# Logic Design 22/10/21

Παρασκευή, 22 Οκτωβρίου 2021 09:52

Καλημέρα!

## Άλγεβρα Boole

Δίτιμη Άλγεβρα Boole

$B = \{0, 1\}$ $(+)(\cdot)$

De Morgan

$(x \cdot y)' = x' + y'$

$(x+y)' = x' \cdot y'$

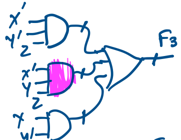
$(x \cdot y \cdot z)' = x' + y' + z'$

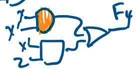
| x y z | F3 | F4 | F5 |
|---|---|---|---|
| 0 0 0 | 0 | 0 | 1 ✓ |
| 0 0 1 | 1 | 1 | 0 |
| 0 1 0 | 0 | 0 | 0 |
| 0 1 1 | 1 | 1 | 1 ✓ |
| 1 0 0 | 1 | 1 | 0 |
| 1 0 1 | 1 | 1 | 0 |
| 1 1 0 | 0 | 0 | 1 ✓ |
| 1 1 1 | 0 | 0 | 0 |

$F_3 = x'y'z + x'yz + xy'$

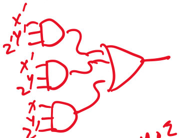
$F_4 = xy' + x'z$

$F_3 = F_4$



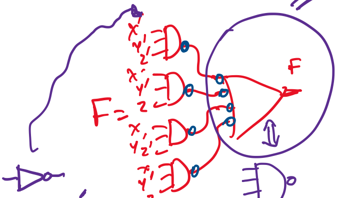


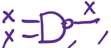
$F_5 = x'y'z' + x'yz + xyz'$



**NAND**

ONLY NAND INSIDE

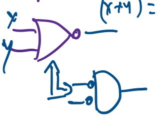




$(x \cdot x)' = x'$

**NOR**

$(x+y)' = (x' \cdot y')$



$F_6 = m_2 + m_3 + m_4 + m_5 + m_6 + m_7$

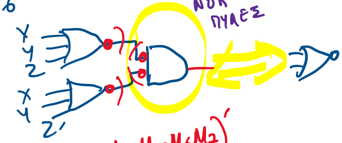
$F_6 = M_0 \cdot M_1$

ΟΡΙΣΜΟΣ

ΕΛΑΧΙΣΤΟΡΟΙ

| F | x y z | ΕΛΑΧΙΣΤΟΡΟΣ | | F6 | ΜΕΓΙΣΤΟΡΟΣ | |
|---|---|---|---|---|---|---|
| 1 | 0 0 0 | $x' \cdot y' \cdot z'$ | $m_0$ | 0 | $x+y+z$ | $M_0$ |
| 1 | 0 0 1 | $x' \cdot y' \cdot z$ | $m_1$ | 0 | $x+y+z'$ | $M_1$ |
| 0 | 0 1 0 | $x' \cdot y \cdot z'$ | $m_2$ | 1 | $x+y'+z$ | $M_2$ |
| 0 | 0 1 1 | $x' \cdot y \cdot z$ | $m_3$ | 1 | $x+y'+z'$ | $M_3$ |
| 1 | 1 0 0 | $x \cdot y' \cdot z'$ | $m_4$ | 1 | $x'+y+z$ | $M_4$ |
| 1 | 1 0 1 | $x \cdot y' \cdot z$ | $m_5$ | 1 | $x'+y+z'$ | $M_5$ |
| 0 | 1 1 0 | $x \cdot y \cdot z'$ | $m_6$ | 1 | $x'+y'+z$ | $M_6$ |
| 0 | 1 1 1 | $x \cdot y \cdot z$ | $m_7$ | 1 | $x'+y'+z'$ | $M_7$ |

NOR ΠΥΛΕΣ



$F_6 = (M_2 M_3 M_4 M_5 M_6 M_7)'$

$F' = m_2 + m_3 + m_6 + m_7$

$F = m_0 + m_1 + m_4 + m_5$

$= x'y'z' + x'y'z + xy'z' + xy'z$

$F = M_0' + M_1' + M_4' + M_5'$

$F' = (M_0' + M_1' + M_4' + M_5')' =$ $M_0 \cdot M_1 M_4 M_5$

$(F')' = (m_2 + m_3 + m_6 + m_7)'$

$= m_2' \cdot m_3' \cdot m_6' \cdot m_7'$

$= M_2 \cdot M_3 M_6 M_7$

DE MORGAN

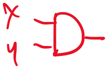
$(m_i)' = M_i$

$(x' \cdot y \cdot z)' = (x')' + y' + z' = x + y' + z'$

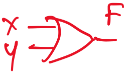
Ο ΕΛΑΧΙΣΤΟΡΟΣ ΓΙΝΕΤΑΙ 1 μόνο για μία είσοδο κ' για όλη τις άλλη είναι 0

Ο ΜΕΓΙΣΤΟΡΟΣ ΓΙΝΕΤΑΙ 0 μόνο για μία είσοδο.. κ' για όλη τις άλλη είναι 1

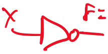
## ΛΟΓΙΚΕΣ ΠΥΛΕΣ



$F = x \cdot y$

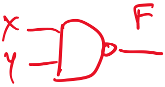


$F = x + y$



$F = x'$

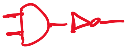
- NAND



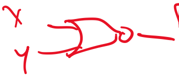
$F = (xy)'$

| x | y | F |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

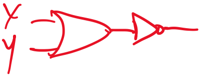
- ⇔



- NOR

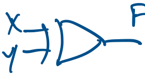


$F = (x + y)'$



| x | y | F |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

- XOR



$F = x'y + xy'$

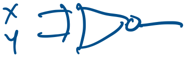
$F = x \oplus y$

| x | y | F |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

- ΙΣΟΔΥΝΑΜΙΑ

Η

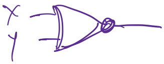
ΑΠΟΚΛΕΙΣΤΙΚΟ ΟΥΤΕ



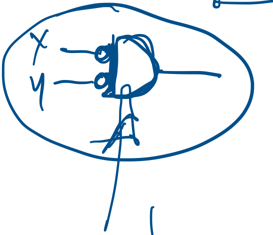
$F = xy + x'y'$

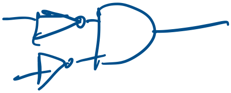
| x y | F |
|---|---|
| 0 0 | 1 |
| 0 1 | 0 |
| 1 0 | 0 |
| 1 1 | 1 |

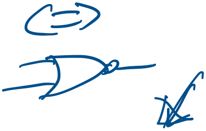
NOR

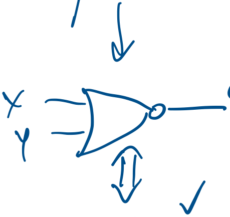


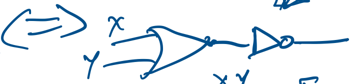
$$\boxed{(x+y)' = x'\cdot y'}$$



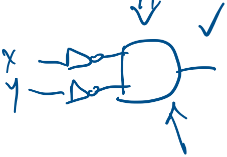






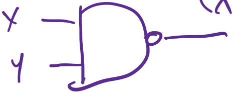


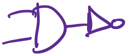
| x y | F |
|---|---|
| 0 0 | 1 |
| 0 1 | 0 |
| 1 0 | 0 |
| 1 1 | 0 |

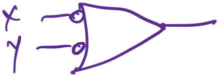


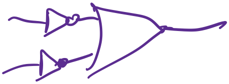
NAND

$(x\cdot y)' = (x'+y')$









AND, OR, NOT

1. **Συνάρτηση** σαν άθροισμα γινομένων

2. σαν γινόμενο αθροισμάτων

3. με πύλες NAND μόνο

4. με πύλες NOR μόνο

## ΑΣΚΗΣΗ

F

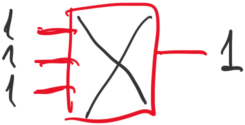
Δίνεται ένας τριψήφιος αριθμός

x y z    0, 1 . . 7

000
001
.
111

Να υλοποιηθεί κύκλωμα

1. 2. 3. 4

1 όταν ο αριθμός είναι prime
0