

NATIONAL TECHNICAL UNIVERSITY OF ATHENS

School of Civil Engineering – Geotechnical Department

Computational Methods in the Analysis of Underground Structures

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Lecture Series in Postgraduate Programs:

- 1. Analysis and Design of Structures (DSAK)
- 2. Design and Construction of Underground Structures (SKYE)

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LECTURE 3: Analysis of tunnel excavation with the convergence-confinement method

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History of tunnel design methods

Until the 1940's, design of tunnel support was a purely empirical art, applying techniques proven successful, or improving techniques proven unsuccessful (failures)

In 1946, Terzaghi proposed a set of loads for the analysis of tunnel supports:

| Rock Condition | RQD | Rock Load H _p | |
|---------------------------------------------|--------|------------------------------------------------|--|
| 1. Hard and intact | 95-100 | Zero | |
| 2. Hard stratified or schistose | 90-99 | 0–0.5 <i>B</i> | |
| 3. Massive, moderately jointed | 85-95 | 0–0.25 <i>B</i> | |
| 4. Moderately blocky and seamy | 75-85 | $0.25 B - 0.20 (B + H_t)$ | |
| 5. Very blocky and seamy | 30-75 | $(0.20-0.60) (B + H_t)$ | |
| 6. Completely crushed but chemically intact | 3–30 | $(0.60-1.10) (B + H_t)$ | |
| 6a. Sand and gravel | 0-3 | $(1.10-1.40) (B + H_t)$ | |
| 7. Squeezing rock, moderate depth | NA | $(1.10-2.10) (B + H_t)$ | |
| 8. Squeezing rock, great depth | NA | $(2.10-4.50) (B + H_t)$ | |
| 9. Swelling rock | NA | Up to 80m irrespective of value of $(B + H_t)$ | |





In the 1970's, other empirical methods were proposed, such as the Rock Mass Rating (RMR) and the Q-system. These methods provided the tunnel support (thickness of shotcrete and density and length of rockbolts) from empirical Tables, based on a rock mass classification

History of tunnel design methods

Purely empirical methods of tunnelling (1850's – 1946)



Terzaghi loads (1946)



Empirical design methods Q-system (1974), RMR (1976)

Convergence-Confinement method (Pacher, 1964 or Panet, 1974)



Conventional tunnel excavation (New Austrian Tunnelling Method)





- As the tunnel is advanced, the tunnel wall converges inwards (u_R)
- Due to (u_R) , the radial stress (σ_r) reduces, and the circumferential stress (σ_{θ}) increases, creating the "arch effect" which supports the crest of the tunnel (i.e., the ground supports itself via the compressive stress in the arch)
- Above stress changes end about 2D behind the tunnel face



Reduction of the radial stress (σ_r) with tunnel excavation (due to the inward wall convergence by tunnel excavation). Reduction of σ_r cannot be avoided.





Reduction of σ_r increases the circumferential stress (σ_{θ}). The increased σ_{θ} causes ground arching \rightarrow ground supports itself. An eventual reduction of σ_{θ} is due to ground yielding (as increase of σ_{θ} eventually causes yielding), and results in moving the ground arch away from the tunnel

Conventional tunnel excavation (New Austrian Tunnelling Method)

Ground arching



Questions:

Does the ground have sufficient strength to take the increased circumferential stress (σ_{θ}) without yielding?

M-C failure: No yielding $\rightarrow \sigma_{\theta,max}^{\mu} < k p_s + \sigma_{cm}$

No yielding $\rightarrow p_s > \left(\frac{N_s - 1}{k + 1}\right) \sigma_{cm}$





If N_s <1 \rightarrow p_s=0 (support is not required) If N_s >1 and $p_s > \left(\frac{N_s - 1}{k + 1}\right)\sigma_{cm} \rightarrow$ no ground yielding If $p_s < \left(\frac{N_s - 1}{k + 1}\right)\sigma_{cm} \rightarrow$ ground is yielding

> ... but the tunnel is stable, unless p_s is too small and the ground disintegrates (due to very large u_R)

Principle of NATM:



- As the tunnel face advances, the surrounding ground deforms causing radial wall convergence (u_R) and horizontal (inward) face extrusion (u_F)
- Due to ground deformation, the initial geostatic stresses ($\sigma_{10} = p_0$, $\sigma_{30} = K_0 p_0$) change, with significant reduction of the radial stress ($\sigma_r = \sigma_3$) and corresponding increase of the circumferential stress ($\sigma_{\theta} = \sigma_1$). Thus, a "load bearing" ground zone (grey zone) develops around the tunnel, which carries the ground stresses after redistribution due to tunnel excavation.
- In an unsupported tunnel, when the wall convergence reaches maximum (far behind excavation face), the radial pressure at the tunnel wall becomes zero ($\sigma_r=0$). If the ground remains linearly elastic, the maximum circumferential stress at the tunnel wall is (Kirsch stresses):

$$\sigma_{\theta} = (3 - K_o) p_o \quad \text{if } K_o \le 1$$

$$\sigma_{\theta} = (3K_o - 1) p_o \quad \text{if } K_o > 1$$

Note: In an unsupported tunnel, the ground remains linearly elastic if $\sigma_{\theta} < \sigma_{cm}$, where σ_{cm} = unconfined ground strength



• If the ground remains linearly elastic, i.e., if the "load bearing" ground zone can undertake the new ground stresses (σ_1 , σ_3) without failure (exceedance of the yield criterion) at the most adverse location (tunnel wall), theoretically there is no need for tunnel support.

However, even in this case, support is required to prevent ground disintegration (detachment of blocks from the tunnel wall), due to the presence of discontinuities (structural failure, continuum mechanics not applicable).

- Tunnel support with shotcrete and/or rockbolts:
 - Applies pressure p_s on the ground \rightarrow increases the radial stress at the tunnel wall ($\sigma_r = \sigma_3 =$ p_s) \rightarrow improves the capacity (strength) of the "load bearing" ground zone.
 - Reduces the wall convergence (u_p) , relative to the unsupported tunnel \rightarrow prevents ground disintegration (detachment of rock blocks for the roof).





• If the load bearing zone can undertake the new stresses of the red Mohr circle (σ_1 , $\sigma_3 = 0$), i.e, if the red Mohr circle is below the M-C failure envelope, theoretically there is no need for tunnel support, since the ground around the tunnel remains elastic (no yielding) even at the most adverse location (tunnel wall). This is the case if (for $K_0 \le 1$):

$$\sigma_1 < \sigma_{cm} \implies \sigma_{cm} > (3 - K_o) p_o$$

• Support pressure (p_S) increases σ_3 (= p_S, blue Mohr circle), thus improving the capacity of the load bearing zone, which can undertake a maximum stress (σ_1):

$$\sigma_1 < k p_S + \sigma_{cm} \qquad k = \tan^2 \left(45 + \frac{\phi}{2} \right)$$

The support aims to increase the radial stress $\sigma_r = p_s$ (compared to $\sigma_r = 0$ in unsupported tunnel), so that the load bearing zone can undertake (without yielding) the increased circumferential stress: $\sigma_{\theta} = \sigma_1 = (3-K_0) p_0 - p_s < k p_s + \sigma_{cm} \rightarrow n_s > \left(\frac{N_s - 1}{N_s}\right) \sigma$

Required support pressure (p_s) to prevent ground yielding (for $K_o \le 1$):

Principle of NATM:



What will happen if the circumferential stress $\sigma_1 = \sigma_{\theta}$ exceeds the yield stress? i.e., if:

 $\sigma_1 > \sigma_{cm}$ (unsupported tunnel)

 σ_{1} > $k p_{S} + \sigma_{cm}$ (supported tunnel)

In the area close to the tunnel wall where σ_1 exceeds the yield stress, stresses will be redistributed and σ_1 will be reduced to the yield stress (this zone is called "plastic zone"). For stress equilibrium, stresses will be increased beyond the plastic zone. Furthermore, due to the increased plastic strains in the plastic zone, tunnel wall convergence will increase (compared to the case where there is no plastic zone, where σ_1 at the tunnel wall does not exceed the yield stress).

Condition for no plastic zone (for $K_0 \le 1$):

Overstress factor: $N_s = \frac{(3-K_o)p_o}{\sigma_{cm}}$

Condition for no plastic zone in an unsupported tunnel ($p_s=0$): $N_s < 1$

 $p_{s} > \left(\frac{N_{s}-1}{k+1}\right)\sigma_{cm}$

Convergence - deconfinement curve:



The various sections along the tunnel axis correspond to gradually increasing wall convergence (u_R) as the tunnel excavation face advances.

In an equivalent 2D model (upper right), the gradually increasing wall convergence (u_p) can be modelled by a fictitious reduction of an "internal pressure" (p) starting from the initial (geostatic) value ($p = p_0$) when $u_R = 0$, and gradually reducing ($p < p_0$) as u_R increases. 2D analysis in a linearly elastic ground ($N_s < 1$) gives the following relationship between pressure reduction/deconfinement (p) wall convergence u_R (convergence - deconfinement curve):

$$u_{R} = \left(1 - \frac{p}{p_{o}}\right) R\left(\frac{p_{o}}{2G}\right) \Rightarrow u_{R} = \lambda R\left(\frac{p_{o}}{2G}\right) \qquad \text{where:} \\ \lambda = \text{deco}$$

 $p = (1 - \lambda) p_o$

nfinement coefficient

 $u_{R\infty} = R$ For full deconfinement (p=0), i.e., for $\lambda = 1$:

The three functions of tunnel support (depending on the value of N_s) :

1. Ground is self-supporting / elastic ($N_S \le 1$):

For
$$K_o \leq 1$$
: $N_s = \frac{(3 - K_o) p_o}{\sigma_{cm}} \leq 1 \implies \sigma_{cm} \geq (3 - K_o) p_o$

Ground strength (σ_{cm}) is appreciable. Support is not required for ground stability (since ground remains elastic). There is risk of block detachment and fall (structural instability) from the tunnel roof (risk of accidents) and risk of ground erosion due to water seepage. A small thickness of shotcrete (about 2-5cm) and sporadic rockbolts can protect against these risks.



The three functions of tunnel support (depending on the value of N_s) :

2. Ground with limited support requirement ($N_s = 1 \div 4$):

Ground strength (σ_{cm}) is moderate. If tunnel is left unsupported, the load bearing ground zone (around the tunnel) will yield (i.e., u_R will increase). Limited support (rockbolts and shotcrete) will increase ground capacity and avoid yielding. Closed invert is not required.



The three functions of tunnel support (depending on the value of N_s) :

2. Ground with significant support requirement ($N_S > 4$):

Significant support (with rockbolts and shotcrete) is required to increase the strength of the load bearing zone around the tunnel. Closed invert is required.

For
$$K_o \le 1$$
: $N_s = \frac{(3-K_o)p_o}{\sigma_{cm}} > 4 \implies \sigma_{cm} < \frac{1}{4}(3-K_o)p_o$



Example: γ =20 kN/m3, H= 50m $\rightarrow p_o = 20x50 = 1000$ kPa , K_o = 0.50 $\sigma_{cm} < 0.25 \times (3-0.50) \times 1000 \rightarrow \sigma_{cm} < 625$ kPa So, N_s > 4 $\rightarrow \sigma_{cm} < 625$ kPa If $\sigma_{cm} > 2500$ kPa \rightarrow Case 1 (elastic). If $\sigma_{cm} = 625$ to 2500 kPa \rightarrow Case 2

Convergence - deconfinement curve:



For $\lambda > \lambda_e$ (after plasticity starts), wall convergence u_R increases more than in linear elasticity (the convergence – confinement curve becomes curved) Support stiffness K_{sn} is given in Tables, for various types of support

Convergence - Confinement (or Ground Reaction) curve: GRC = Ground Reaction Curve: $U_r(x) / U_{r,max}$ versus p_i / p_o LDP = Longitudinal Displacement Profile or Chern curve: $U_r(x) / U_{r,max}$ versus x / R Combination of the GRC and LDP curves provides the relation: p_i / p_o versus x / R which is required in 2D numerical analyses.



Analysis of tunnel excavation with the convergence-confinement method

The convergence-confinement method (Pacher, 1964; Panet 1974), was the first rational attempt to calculate the ground pressure exerted on the tunnel support (**support pressure** σ_s), and thus design the tunnel support to sustain this pressure.

The method exploits the fact that a large fraction of the overburden ground pressure is undertaken by the ground itself via arching, due to the inward convergence of the tunnel wall.



 $\sigma_{\theta} = \sigma_{cm} + k \sigma_{s} \qquad \mathbf{k} \approx 3$

Tunnel support aims to <u>increase the capacity</u> <u>of the ground arch</u> (i.e., increase its compressive capacity σ_{θ} , by providing lateral confinement σ_s) \rightarrow the arch undertakes larger loads, rather than the support <u>undertaking</u> <u>the ground loads</u>



Convergence-confinement method – Elastic ground



 λ_d = deconfinement coefficient at location (Δ) where support is installed Support stiffness K_{sn} is given in Tables, for various types of support

Convergence-confinement method – Elastoplastic ground



Analysis of tunnel excavation with the convergence-confinement method





Influence of the temporary support on the convergenceconfinement curve

CASE 1: Support with rockbolts only



Influence of the temporary support on the convergenceconfinement curve

CASE 2: Support with rockbolts and shotcrete

 u_{R}

Analysis of tunnel excavation with the convergence-confinement method

The convergence-confinement method calculates the required support pressure (p_S) to prevent failure of the ground arch.

With safety factor (FS), the required ultimate support pressure is: $p_{Su} = (FS) p_S$

Usually, support consists of shotcrete (S), anchors (A) and steel ribs (R): $p_{Su} = p_u^S + p_u^A + p_u^R$

Calculation of shotcrete capacity:

$$p_u^s\left(\frac{b}{2}\right) = \tau^s\left(\frac{d}{\sin\alpha^s}\right) \Rightarrow \quad p_u^s = \frac{d\cdot\tau^s}{(b/2)\sin\alpha^s}$$

d= shotcrete thickness

$$b = 2R\cos\alpha^S$$
 $\alpha^S = \alpha^R = 45 - \varphi/2$

 $\tau^{s} = 0.20 f_{c}$

 τ^{s} = shear strength of shotcrete f_{c} = compressive strength of shotcrete



Analysis of tunnel excavation with the convergence-confinement method

Anchor (rockbolt) capacity: $p_u^A = \frac{f^A \cdot \beta_S}{l \times t}$

 $f^{A} = \pi D^{2}/4$ = anchor section area (*D* = anchor diameter)

 β_S = yield stress of anchor steel ($l \ge t$) = anchor spacing (length x width)

Steel set (rib) capacity:

$$p_u^R = \frac{\left(f^R / L^R\right) \cdot \tau^R}{\left(b/2\right) \sin \alpha^S}$$

 f^{R}/L^{R} = steel set section area per unit length of tunnel L^R = spacing of steel sets

 $\tau^{R} = \tau^{S} (E_{R}/E_{S})$ = shear stress of the steel sets at yielding of shotcrete

 τ^{s} = shear strength of shotcrete

 E_R/E_S = modulus of steel sets / shotcrete



Starting from the 1990's, increased computer power permitted the use of numerical analyses in tunnelling (mainly via the finite element method). Finite element analyses provided the loads on tunnel supports, thus permitting to check their adequacy, or the need to increase their capacity.

With the widespread use of numerical analyses in tunnelling, the convergenceconfinement method is no longer used for the calculation of tunnel support requirements, but it is still useful in 2D numerical analyses (provide the deconfinement coefficient, λ , at the position where tunnel support is installed).

Numerical analyses required more elaborate models for the behaviour of the ground (stress-strain-strength relationships), such as the Hoek-Brown failure criterion for rockmasses. These models required methods to estimate their input parameters (ground stiffness and strength parameters).

- In soils, these parameters are usually obtained by testing.
- In rock, testing rockmass samples to obtain parameters is not possible → empirical methods developed to provide rockmass parameters from rockmass classifications. The Geological Strength Index (GSI) (Hoek, 1994) is one such method.

Analysis of tunnel excavation with numerical methods

Erroneous (but common) method for modelling tunnel excavation and support:



Simultaneous excavation and support

Initial condition (geostatic)

This method is equivalent to excavation AFTER the installation of support. It gives too small convergence and too high pressure on the support. Analysis of tunnel excavation with numerical methods

Tunnel wall convergence occurs during (A) and (B)





The method requires the use of a deconfinement coefficient (λ) to reduce the pressure in the area to be excavated, and simulate pre-convergence (u_R up to support installation)

Selection of the suitable deconfinement coefficient (λ) Combination of GRC (λ -u_R) and Chern (x- u_R) curves \rightarrow (λ – x)



Importance of the overstress factor (N_s) in tunnel excavation

$$N_s = \frac{2p_o}{\sigma_{cm}}$$

 $- p_o = \gamma H = \text{geostatic pressure}$

 $\leftarrow \sigma_{cm} =$ ground strength

$$\sigma_{cm} = (0.019 \ \sigma_{ci}) \exp\left(\frac{GSI}{20}\right) \qquad \text{(Hoek, 1999)}$$

Tunnel depth (H) and ground strength (σ_{cm}) or rockmass quality (GSI) are or equal importance: Increasing depth (H) is equivalent to reducing rockmass quality (GSI)

Example (
$$\sigma_{ci} = 15 MPa$$
, $\gamma = 24 kN/m^3$)
GSI = 25, H=50m $\Rightarrow \sigma_{cm} = 1 \text{ MPa}$, $p_o = 1.2 \text{ MPa}$, $N_s = 2.40$
GSI = 47, H=150m $\Rightarrow \sigma_{cm} = 3 \text{ MPa}$, $p_o = 3.6 \text{ MPa}$, $N_s = 2.40$

Conclusion: Tunnel design sections correspond to ranges of N_s (not on ranges of rock mass quality)





Identical methods: reduction of p or E cause same effect (same wall convergence u_R)



Example : $\lambda = 0.70 \implies p = 30\% p_o$

Example : $\lambda=0.70 \implies E = 10\% E_o$ Analysis of tunnel excavation with numerical methods Deconfinement method: Reduction of p or E (via λ)

Values of reduced (p) and reduced (E) for various values of λ

| λ | p/p_o | Τιμές του <i>Ε/Ε_ο</i> για | | | |
|--------|---------|--------------------------------------|-----------------|-----------------|--|
| | | <i>v</i> = 0.25 | <i>v</i> = 0.30 | <i>v</i> = 0.35 | |
| 0.20 | 0.80 | 0.571 | 0.533 | 0.480 | |
| 0.30 | 0.70 | 0.438 | 0.400 | 0.350 | |
| 0.40 | 0.60 | 0.333 | 0.300 | 0.257 | |
| 0.50 | 0.50 | 0.250 | 0.222 | 0.187 | |
| 0.60 | 0.40 | 0.182 | 0.160 | 0.133 | |
| (0.70) | →(0.30) | 0.125 | →(0.109) | 0.090 | |
| 0.80 | 0.20 | 0.077 | 0.067 | 0.054 | |
| 0.90 | 0.10 | 0.036 | 0.031 | 0.025 | |
| | | | | | |

$$\frac{E}{E_o} = \frac{(1-2\nu)(1-\lambda)}{(1-2\nu)+\lambda}$$

 $\lambda = 1 - p/p_o$

Use of the Chern-Panet curves with internal pressure (p) reduction

- 1. Calculate the overstress factor: $N_s = 2 p_o / \sigma_{cm}$
- Using the suitable Chern-Panet curve and the value of (x) corresponding to the location of support installation, calculate the deconfinement coefficient (λ) and the reduced pressure (p):

$$p = p_o (1 - \lambda)$$

 In the numerical model, reduce the internal pressure from (p_o) to (p). The resulting ground stresses and tunnel wall convergence correspond to the location where the tunnel support will be applied.



- 4. Apply the tunnel support and then reduce the internal pressure to zero. The model calculates the ground stresses and forces on the support measures.
- If tunnel excavation includes more than one phases, repeat steps (3) and (4) for each phase in sequence.

Use of the Chern-Panet curves with internal pressure (p) reduction Example:

h = 100 m, $\gamma = 25 \text{ kN/m^3}$, $K_o = 0.50$ $p_o = 0.5*(1+K_o) \gamma h = 0.50*(1+0.50)*25*100 = 1875 kPa$ $N_s = 2 p_o / \sigma_{cm}$ (σ_{cm} = rock mass strength) $\sigma_{ci} = 40 MPa$, GSI = 33 $\sigma_{cm} = \left(\frac{\sigma_{ci}}{52.63}\right) \exp\left(\frac{GSI}{20}\right)$ Thus: $\sigma_{cm} = 1 MPa$ Thus: $N_{\rm s} = 3.75$ R=5m, $x = -2m \rightarrow \lambda = 0.72$ Thus: $p = p_0(1-\lambda) = 1.875*0.28 = 0.525 MPa$ **Results**:

- 1. Model up to the installation of support with pressure reduction from 1.875 MPa to 0.525 MPa.
- Support installation and reduction of the internal pressure from 0.525 MPa to zero.

Use of the Chern-Panet curves with reduction of the E-modulus

1. Calculate the overstress factor: $N_s = 2 p_o / \sigma_{cm}$

- 2. Using the suitable Chern-Panet curve and the value of (x) corresponding to the location of support installation, calculate the deconfinement coefficient (λ) and the reduced E-modulus: $\frac{E}{E_{\alpha}} = \frac{(1-2\nu)(1-\lambda)}{(1-2\nu)+\lambda}$
- 3. In the numerical model, reduce the internal E-modulus from (E_o) to (E). The resulting ground stresses and tunnel wall convergence correspond to the location where the tunnel support will be applied.



- 4. Apply the tunnel support and then reduce the internal Emodulus to zero. The model calculates the ground stresses and forces on the support measures.
- If tunnel excavation includes more than one phases, repeat steps (3) and (4) for each phase in sequence.

Use of the Chern-Panet curves with reduction of the E-modulus Example:

 $\begin{array}{l} h = 100 \ m \ , \ \gamma = 25 \ kN/m^3 \ , \ K_o = 0.50, \ v = 0.30 \\ p_o = 0.5*(1+K_o) \ \gamma \ h = 0.50*(1+0.50)*25*100 = 1875 \ kPa \\ N_s = 2 \ p_o \ / \ \sigma_{cm} \quad (\sigma_{cm} = \text{rock mass strength}) \\ \sigma_{ci} = 40 \ MPa \ , \ GSI = 33 \\ \text{Thus:} \ \sigma_{cm} = 1 \ MPa \qquad \qquad \sigma_{cm} = \left(\frac{\sigma_{ci}}{52.63}\right) \exp\left(\frac{GSI}{20}\right) \\ \text{Thus:} \ N_s = 3.75 \end{array}$

R=5m, $x = -2m \rightarrow \lambda=0.72$, Thus: $E/E_o=0.10$ Results:

- 1. Model up to the installation of support, with E-modulus reduction from E_o to E=0.10 E_o .
- 2. Support installation and reduction of the E-modulus to zero.

Analysis of tunnel excavation with numerical methods 2D numerical analysis of tunnel excavation and support with computer software RS2 (Rocscience Inc)





Analysis of tunnel excavation with numerical methods 3D numerical analysis of tunnel excavation and support with computer software RS3 (Rocscience Inc)



Analysis of tunnel excavation with numerical methods 3D numerical analysis of tunnel excavation and support with computer software RS3 (Rocscience Inc)



Ground displacements