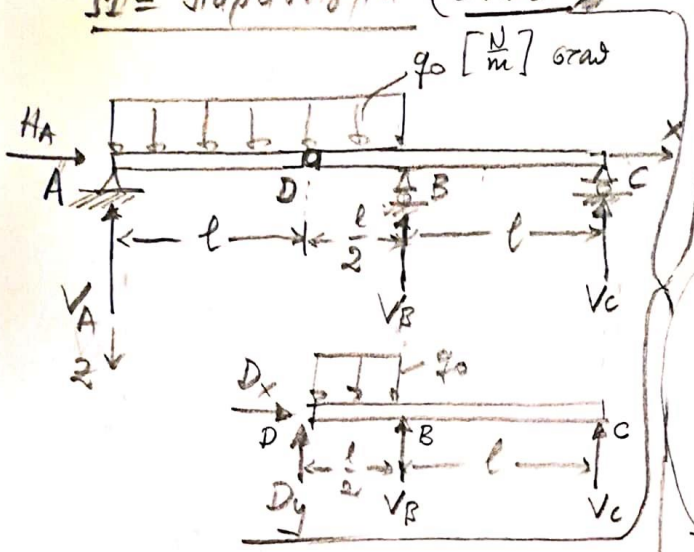
11ο Παράδειγμα (Gerber)



Ανιδράσεις στηρίξεων

(DBC): $\Sigma M_D = 0 \Rightarrow \frac{\ell}{2} V_B + \frac{3\ell}{2} V_C - q_0 \frac{\ell}{2} \frac{1}{2} \frac{\ell}{2} = 0$

$\Rightarrow V_B + 3V_C = \frac{q_0 \ell}{4} \Rightarrow V_B = \frac{q_0 \ell}{4} - 3V_C$

$\Sigma F_x = 0 \Rightarrow D_x = 0$

(ADBC): $\Sigma M_A = 0 \Rightarrow \frac{3\ell}{2} V_B + \frac{5\ell}{2} V_C - \frac{3\ell}{2} \frac{1}{2} \frac{3\ell}{2} q_0 = 0 \Rightarrow 3\frac{q_0 \ell}{4} - 3V_C + 5V_C - \frac{9}{4} q_0 \ell = 0$

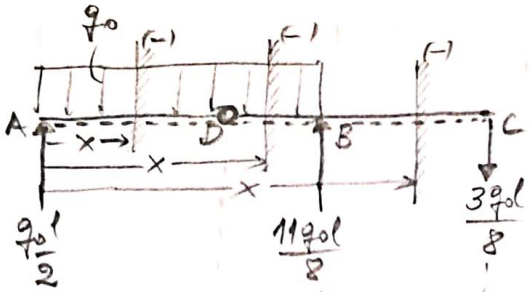
$\Rightarrow 4V_C = -\frac{3 q_0 \ell}{4/2} \Rightarrow \boxed{V_C = -\frac{3 q_0 \ell}{8} \left(\downarrow\right)}$

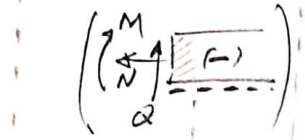
$V_B = \frac{q_0 \ell}{4} - 3\left(-\frac{3 q_0 \ell}{8}\right) \Rightarrow \boxed{V_B = \frac{11 q_0 \ell}{8}}$

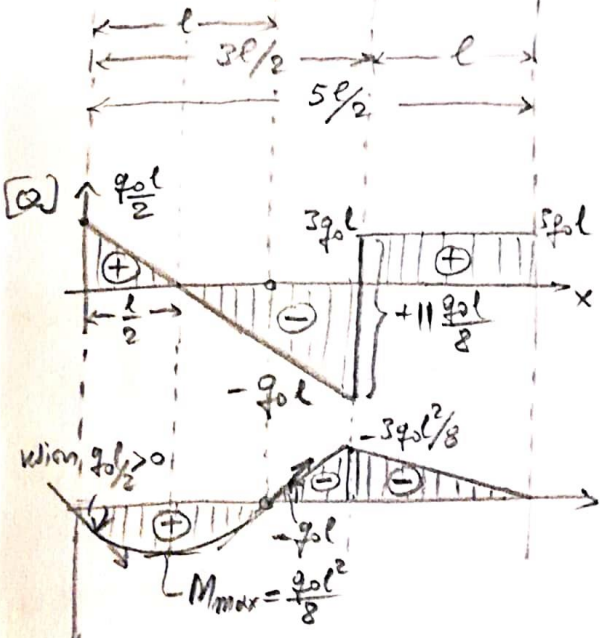
$\Sigma F_z = 0 \Rightarrow V_A = \frac{3\ell}{2} q_0 - V_B - V_C =$

$= \frac{3\ell}{2} q_0 - \frac{11 q_0 \ell}{8} - \left(-\frac{3 q_0 \ell}{8}\right) = q_0 \ell \frac{12 - 11 + 3}{8} \Rightarrow$

$\Rightarrow \boxed{V_A = q_0 \ell / 2}$







Η εσωτερική άρθρωση D δεν είναι σημείο μεταβολής για αξονικές και τέμνουσες. Είναι όμως σημείο μηδενισμού της εσωτερικής ροπής.

- **Τμήμα AB**, $0 \le x \le \frac{3\ell}{2}$

Τέμνουσα ($\frac{dQ}{dx} = -q(x)$)

$Q(x) = Q(0) - \int_0^x q(x)dx = \frac{q_0 \ell}{2} - q_0 x$ (γραμμ.)

$Q(\frac{3\ell}{2}) = \frac{q_0 \ell}{2} - q_0 \frac{3\ell}{2} = -q_0 \ell.$

Σημείο μηδενισμού $Q(x)$: $\frac{q_0 \ell}{2} - q_0 x = 0 \Rightarrow x = \frac{\ell}{2}$.

Ροπή ($\frac{dM}{dx} = Q(x)$)

$M(x) = M(0) + \int_0^x Q(x)dx = 0 + \frac{q_0 \ell}{2} \int_0^x dx - q_0 \int_0^x x dx =$

$= \frac{q_0 \ell}{2} x - \frac{q_0 x^2}{2}$ (παραβ.)

$M(\ell) = \frac{q_0 \ell^2}{2} - \frac{q_0 \ell^2}{2} = 0$ ✓ (άρθρ. D)

$M(3\ell/2) = \frac{q_0}{2} \frac{3\ell}{2} - \frac{q_0}{2} \frac{9\ell^2}{4} = \frac{6 q_0 \ell^2}{8} - \frac{9 q_0 \ell^2}{8} = -\frac{3 q_0 \ell^2}{8}$

Κλίση $M(x)$: $Q(x=0) = \frac{q_0 \ell}{2} > 0$, $Q(x=\ell) = -\frac{q_0 \ell}{2} < 0$, $Q(x = 3\ell/2) = -q_0 \ell < 0$, $Q(\frac{\ell}{2}) = 0 \Rightarrow$

$\Rightarrow M_{max} = M(\frac{\ell}{2}) = \frac{q_0 \ell^2}{8}$

Καμπυλότητα $M(x)$: $-q_0 < 0$ σταθ.

- **Τμήμα BC** Τέμνουσα: $\frac{dQ}{dx} = -q = 0 \Rightarrow Q = \text{σταθ.} = Q(\frac{3\ell}{2}) + \frac{11 q_0 \ell}{8} = -q_0 \ell + \frac{11 q_0 \ell}{8} = \frac{3 q_0 \ell}{8}$

Ροπή: $\frac{dM}{dx} = Q = \frac{3 q_0 \ell}{8} = \text{σταθ.} \Rightarrow M(x) = M(\frac{3\ell}{2}) + \frac{3 q_0 \ell}{8}(x - \frac{3\ell}{2})$, γραμμ., $M(\frac{5\ell}{2}) = -\frac{3 q_0 \ell^2}{8} + \frac{3 q_0 \ell}{8} \ell =$