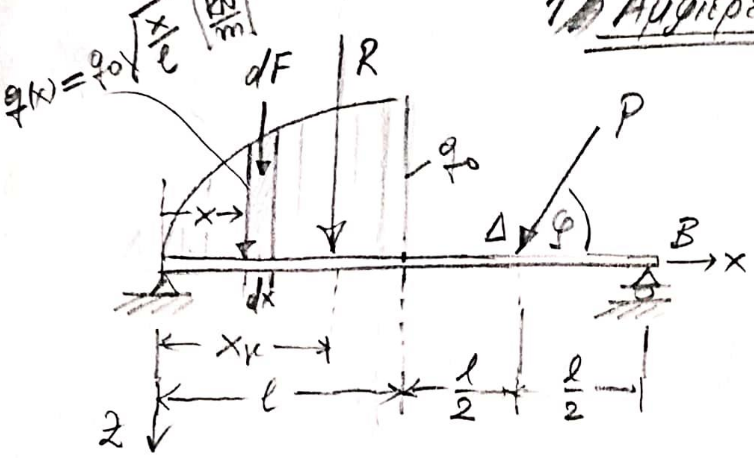
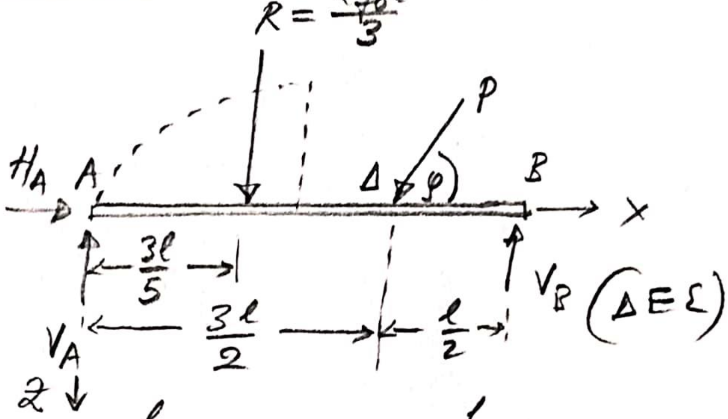
2024–25
Πολ. Μηχαν.
5ο Μαθ

ΙΣΟΡΡΟΠΙΑ ΙΣΟΣΤΑΤΙΚΩΝ ΦΟΡΕΩΝ ΣΕ ΔΥΟ ΔΙΑΣΤΑΣΕΙΣ

1) Αμφιέρειστη δοκός





$$dF = q(x)\,dx \quad [kN], \qquad R = \int dF = \int_0^l q(x)\,dx = \frac{q_0}{\sqrt{l}}\int_0^l x^{\frac{1}{2}}dx =$$

$$= \frac{q_0}{\sqrt{l}}\,\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\Bigg|_0^l = \frac{q_0}{\sqrt{l}}\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_0^l = \frac{q_0}{\cancel{\sqrt{l}}}\,\frac{l\cancel{\sqrt{l}}}{\frac{3}{2}} = \frac{2q_0 l}{3}$$

$$x_K R \text{ ή } x_K \int dF = \int x\,dF \Rightarrow x_K \int_0^l q(x)\,dx = \int_0^l x\cdot q(x)\,dx \Rightarrow$$

$$\Rightarrow x_K = \underbrace{\frac{\int_0^l x\,q(x)\,dx}{\int_0^l q(x)\,dx}}$$

Αριθμ. $\underbrace{\int_0^l x\,q(x)\,dx} = \int_0^l x\,q_0\sqrt{\frac{x}{l}}\,dx = \frac{q_0}{\sqrt{l}}\int_0^l x^{\frac{3}{2}}dx = \frac{q_0}{\sqrt{l}}\,\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}\Bigg|_0^l =$

$$= \frac{q_0}{\sqrt{l}}\,\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\Bigg|_0^l = \frac{q_0}{\sqrt{l}}\,\frac{l^{\frac{5}{2}}}{\frac{5}{2}} = \frac{q_0}{\cancel{\sqrt{l}}}\,\frac{\cancel{\sqrt{l}}\,l^2}{\frac{5}{2}} = \underbrace{\frac{2q_0 l^2}{5}}$$

Παρατ. $\underbrace{\int_0^l q(x)\,dx = R = \frac{2q_0 l}{3}}$ (Συνισταμένη R της κατανομής)

$$x_K = \frac{\frac{2q_0 l^2}{5}}{\frac{2q_0 l}{3}} = \frac{3l}{5}$$

Ισορροπία

$$\rightarrow \Sigma F_x = 0 \Rightarrow H_A - P\cos\varphi = 0 \Rightarrow \boxed{H_A = P\cos\varphi}$$

$$\downarrow \Sigma F_z = 0 \Rightarrow -V_A - V_B + R + P\sin\varphi = 0 \Rightarrow$$

$$\boxed{V_A + V_B = \frac{2q_0 l}{3} + P\sin\varphi} \quad (1)$$

(2)

$$\circlearrowleft \Sigma M_A = 0 \Rightarrow 2\cancel{l} V_B - \frac{3\cancel{l}}{5} R - \frac{3\cancel{l}}{2} P \sin\varphi = 0 \Rightarrow$$

$$\Rightarrow V_B = \frac{3}{\cancel{2}\cdot 5} \frac{\cancel{2} q_0 l}{\cancel{3}} + \frac{3}{4} P \sin\varphi \Rightarrow \boxed{V_B = \frac{q_0 l}{5} + \frac{3P}{4} \sin\varphi} \quad (2)$$

$$(1),(2) \Rightarrow V_A = \frac{2 q_0 l}{3} - \frac{q_0 l}{5} + P \sin\varphi - \frac{3P}{4} \sin\varphi \Rightarrow$$

$$\Rightarrow \boxed{V_A = \frac{7 q_0 l}{15} + \frac{P}{4} \sin\varphi} \quad (3)$$