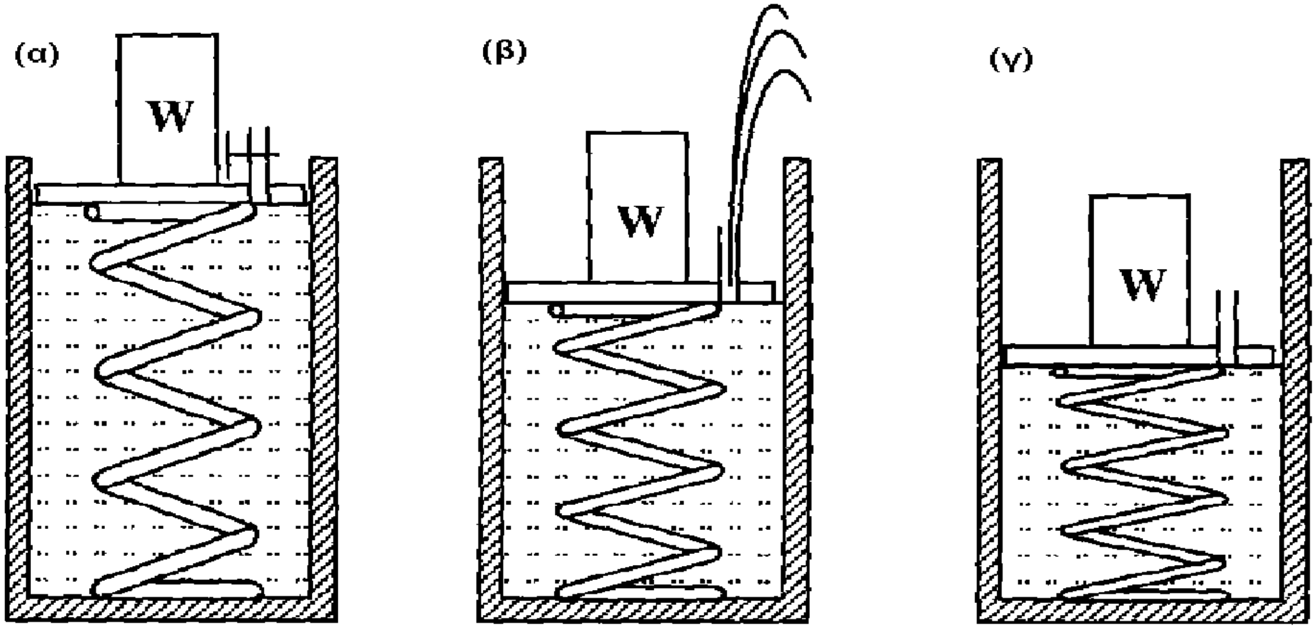
GEOTECHNICAL ENGINEERING IN THE DESIGN OF STRUCTURES:

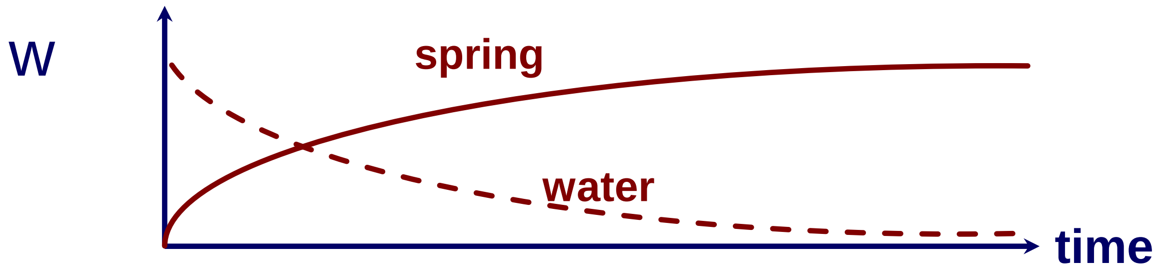
Consolidation-long term settlement

Professor V.N. Georgiannou,
MSc, DIC, Ph.D.

NTUA, 05/10/2022

Spring analogy to soil consolidation



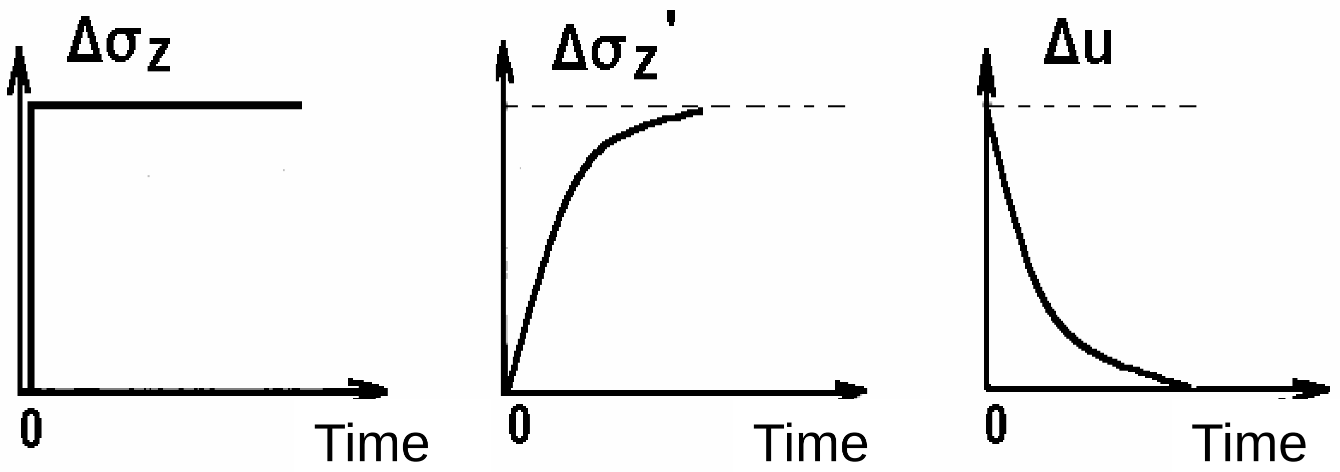


PRINCIPLES OF CONSOLIDATION
ΑΡΧΕΣ ΣΤΕΡΕΟΠΟΙΗΣΗΣ
Saturated material-Κορεσμένο υλικό

Change in vertical stress (induced on top of geostatic stresses)

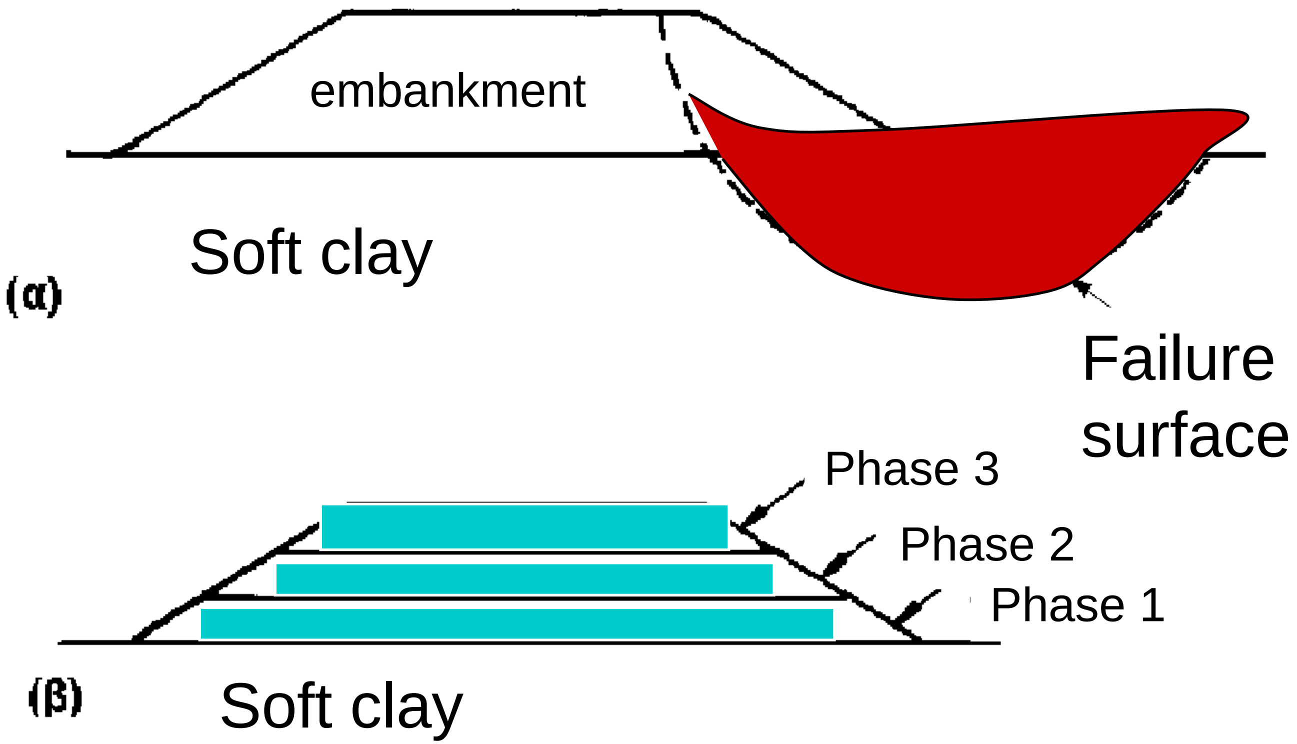
Change in vertical effective stress

Change pore water pressure



Change in load	$\Delta\sigma_z$	t=0
Excess pore pressure completely dissipates	**Δu**	**t=∞**

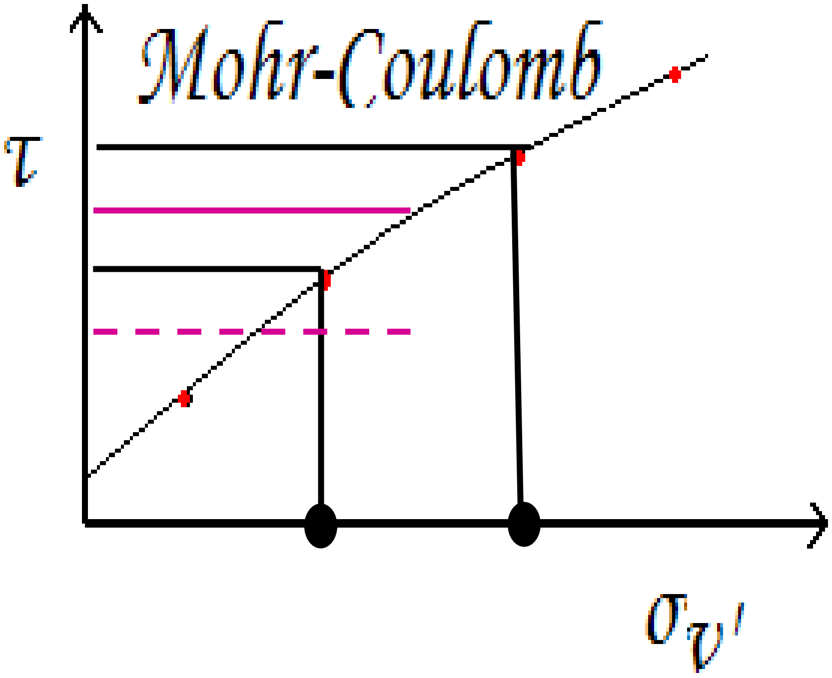
Rate of deformation for fine-grained soils

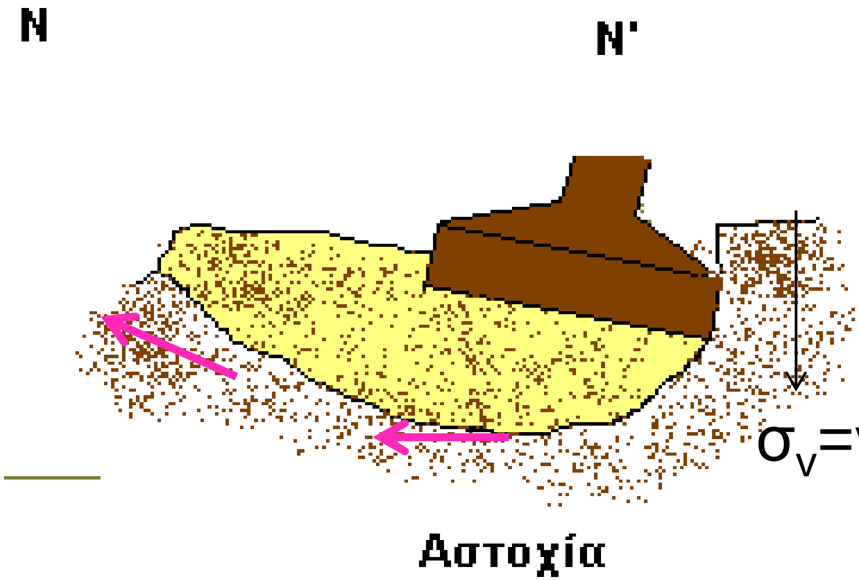


(a) Construction

(b) Construction in stages

Mohr-Coulomb failure criterion

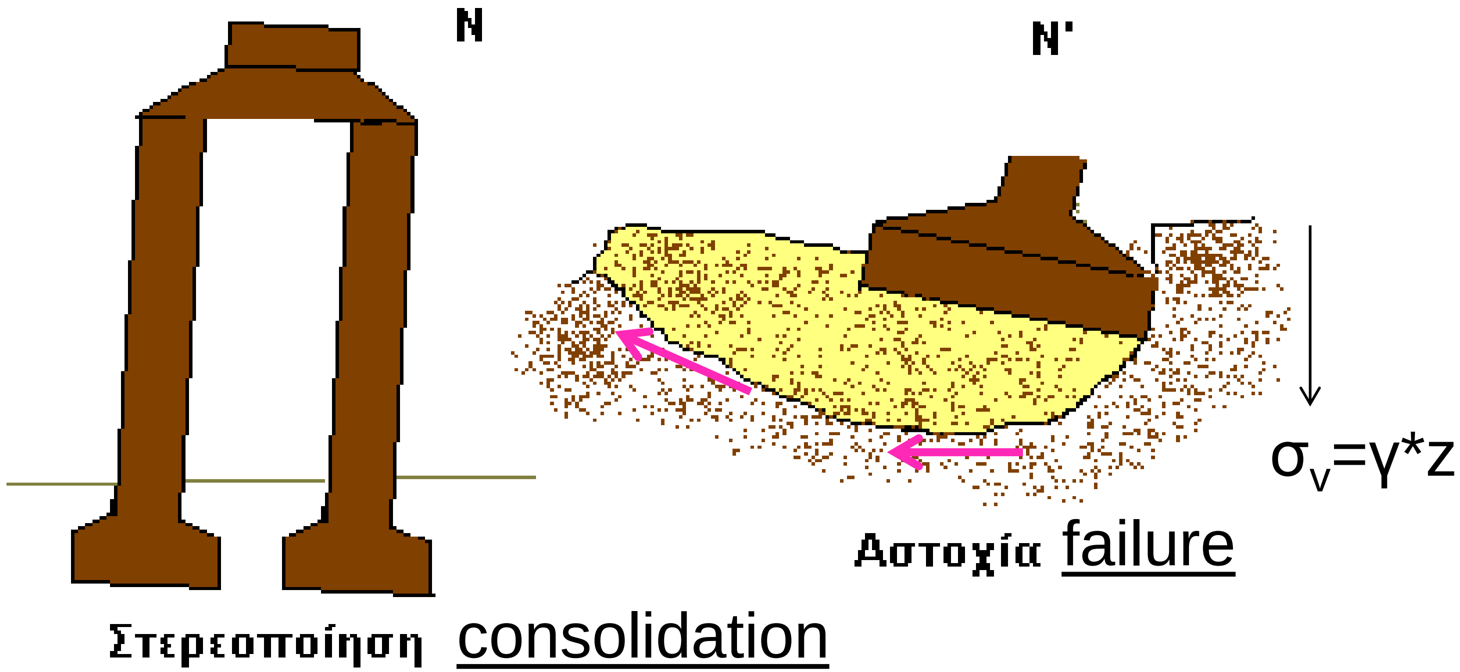




$\sigma_v = \gamma * z$

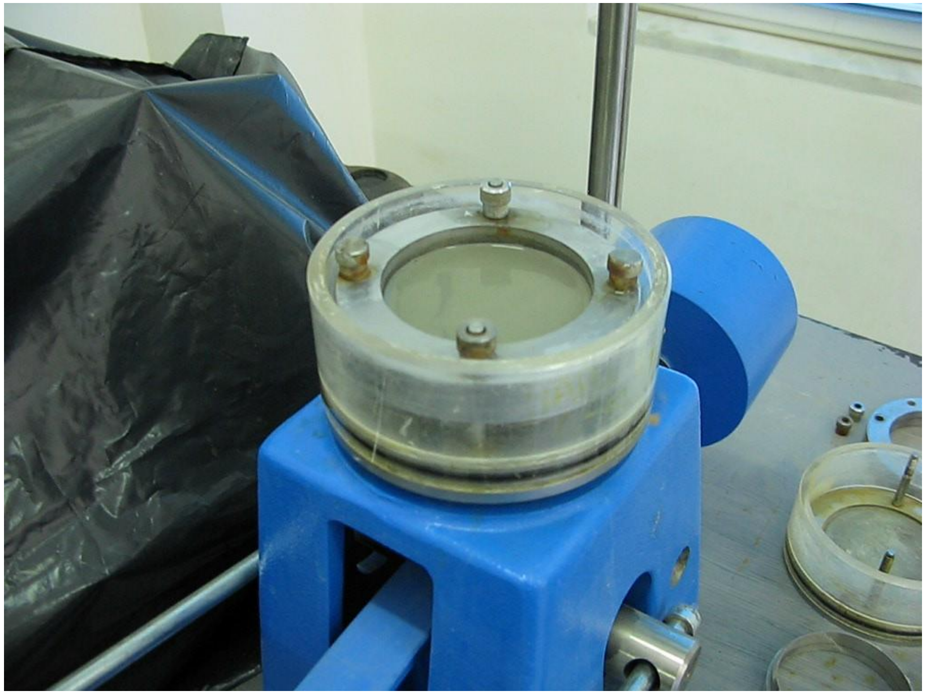
failure

SOIL REACTION TO LOADING

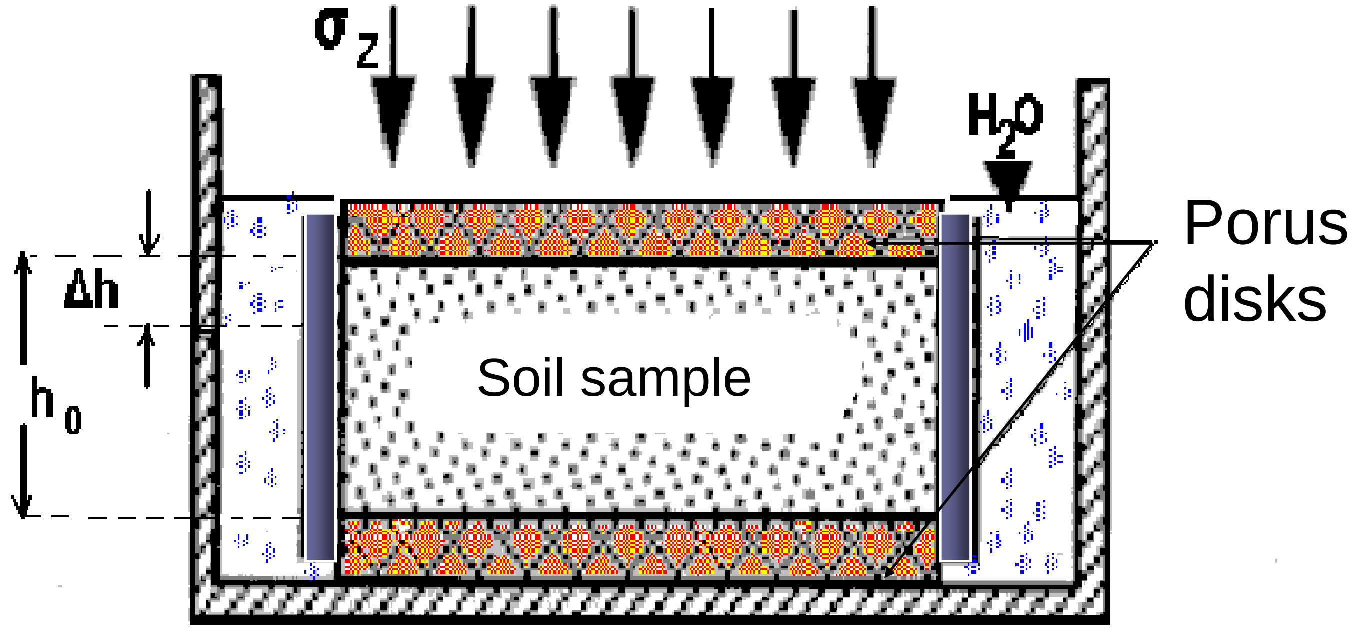


1. Load carried by foundation is N or N’, where N’>N
2. For case (a) soil consolidation takes place
3. For case (b) and for soil obeying the Mohr-Coulomb failure criterion, failure of the soil below the founding plane will occur

oedometer



1-DIMENSIONAL CONSOLIDATION
experimental setup



$$\varepsilon_z = \frac{\Delta h}{h_0}$$

ONE-DIMENSIONAL TEST

- Study of the long term settlement of fine-grained soil layers
- <u>measurenents:</u>
  - Applied vertical stress
  - Sample deformation
  - Time
- <u>Sample:</u>
  - dimensions d=75mm, h=20mm
  - placed in a stiff ring
  - immersed in water
- <u>Loading sequence:</u> vertical stress increase e.g. 50, 100, 200, 400, 800kPa or decrease 400, 50kPa. Each load is applied until settlement is complete

LINEAR ELASTICITY

$$\varepsilon_x = \frac{1}{E}\left(\sigma'_x - \nu\left(\sigma'_y + \sigma'_z\right)\right)$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma'_y - \nu\left(\sigma'_z + \sigma'_x\right)\right)$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma'_z - \nu\left(\sigma'_x + \sigma'_y\right)\right)$$

$$\varepsilon_v = \frac{1}{K}\left(\frac{\sigma'_x + \sigma'_y + \sigma'_z}{3}\right)$$

$$\left\{ \nu = 0.5 \right\}$$

$$K = \frac{E}{3(1-2\nu)}$$

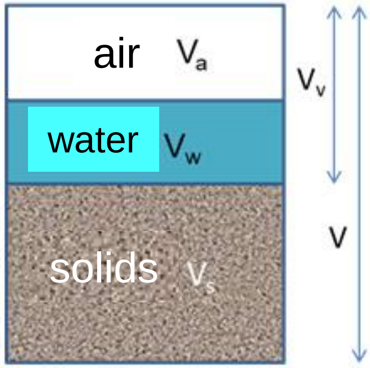
$$p' = K \cdot \varepsilon_v$$

failure

A change in mean effective stress without shear creates volumetric strains

Void ratio, e, is related to axial strain, εz

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \varepsilon_z = \frac{\Delta h}{h_0}$$



Volumetric strain is also

$$V_0 = V_{v0} + V_s$$

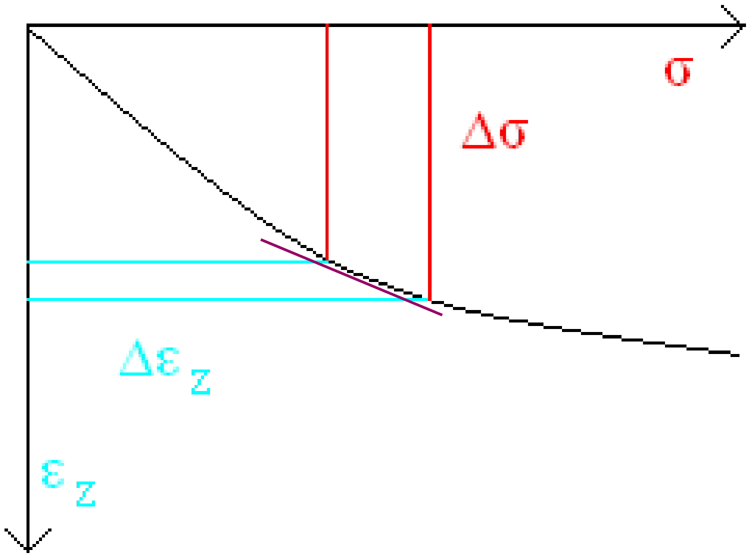
$$\varepsilon_v = \frac{V_0 - V}{V_0} = \frac{V_0 / V_s - V / V_s}{V_0 / V_s} = \frac{e_0 - e}{1 + e_0}$$

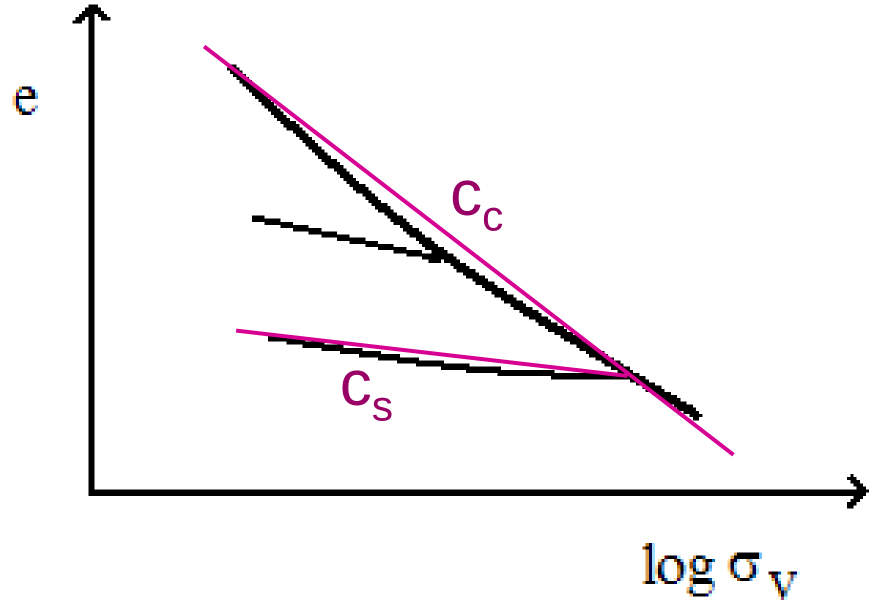
Where e is the present void ratio, e_0 the initial void ratio, V the present volume, V_0 the initial volume, and Vs the total volume of soil particles alone.
Therefore, e is related to Δh through

$$e = e_0 - \varepsilon_v (1 + e_0) = e_0 - \frac{\Delta h}{h_0} (1 + e_0)$$

Stress-strain curves

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \Delta h / h_0 = \Delta e / 1 + e_0$$



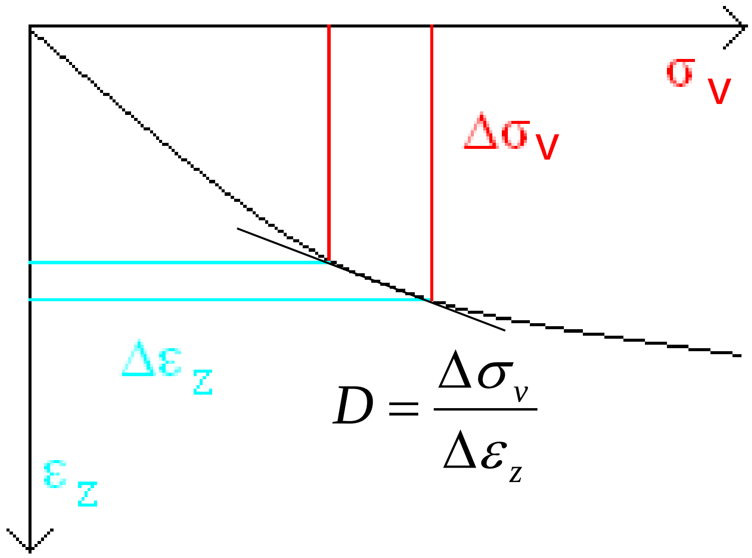


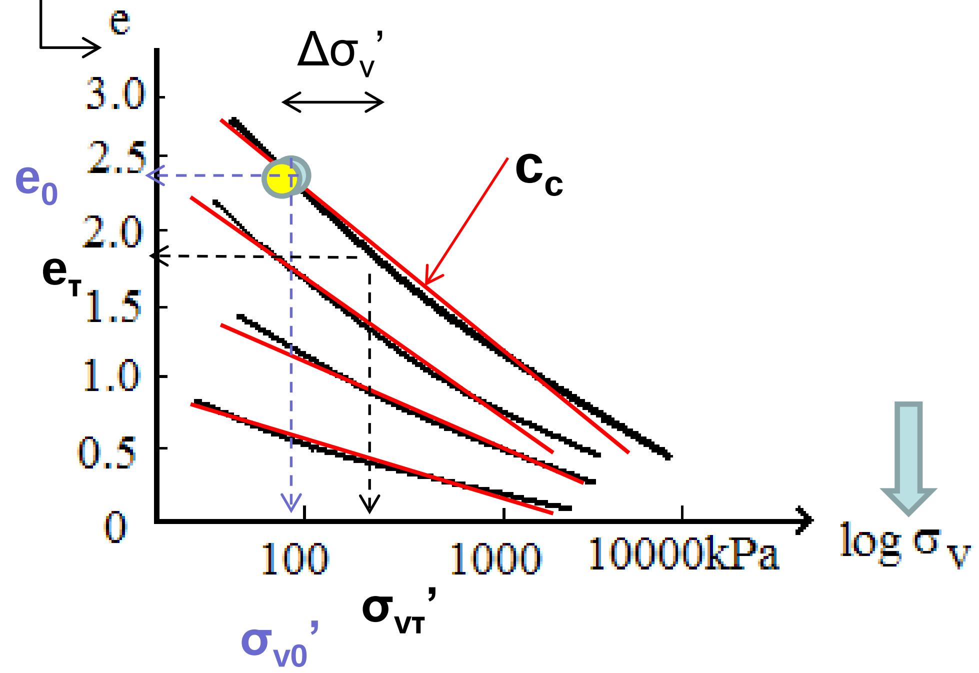
Modulus of 1-dimensional compression = $D = \Delta\sigma / \varepsilon_z$

settlement: $\Delta h = h_0 * \Delta\sigma / D$ **or** $h_0 * Cc / (1+e_0) * \Delta \log \sigma_v$

Settelement calculation: ρ_c

$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \Delta h/h_0 = \Delta e/1+e_0$





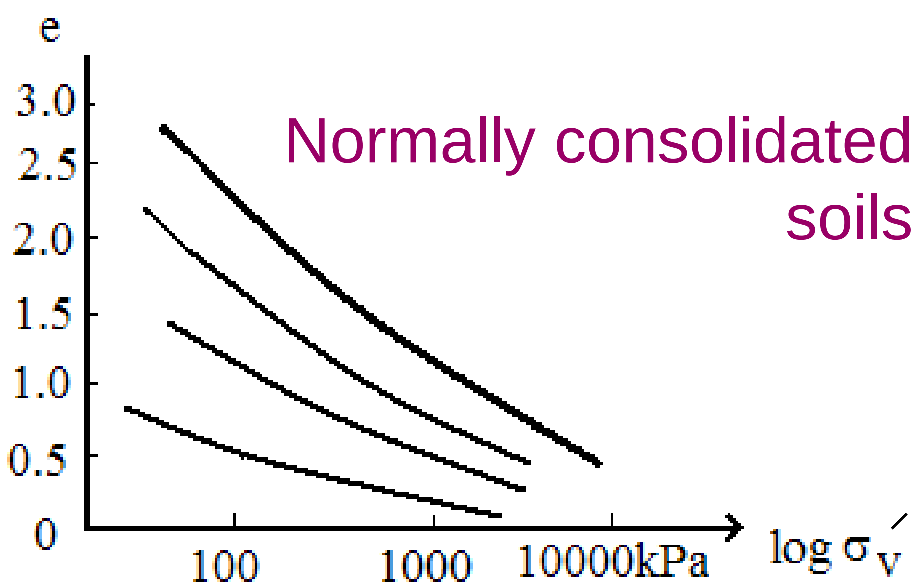
$$\frac{\rho}{H} = \varepsilon_z \rightarrow \rho = H \times \varepsilon_z$$

$$\varepsilon_Z = \frac{\Delta h}{h_0} = \frac{c_c}{1+e_0} \log\left(\frac{\sigma_{v0}' + \Delta\sigma_v'}{\sigma_{v0}'}\right)$$

Soil layer: NC = normally consolidated

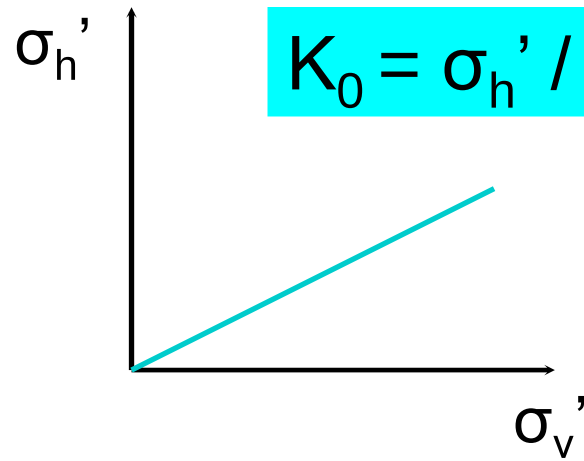
Normal compression lines for various clay soils

Normally consolidated soils



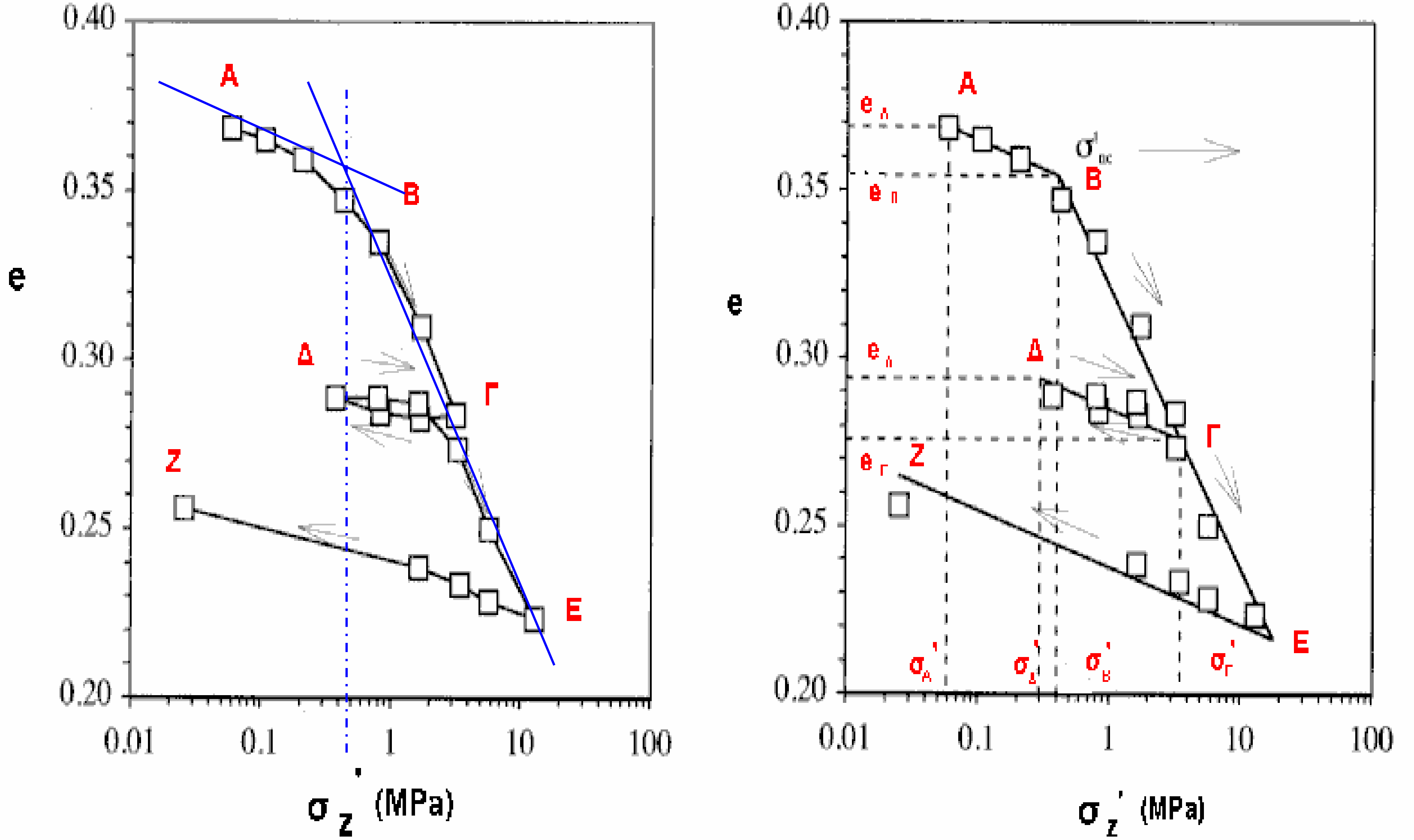
? σ_h'

$K_0 = \sigma_h' / \sigma_v'$



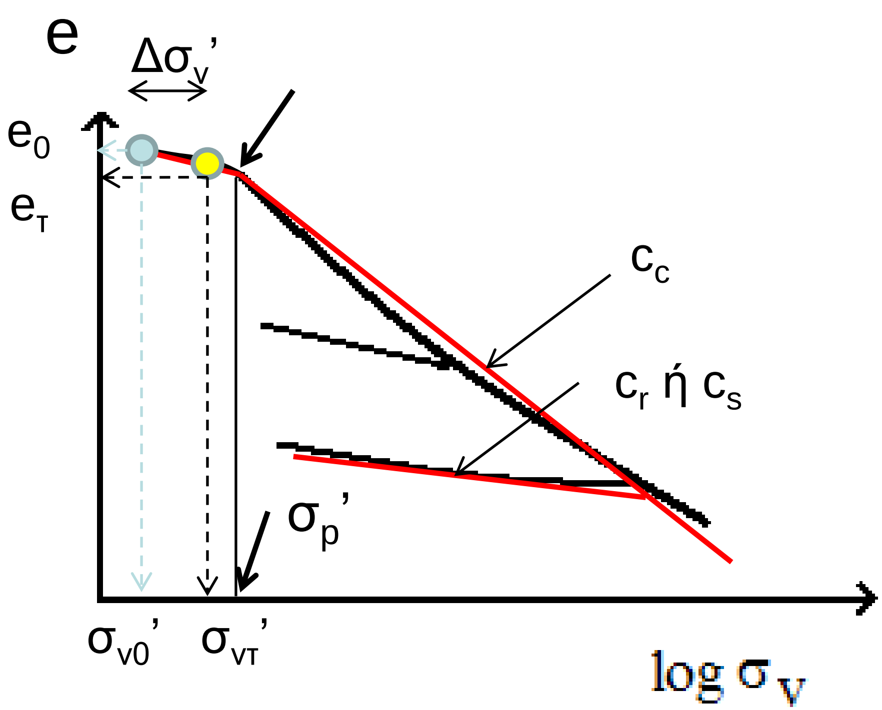
Normally consolidated soils

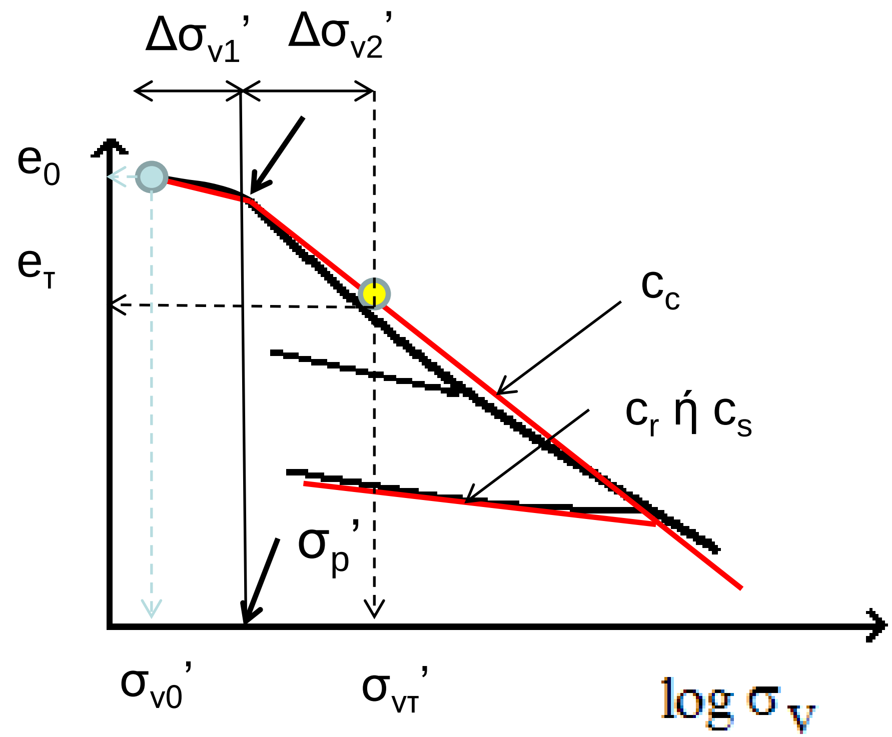
Experimental results during one dimensional consolidation tests



Consolidation test results: idealized representation of stress - strain response for OVERCONSOLIDATED CLAYS

Settelement calculation: ρ_c





$$\rho = H\,\frac{c_r}{1+e_0}\log\left(\frac{\sigma_{v0}'+\Delta\sigma_v'}{\sigma_{v0}'}\right)$$

$$\rho = H\,\frac{c_r}{1+e_0}\log\left(\frac{\sigma_{v0}'+\Delta\sigma_{v1}'}{\sigma_{v0}'}\right)$$

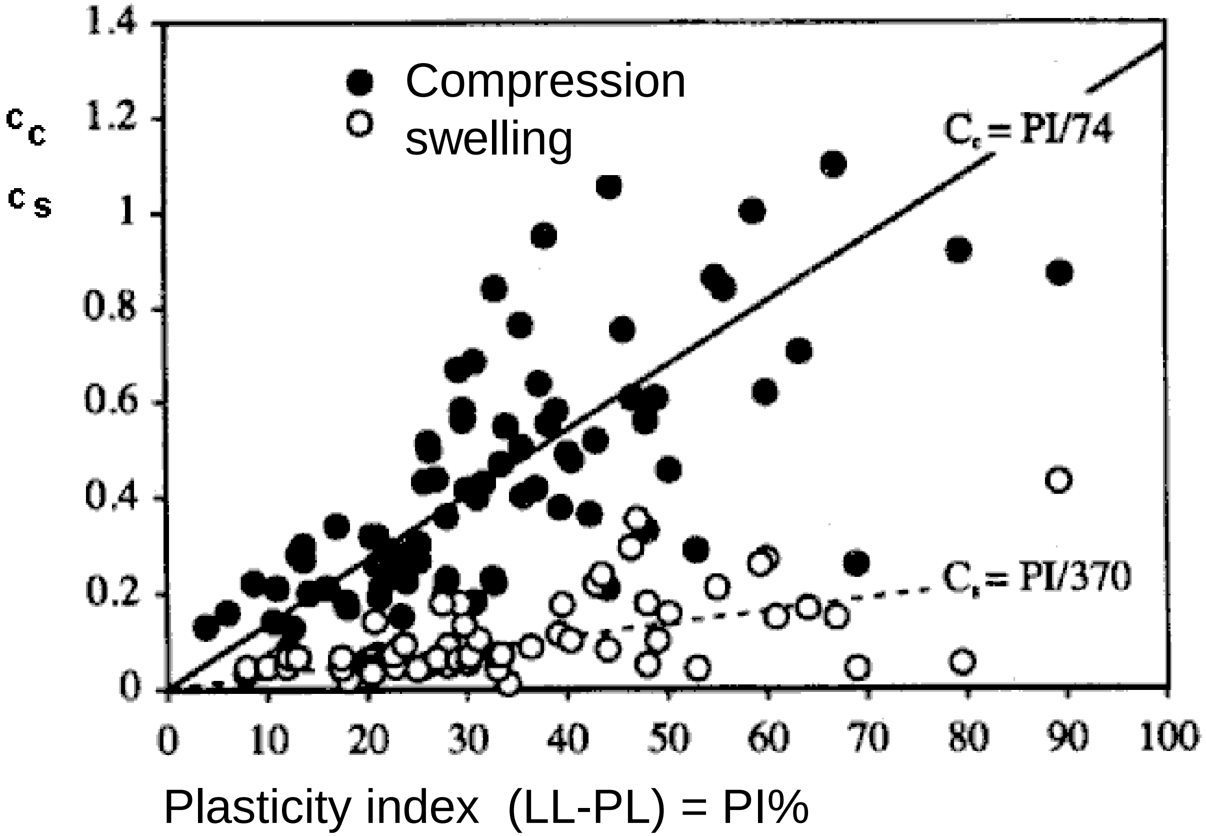
$$+H\,\frac{c_c}{1+e_0}\log\left(\frac{\sigma_p'+\Delta\sigma_{v2}'}{\sigma_p'}\right)$$

Soil layer: overconsolidated where σ_p' is the preconsolidation pressure

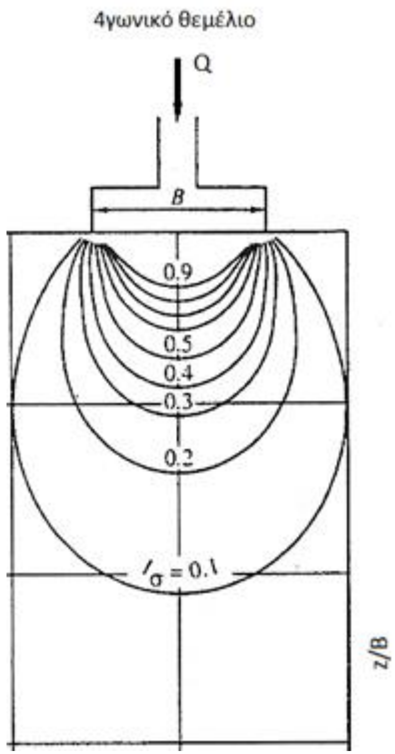
$\sigma_{v0}' < \sigma_p'$

$$OCR = \frac{\sigma_p'}{\sigma_{v0}'}$$

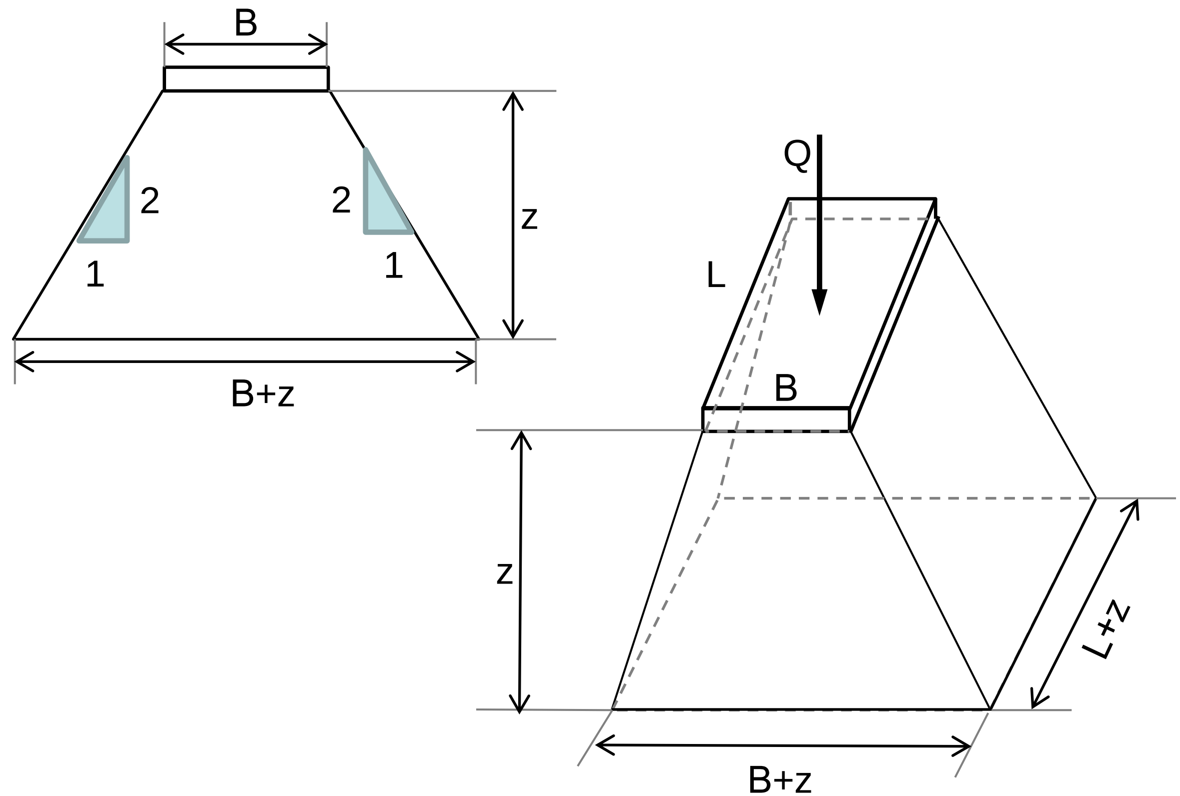
Compressibility indices: compression index Cc, swelling index Cs



Settlement ρ_c of a soil layer



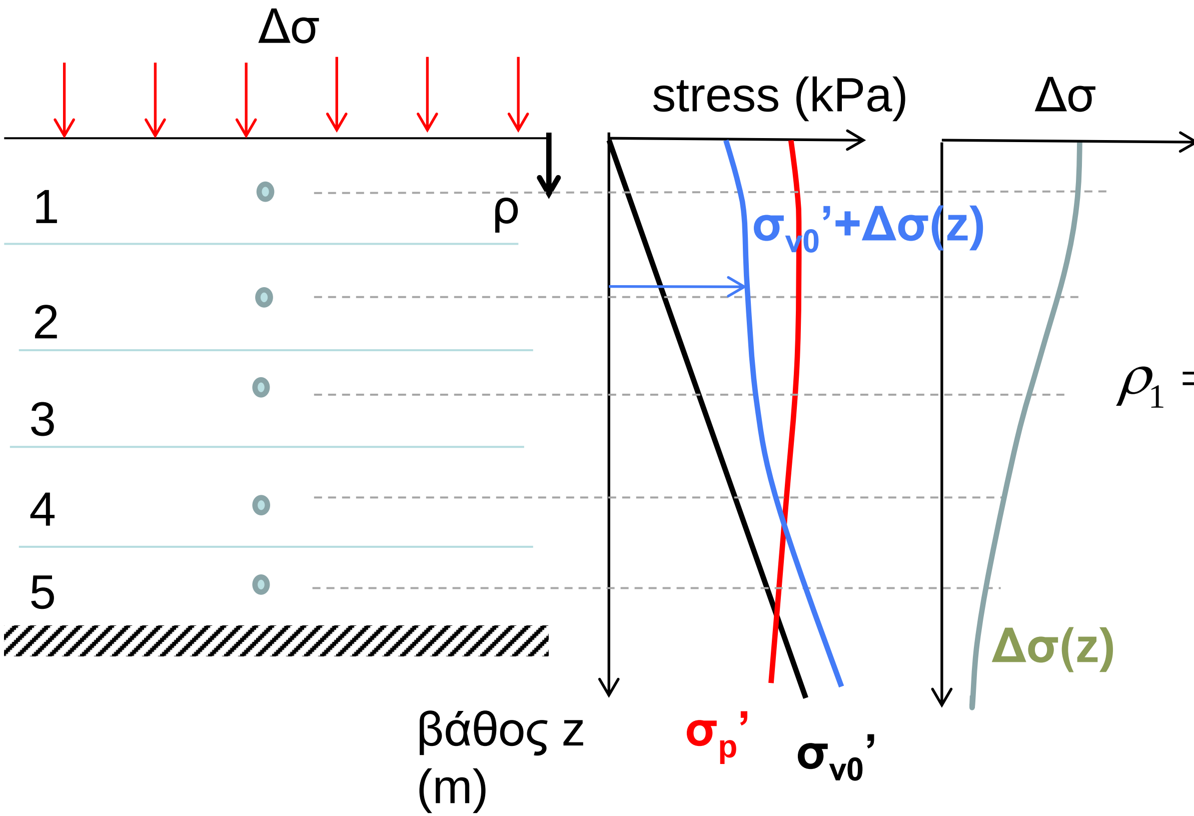
Stress reduction with depth



Simplified stress distribution with slope 2:1

$$\Delta\sigma_z' = Q / \{(B+z) x (L+z)\}$$

Settlement calculation in soil layers: ρ_c



$$\rho_1 = H_1 \frac{c_r}{1+e_0} \log\left(\frac{\sigma_{v0}'+\Delta\sigma_z}{\sigma_{v0}'}\right)$$

$$\rho_c = \sum_1^5 \rho$$

Sub/ layer	thickness(m)	Centre of sublayer (m)	σ_{v0}' (kPa)	Δσ(z) (kPa)	σ_{v0}'+Δσ(z) (kPa)	σ_p' (kPa)	c_c, c_r	ρ (m)
1	H_1				< σ_p'		Cr	
2					<		Cr	
3					<		Cr	
4					>		Cc	
5					>		Cc	

CONSOLIDATION THEORY

ASSUMPTIONS

- **Soil fully saturated**
- **Soil particles and water are incompressible**
- **Soil layer is homogeneous and laterally confined**
- **Relationship between e and σ' is linear during a stress increment**
- **Darcy's law describes the flow of water through soil**
- **The permeability coefficient k remains constant**
- **The soil's own weight has negligible effects**

Consolidation theory

- Darcy

$$\upsilon_y = -k \frac{\partial h}{\partial y}$$

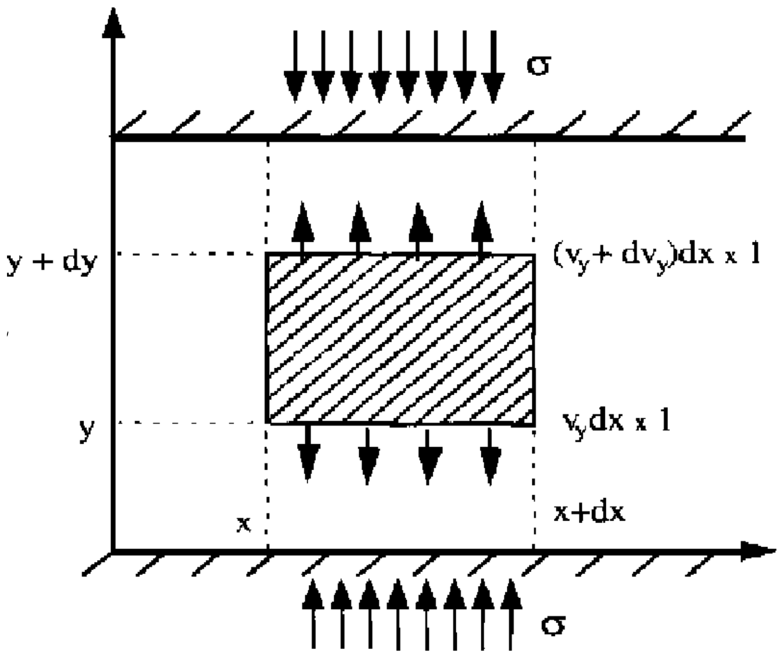
Volume of water stored or lost by the element per unit of time

$$\frac{dV_w}{dt} = \left(\upsilon_y + \frac{\partial \upsilon_y}{\partial y} \cdot dy \right) \cdot dx \cdot 1 - \upsilon_y \cdot dx \cdot 1$$

$$= \frac{\partial \upsilon_y}{\partial y} \cdot dy \cdot dx = k \frac{\partial^2 h}{\partial y^2} \cdot dx \cdot dy$$

$$h = h_{\gamma\varepsilon\omega\mu} + \frac{1}{\gamma_w} \cdot (u_0 + u) \Rightarrow \frac{\partial^2 h}{\partial y^2} = \frac{1}{\gamma_w} \left(\frac{\partial^2 u}{\partial y^2} \right)$$

$$\frac{dV_w}{dt} = \frac{k}{\gamma_w} \cdot \frac{\partial^2 u}{\partial y^2} \cdot dx \cdot dy \qquad \ldots\ldots(1)$$



Consolidation theory

- If the soil behaves elastically

$$\frac{de}{1+e_0} = m_v \cdot d\sigma'$$

$$\frac{de}{1+e_0} = \frac{d(V_v - V_s)}{V_0} = \frac{dV_v}{V_0} = \frac{dV_v}{dx \cdot dy \cdot 1}$$

where
V_0 Initial volume
V_s Solid volume
V_v Void volume

hence $$\frac{dV_v}{dt} = m_v \cdot \frac{\partial \sigma'}{\partial t} \cdot dx \cdot dy$$

- For constant total stress

$$\frac{\partial \sigma}{\partial t} = \frac{\partial \sigma'}{\partial t} + \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial \sigma'}{\partial t} = -\frac{\partial u}{\partial t} \qquad \ldots\ldots(3)$$

Where u is excess pore water pressure

$$\frac{dV_v}{dt} = -m_v \cdot \frac{\partial u}{\partial t} \cdot dx \cdot dy \qquad \ldots\ldots.(4)$$

- When the soil element remains fully saturated

$$\frac{dV_v}{dt} = \frac{dV_w}{dt}$$

Equations 1 & 4 give the consolidation equation

$$\frac{k}{\gamma_w} \cdot \frac{\partial^2 u}{\partial y^2} = m_v \cdot \frac{\partial u}{\partial t}$$

CONSOLIDATION EQUATION

$$c_v \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial t}$$

$$c_v = \frac{k}{m_v \cdot \gamma_w}$$

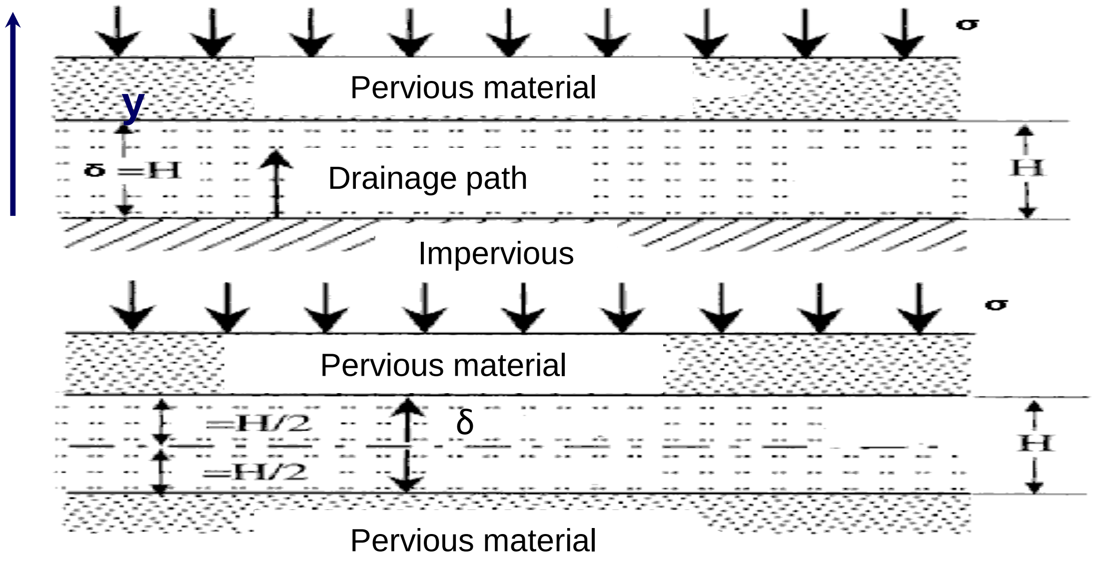
where c_v =coefficient of consolidation

$k =$ Soil permeability

$u =$ Excess pore water pressure

$m_v =$ Compressibity

Consolidation of a Soil Layer under Constant Load



Definition of (a) single- and (b) double-drainage consolidation problems

Initial conditions

$u(y, 0) = u_i = \Delta\sigma_v,\ 0 < y < H$

Boundary conditions

$u(0, t) = u(H, t) = 0,\ t > 0$

δ=drainage path

Dimentionless time factor

$$T_v = \frac{c_v * t}{\delta^2}$$

$$u(y,t) = u_{\alpha\rho\chi} \sum_{n=0}^{\infty} \frac{4}{(2n+1)\pi} \sin((2n+1)\pi \frac{y}{H}) e^{\frac{-((2n+1)\pi)^2}{4} * T_v}$$

Degree U of Consolidation – Time Factor (Terzaghi,1923)

Beginning of consolidation $\Delta\sigma'(y, t) = 0$ και $u(y, t) = u_i$

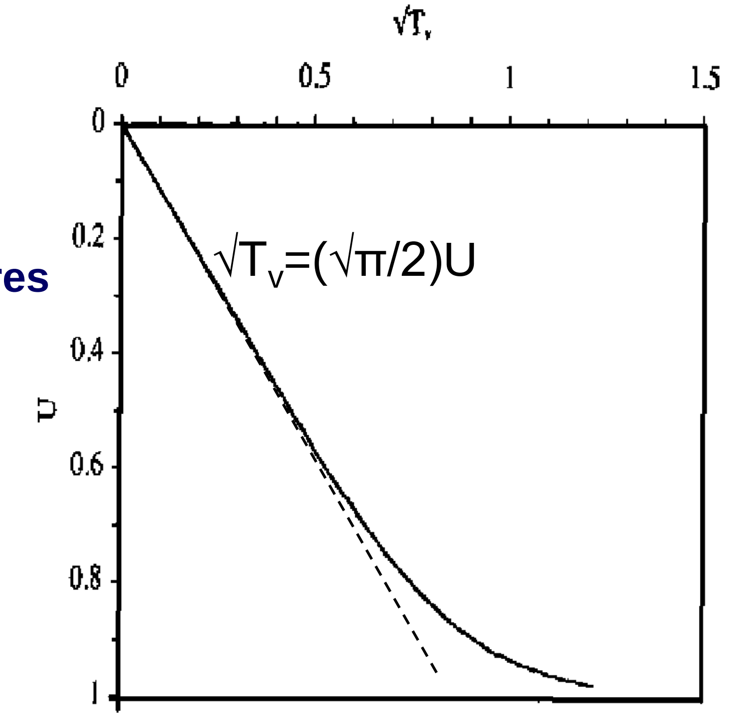
End of consolidation $\Delta\sigma'(y, t) = \Delta\sigma$ και $u(y, t) = 0$

Degree of consolidation $U_y = \dfrac{u_i - u(y, t)}{u_i} = \dfrac{\Delta\sigma'(y,t)}{\Delta\sigma}$

where $U_y = 0$ at the beginning of consolidation
and $U_y = 1$ at the end of consolidation

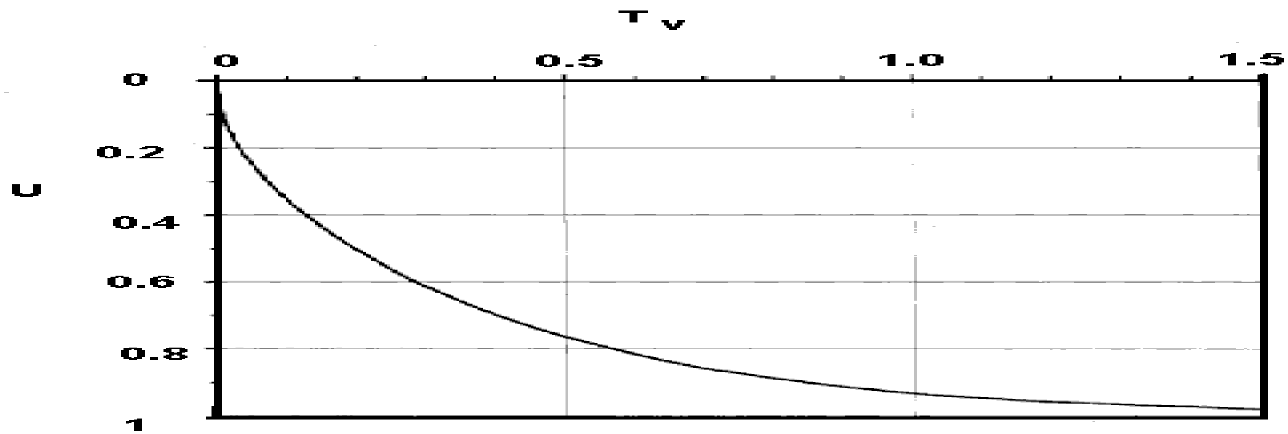
The average degree of consolid. $U = \dfrac{1}{H}\int_0^H U_y\,dy$ **is**

a function of $T_v = \dfrac{c_v * t}{H^2}$ **as shown in the figures**



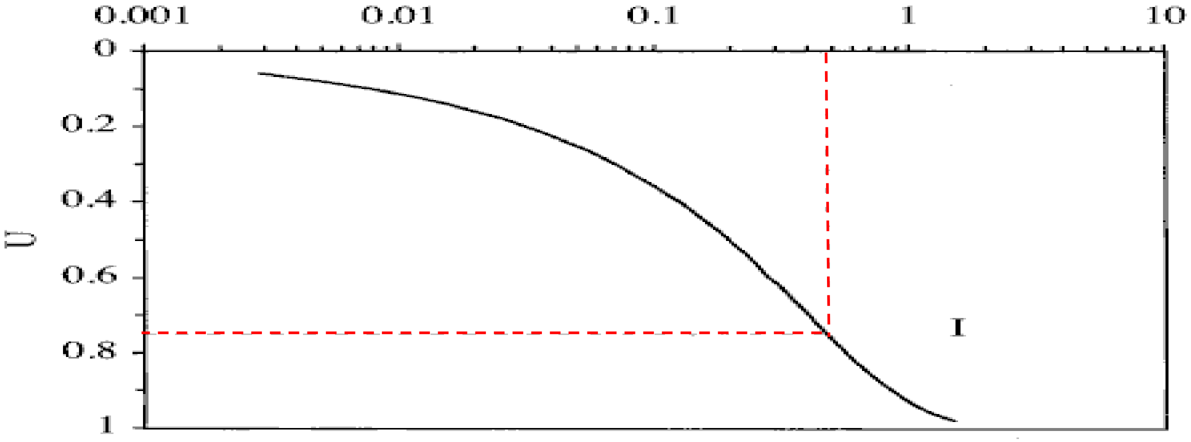
$$U = 1 - \frac{8}{\pi^2}\sum_{m=1}^{\infty}\frac{e^{-m^2\pi^2 T_v/4}}{m^2}$$

Approximation of the Equation of consolidation

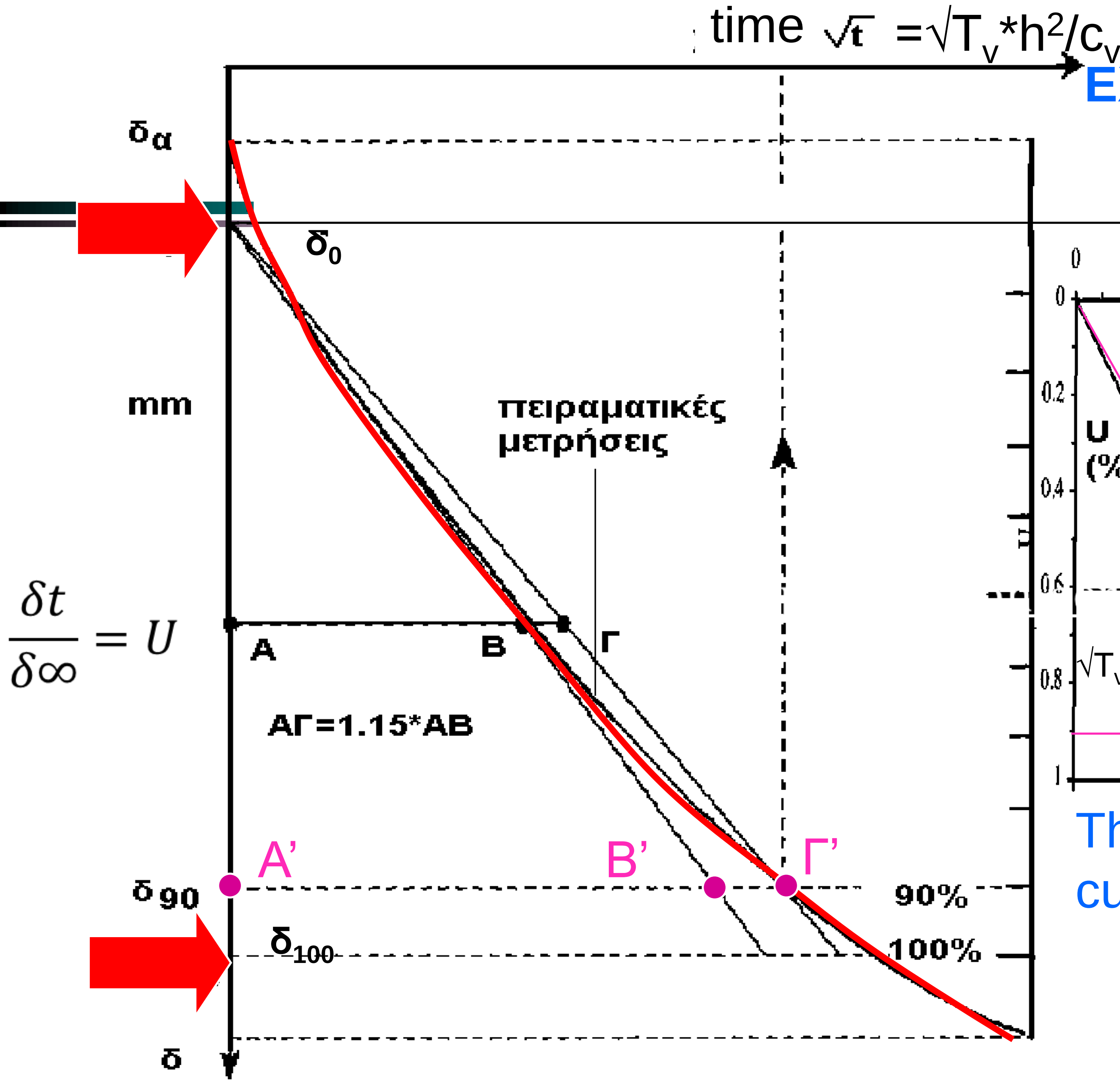


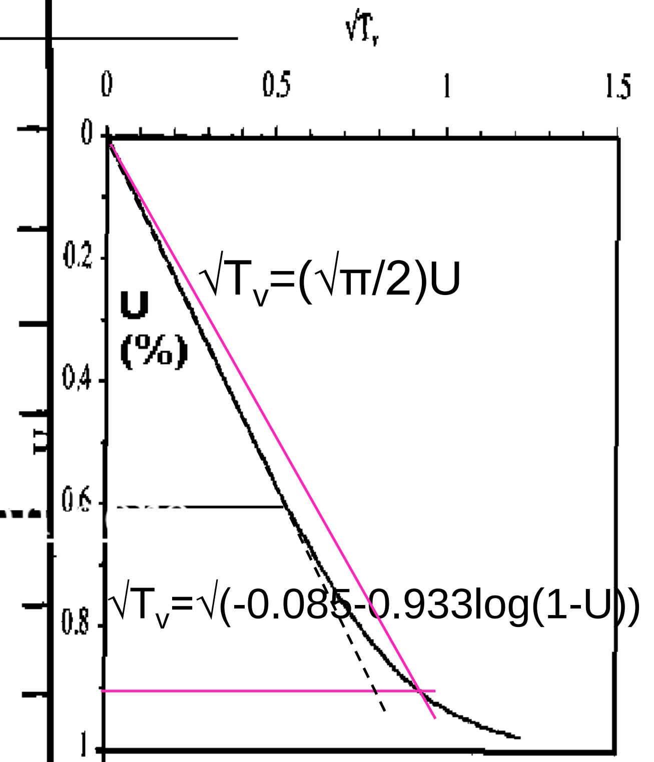
$T_1(U) = \pi/4*(U^2)$, **U<0.6** (Fox,1948)

$T_2(U) = -0.085-0.933 \log(1-U)$, **U>0.6** (Taylor,1948)



EXPERIMENTAL RESULTS





Theoretical curve: U - $\sqrt{T_v}$

Time of Consolidation

- Time at 90% (U>0.9) of consolidation t_c
- From equation of consolidation T_v~1 at U=0.9

$$T_v = \frac{c_v * t}{\delta^2}, \delta = H$$

$$t_c \cong \frac{H^2}{c_v} = \gamma_w * \frac{m_v * H^2}{k}$$

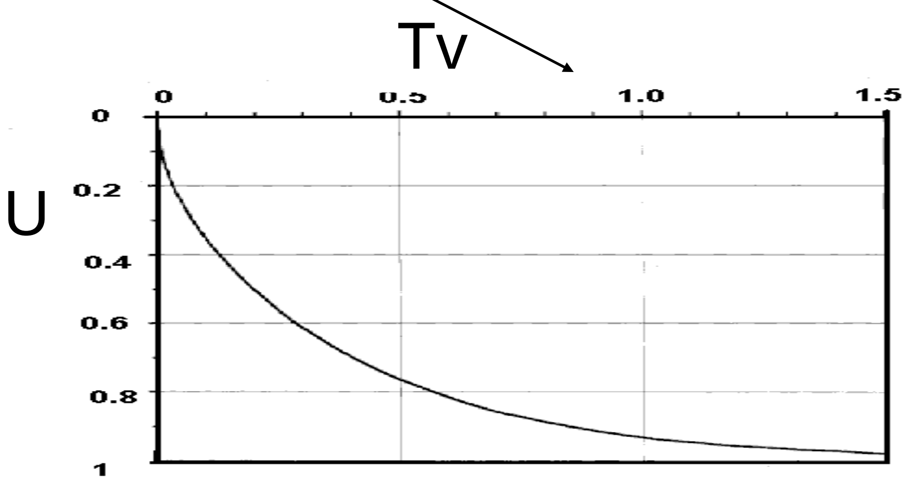
- increases with compressibility
- increases with layer depth
- decreases permeability
- not affected by Δσ

Time of Consolidation for a Soil Layer

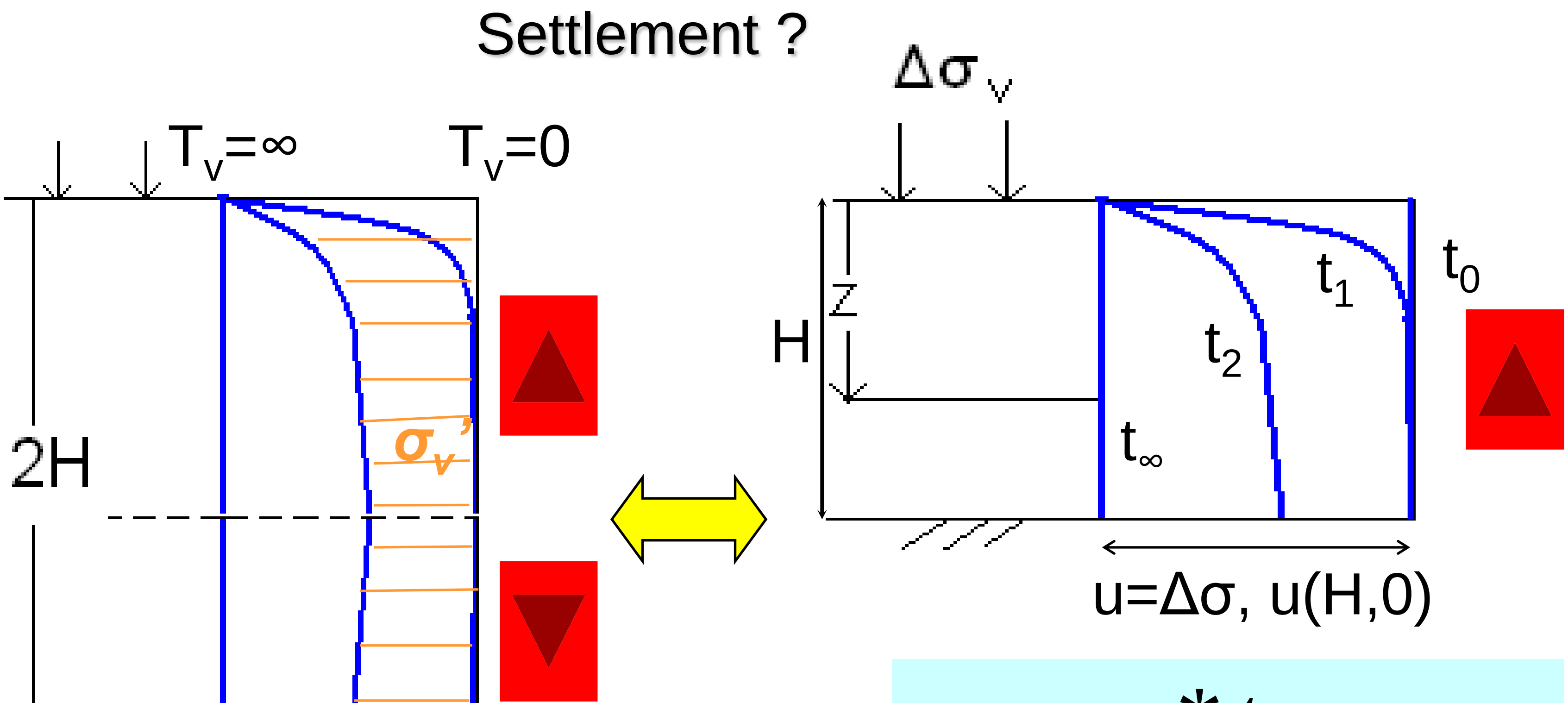
Coefficient of consolidation

$$T_v = \frac{c_v * t}{\delta^2}$$

- time at (90%) of consolidation for soil layer
- length of drainage path in soil layer

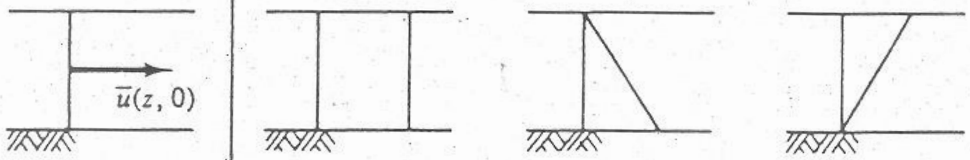


Excess pore pressure variation: u(y,t)



$$T_v = \frac{c_v * t}{\delta^2}, \delta = H$$

Excess pore water pressure distribution

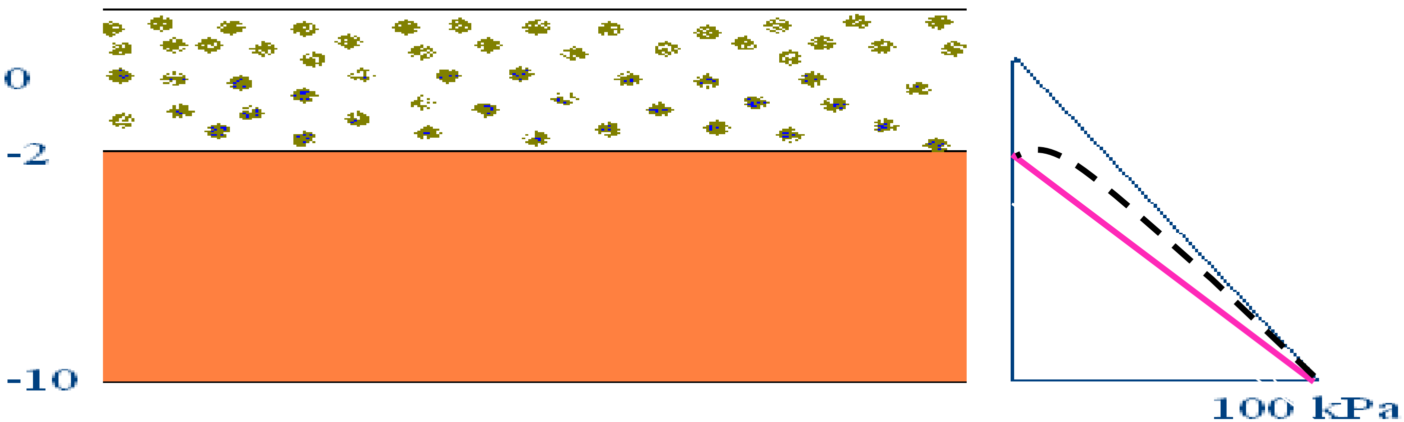


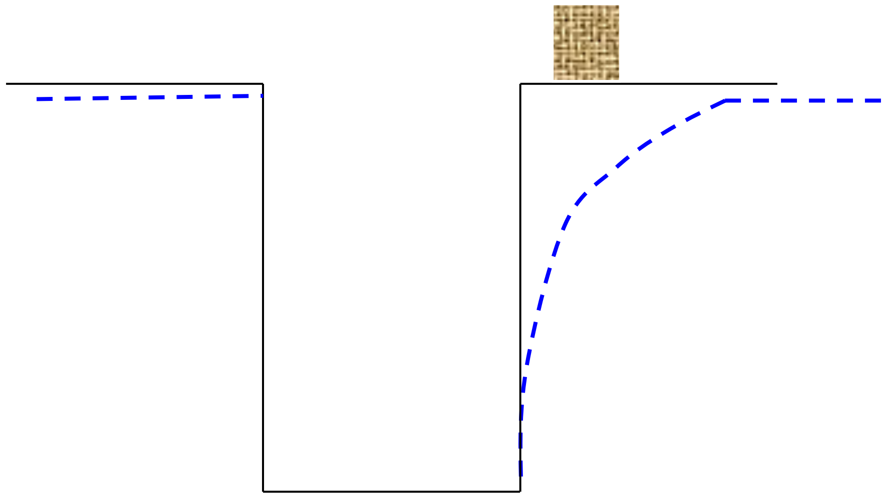
U_t	T_v		
	1	2	3
0.1	0.008	0.047	0.003
0.2	0.031	0.100	0.009
0.3	0.071	0.158	0.024
0.4	0.126	0.221	0.048
0.5	0.196	0.294	0.092
0.6	0.287	0.383	0.160
0.7	0.403	0.500	0.271
0.8	0.567	0.665	0.440
0.9	0.848	0.940	0.720

Ακριβής επίλυση της εξίσωσης μονοδιάστατης στερεοποίησης με στράγγιση ελεύθερη στο ένα άκρο για διαφορετικές κατανομές αρχικής υπερπίεσης πόρων.

Exact solutions for one-dimensional consolidation with one-way drainage for various distributions of initial excess pore water pressure

Triangular distribution of excess pwp: Δu





- Decrease in Δu

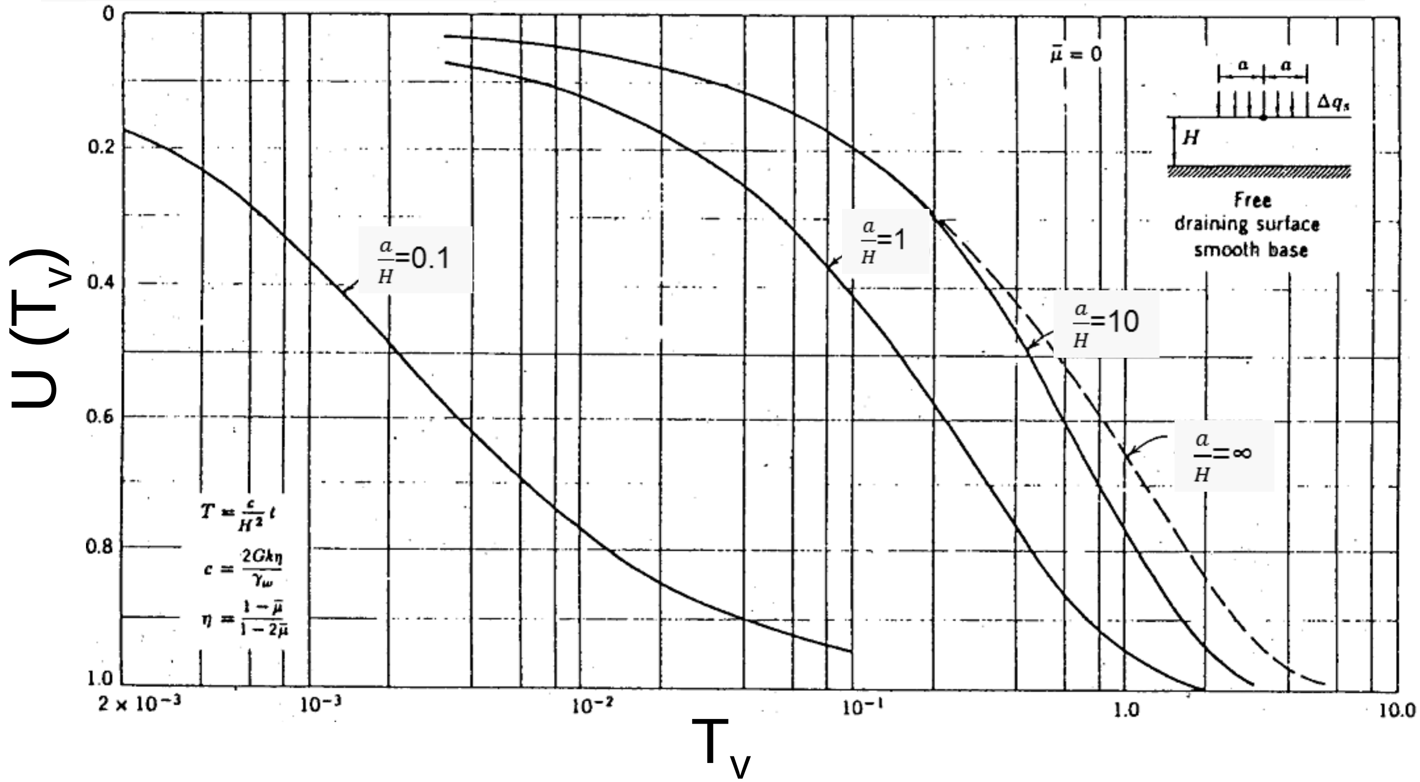
→

- Increase in Effective Stress

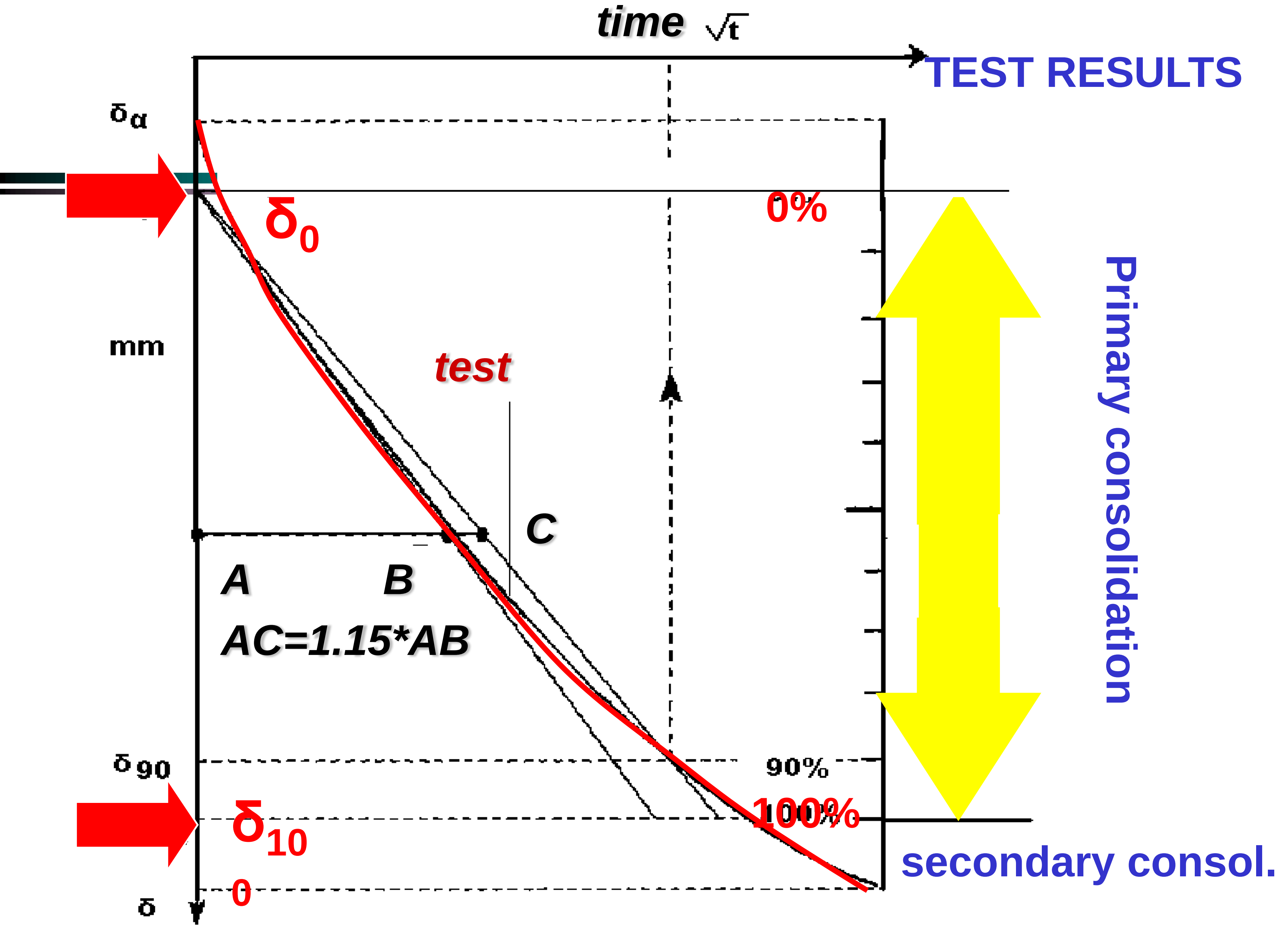
3- dimensional consolidation under circular footing

3 – Διάστατη στερεοποίηση κάτω από κυκλικό θεμέλιο

$\Delta\sigma \neq ct$



Στερεοποίηση κάτω από κυκλικό θεμέλιο, Gibson et al., 1967)



time √t
TEST RESULTS
δα
δ0
0%
mm
test
C
A
B
AC=1.15*AB
Primary consolidation
δ90
90%
δ10
100%
100%
0
δ
secondary consol.

Secondary Consolidation

- In contrast to primary consolidation, secondary compression takes place at constant effective stress and without a dissipation of excess pore pressure
- Assume it starts after primary consolidation
- The change in void ratio resulting from secondary compression from time t_s to time t is $\Delta e_s = C_a \log_{10}(t/t_s)$ (Mersi & Godlewski, 1977), where Ca is the secondary compression index
- The secondary compression axial strain εs is $\varepsilon_s = C_{\alpha\varepsilon} \log_{10}(t/t_s)$, where $C_{\alpha\varepsilon} = C_\alpha/(1+es)$ and e_s is the void ratio at the beginning os secondary compression. e_0 can be used as an approximation
- The secondary compression settlement is $\delta_s = h_s C_{\alpha\varepsilon} \log_{10}(t/t_s)$ where hs is the height of the sample (depth of layer) at the beginning of secondary consolidation
- Cαε has been correlated to the natural water content and for normally consolidated clays the following relation is suggested: $C_{\alpha\varepsilon} = 0.0001^*$ W, where W is the natural water content in percent

ΔΕΥΤΕΡΕΥΟΥΣΑ ΣΤΕΡΕΟΠΟΙΗΣΗ

Secondary consolidation: settlement under CONSTANT effective stress, significant for NC clays

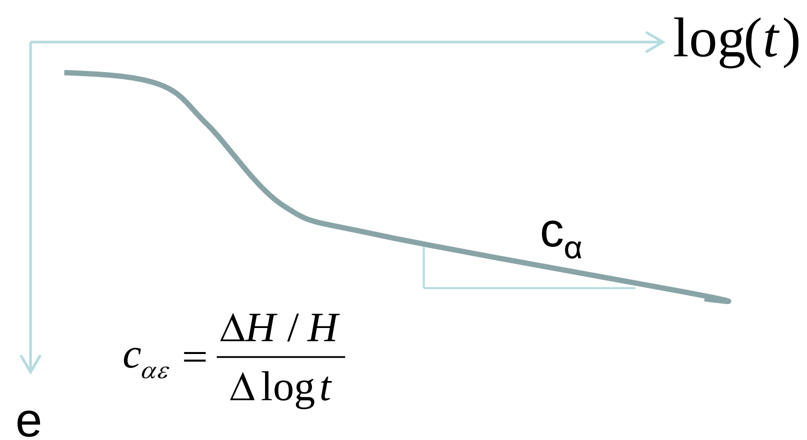
$$\rho_s = H \frac{c_\alpha}{1+e_0} \log \frac{t_2}{t_1}$$

H=layer thickness
t_2= secondary consolidation
t_1= end of primary
c_α= coefficient of secondary consolidation
e_0=void ratio at the beginning of secondary consolidation

c_α/c_c=0.025-0.085

$$c_{\alpha\varepsilon} = \frac{c_\alpha}{1+e_0} = 0.0001w$$

Mesri, 1973, NC clays, w=water content %



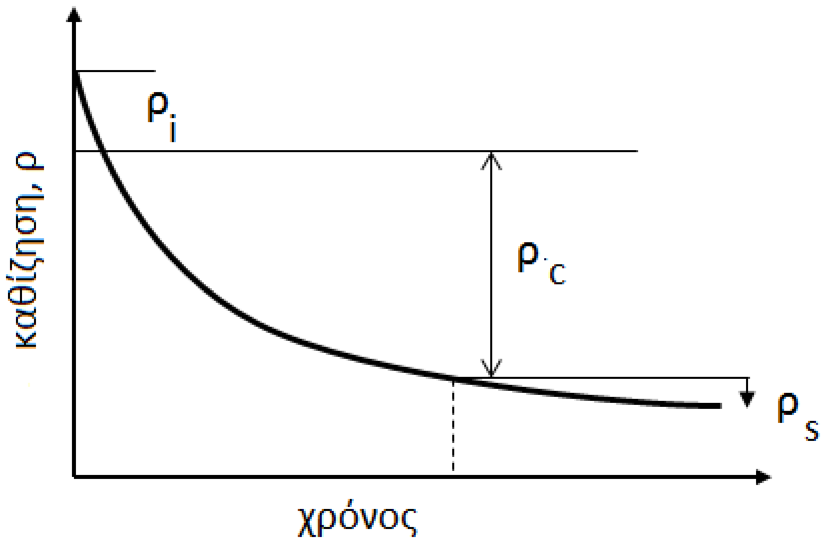
$$c_{\alpha\varepsilon} = \frac{\Delta H / H}{\Delta \log t}$$

- $\Delta e_s = C_a \log_{10}(t/t_s)$ (Mersi & Godlewski, 1977),

ΚΑΘΙΖΗΣΕΙΣ

Settlement

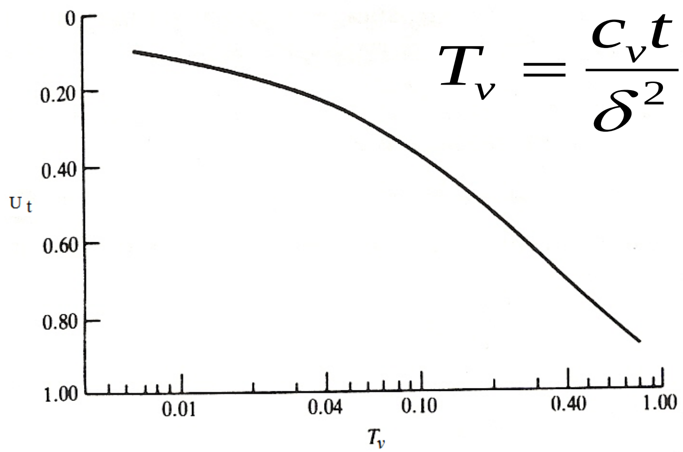
1. Immediate or undrained (ρ_i, m)
2. consolidation (ρ_c, m)
3. secondary consolidation (ρ_s, m)
4. Due to yclic loading



$$\rho_{\max} = \rho_i + \rho_c + \rho_s$$

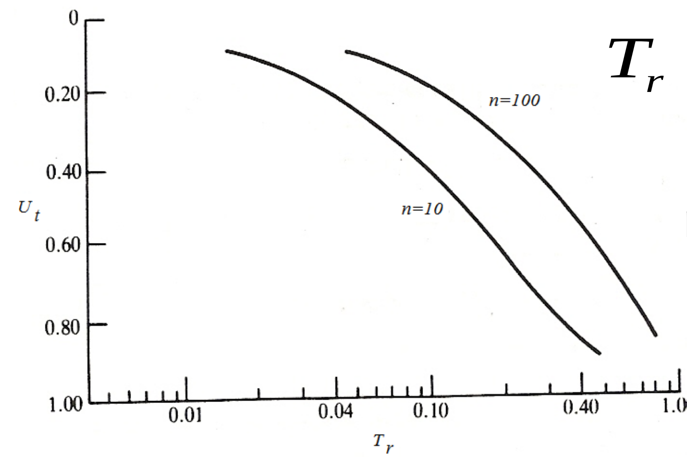
4. cyclic/dynamic loading an increase in settlement of up to 30% compared to static conditions

Vertical and radial consolidation



$$T_v = \frac{c_v t}{\delta^2}$$

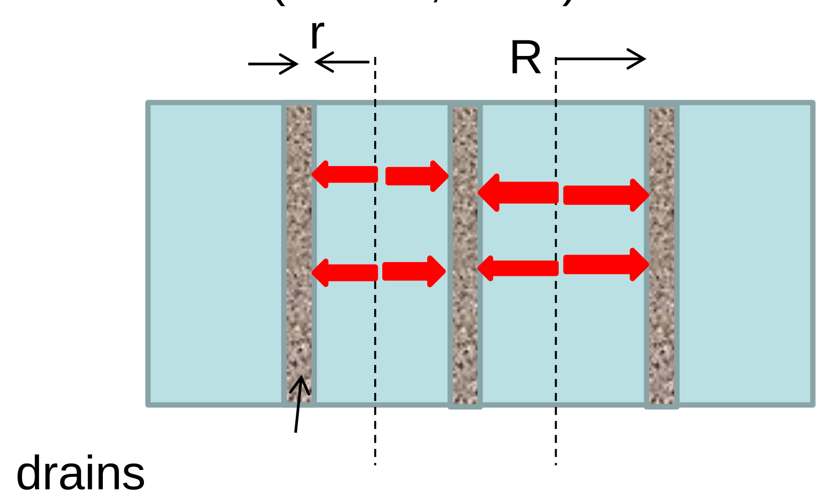
Vertical (Taylor, 1948)



$$T_r = \frac{c_r t}{R^2}$$

n=R/r

Radial (Barron, 1948)



SUMMARY

- The long term settlement of fine grained soil layers due to changes in load is controlled by primary consolidation involving the dissipation of excess pore pressure as the pore water diffuses through the compressible matrix of soil. Due to their viscus nature soils undergo secondary compression after the excess pwp has completely dissipated

- compression index c_c
- swelling index c_s
- preconsolidation stress σ_p'
- consolidation coefficient c_v

$$T_v = \frac{c_v * t}{\delta^2}$$