SEEPAGE: steady-state groundwater flow
ΥΔΑΤΙΚΗ ΡΟΗ ΔΙΑΜΕΣΟΥ ΕΔΑΦΙΚΟΥ ΥΛΙΚΟΥ

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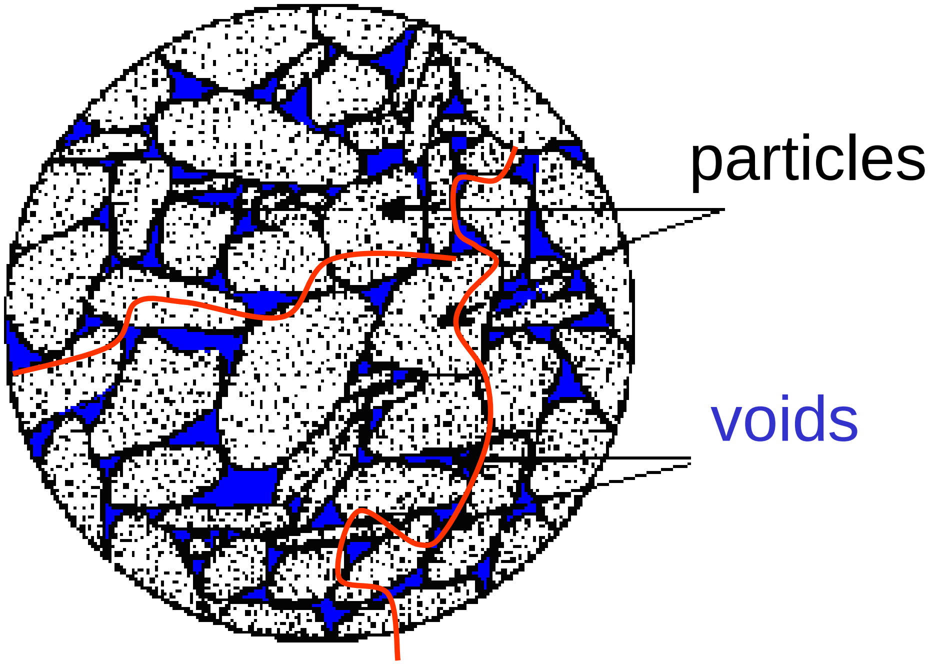
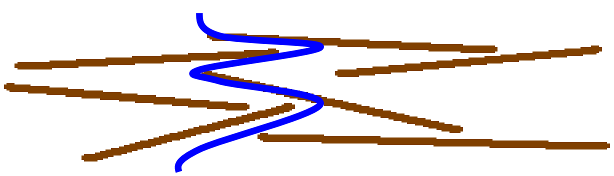
Permeable soils

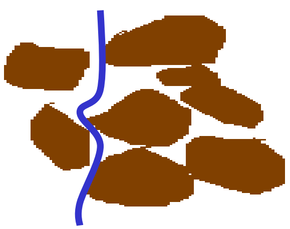
The apparent **velocity** with which groundwater moves through the bulk of the porous medium is called seepage velocity





clay

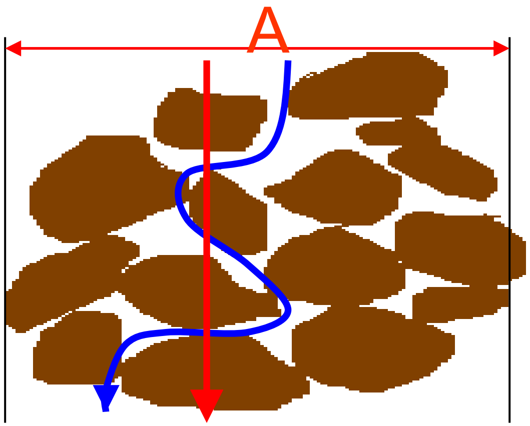
$K<10^{-7}$ m/s



sand

$K>10^{-2}$ m/s

Flow velocity or flux

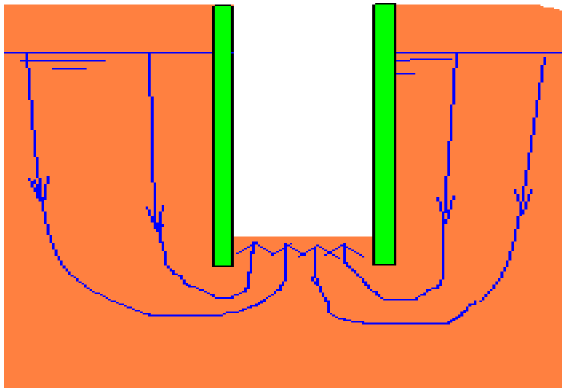


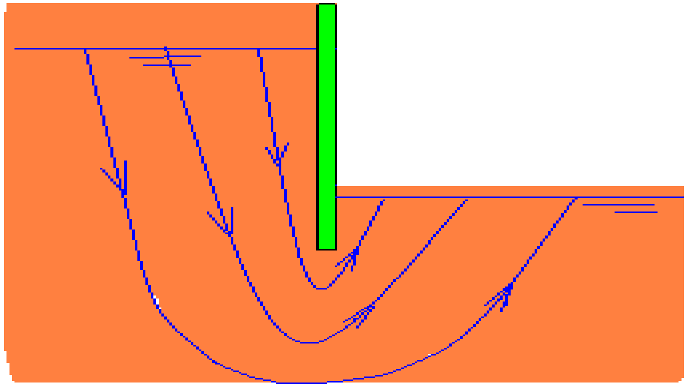
- Gross cross sectional area of flow $u=Q/A$
- Effective cross sectional area of voids $u_s=Q/A_{voids}$

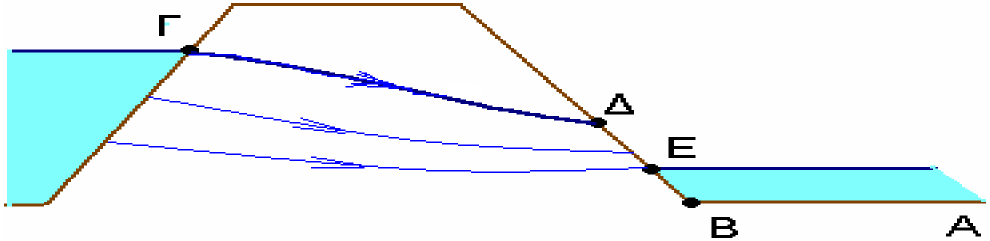
Avoids / A=Vvoids / V= n = porosity

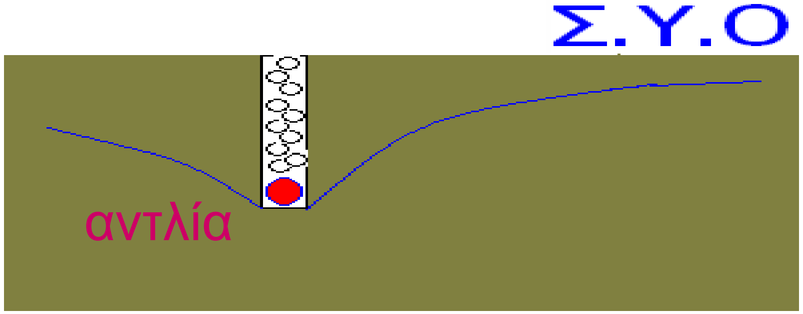
$u_s = u / n$

Flow problems



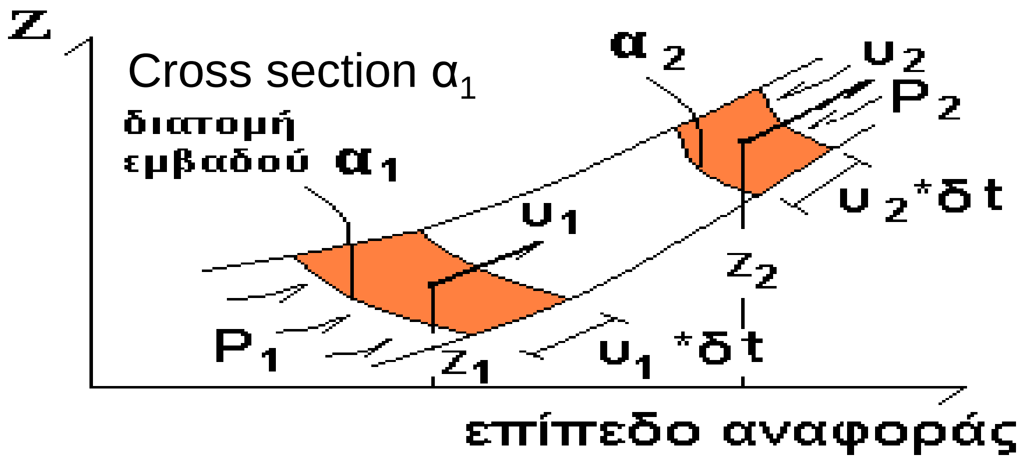






ONE DIMENSIONAL FLOW

Variables at points 1 & 2: pressure, speed, height



Reference plane

$P_1 x(\alpha_1 v_1 \delta t)V - P_2 x V =$
$\rho V g z_2 + 1/2 \rho V (v_2)^2$
$-(\rho V g z_1 + 1/2 \rho V (v_1)^2)$
Bernoulli

Pressure at elevation 1

$$\frac{P_1}{\gamma_w} + Z_1 + \frac{\upsilon_1^2}{2*g} = \frac{P_2}{\gamma_w} + Z_2 + \frac{\upsilon_2^2}{2*g} = h$$

Principle of energy conservation

$\sim 10^{-6}\mu$

Potential+kinetic

Total head

$$h = \frac{u}{\gamma_w} + z + \Delta h$$

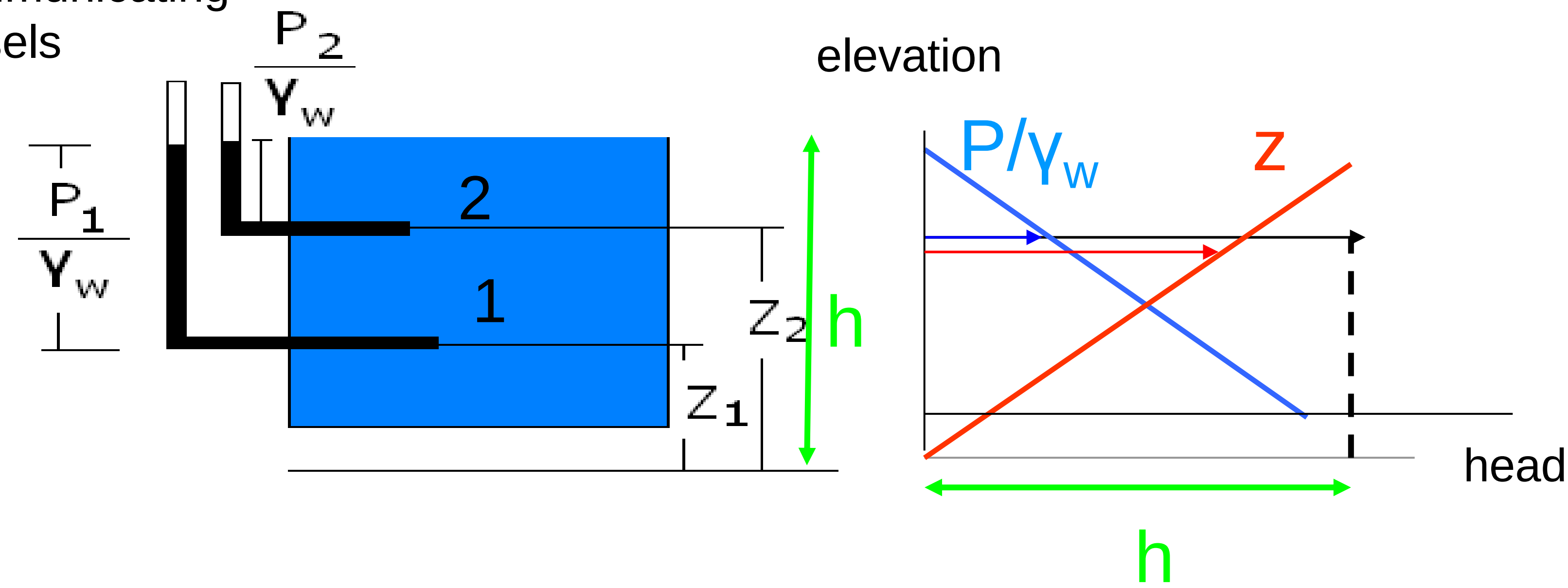
head loss

piezometric + geometric head

control groundwater movement

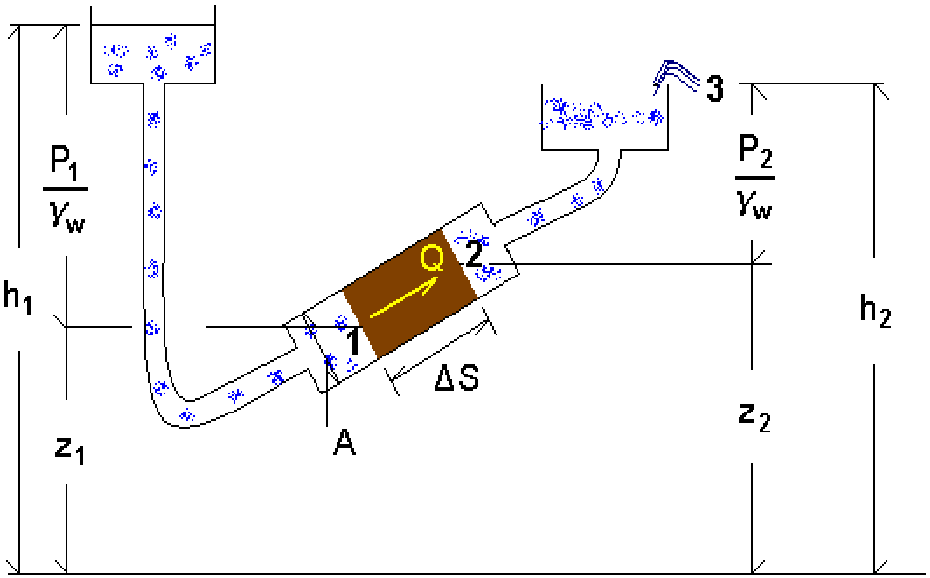
Total head in static water

Communicating vessels



- 1: $P_1/\gamma_w + z_1 =$
- 2: $P_2/\gamma_w + z_2 = h$

DARCY's Law



$$\frac{Q}{A} = k \cdot \frac{h_1 - h_2}{\Delta S}$$

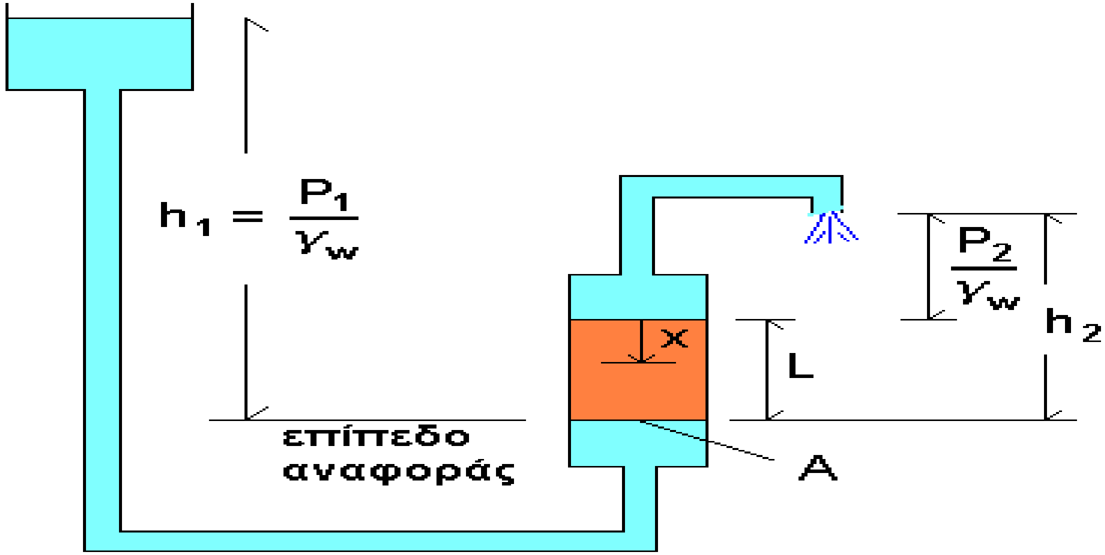
$\frac{Q}{A} = \upsilon =$ Flow velocity

$$i = \frac{h_2 - h_1}{\Delta S} =$$ Hydraulic gradient
υδραυλική κλίση

$\upsilon = -k \cdot i$ και $Q = -k \cdot i \cdot A$

$k =$ hydraulic conductivity, permeability (in velocity units)

Critical Hydraulic Gradient



$$Q = A \cdot k \cdot \frac{h_1 - h_2}{L} = A \cdot k \cdot (-i)$$

$$\sigma_v = p_2 + \gamma \cdot x$$

$$p_2 + \frac{p_1 - p_2}{L} \cdot x$$

$$\frac{p_1 - p_2}{L} = \gamma$$

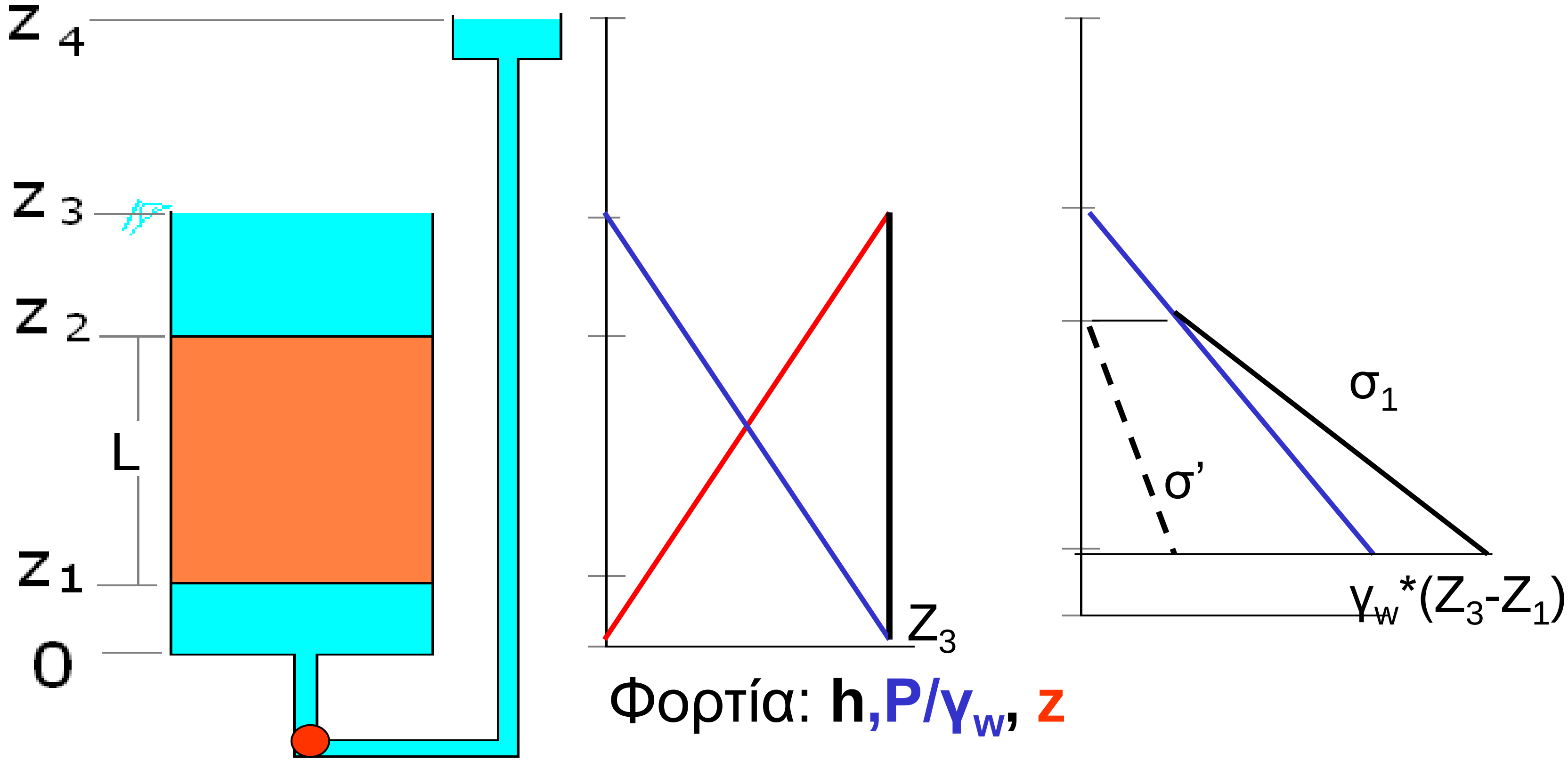
$$i_{crit} = \frac{h_2 - h_1}{L} = \frac{\frac{p_2}{\gamma_w} + L - \frac{p_1}{\gamma_w}}{L} = 1 - \frac{p_1 - p_2}{\gamma_w \cdot L}$$

$$-i_{crit} = \frac{\gamma}{\gamma_w} - 1$$

QUICK SAND

1. Sand has no strength due to LIQUEFACTION or due to SOIL PIPING
2. Can we drown in quick sand?

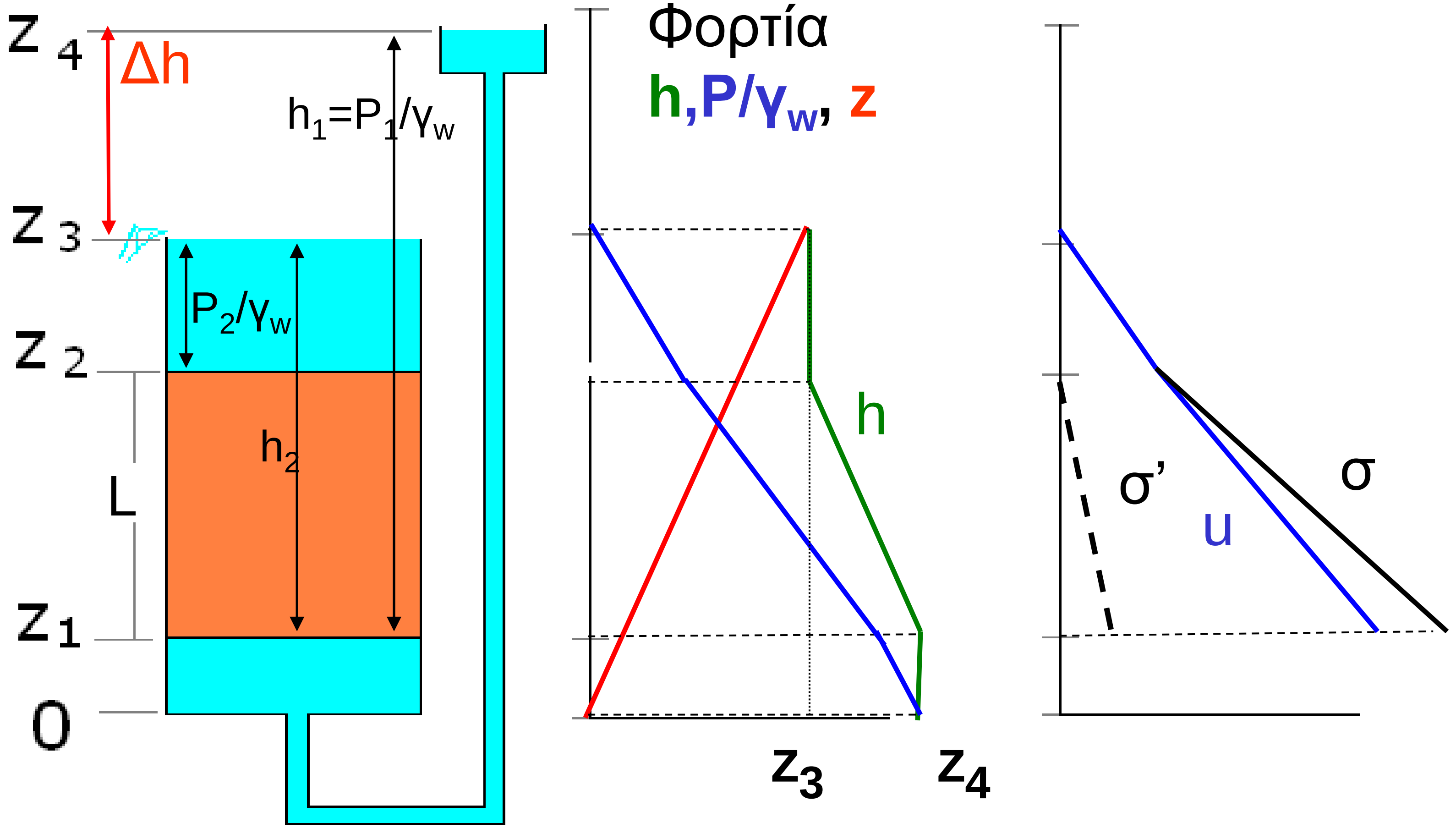
Effective stresses under hydrostatic conditions



- $\sigma_1{}'_{(z1)} = \gamma_w(z_3 - z_2) + \gamma(z_2 - z_1) - \gamma_w(z_3 - z_1) = \gamma(z_2 - z_1) - \gamma_w(z_2 - z_1) = \gamma' L$

$\sigma_{(z1)}$ P ή u

Effective stresses under seepage conditions



- $\sigma_{(z1)} = \gamma_w (z_3 - z_2) + \gamma (z_2 - z_1)$
- $u = \gamma_w (z_4 - z_1)$

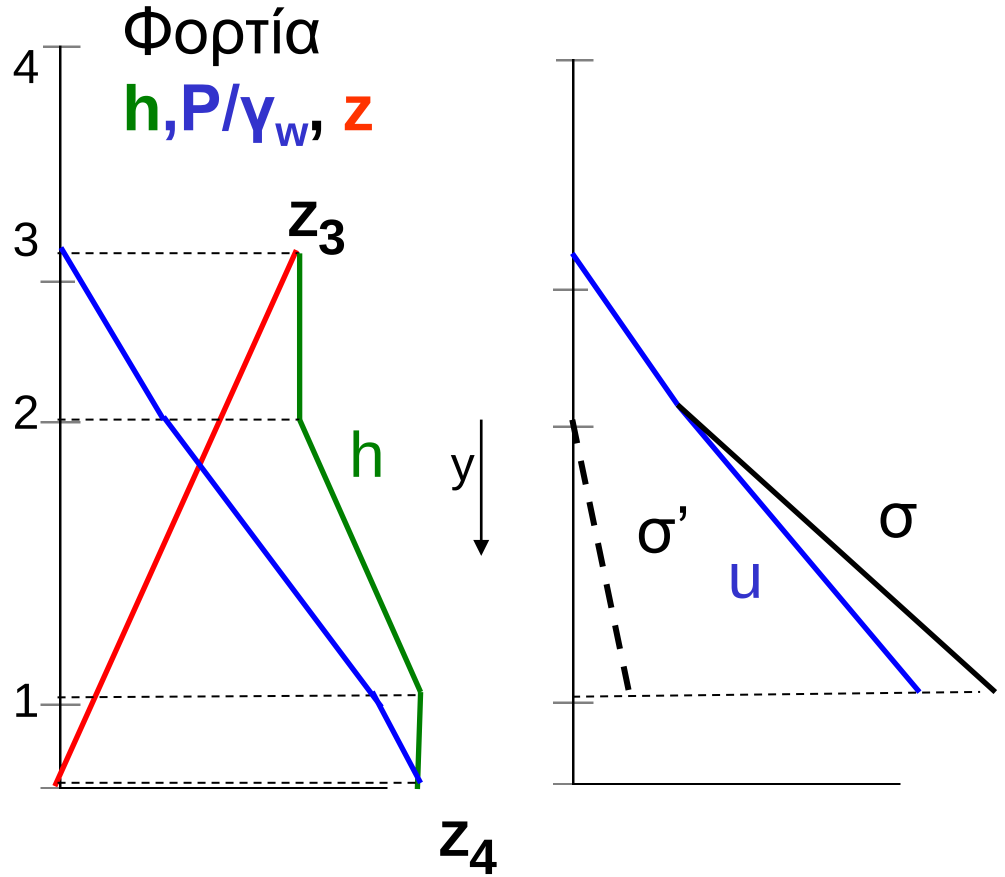
$$\sigma'_{(z1)} = \gamma (z_2 - z_1) - \gamma_w (z_2 - z_1) - \gamma_w (z_4 - z_3) = \gamma' L - \gamma_w \Delta h$$

Effective stresses under seepage conditions

- $\sigma = \gamma_w(z_3 - z_2) + \gamma^* y$
- $u = \gamma_w(z_3 - z_2)$ +

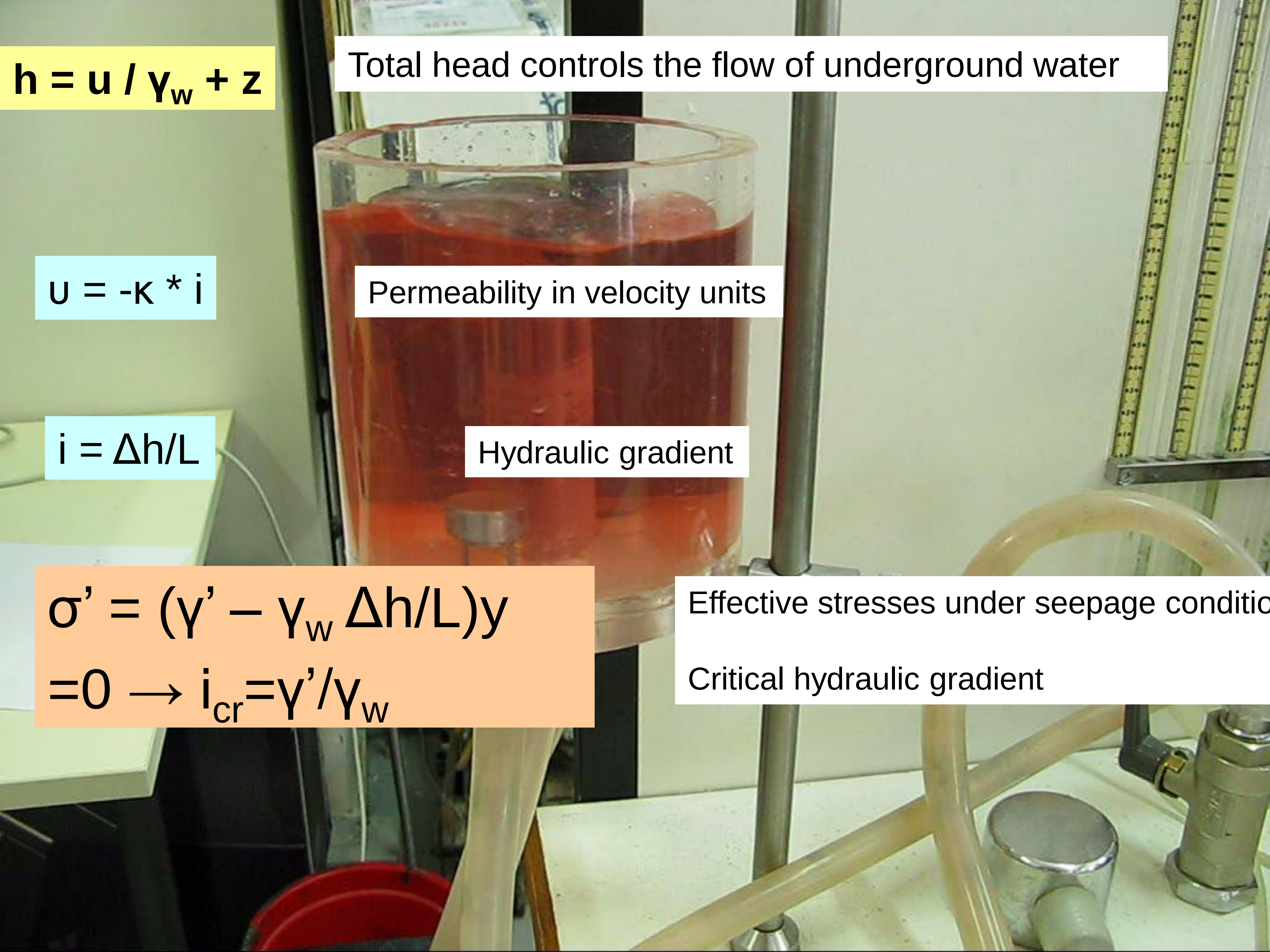
$\gamma_w((z_4 - z_1) - (z_3 - z_2))^* y/L$

σ' = σ-u = $\gamma^* y - \gamma_w(\Delta h - (z_2 - z_1))^* y/L$



$\sigma' = (\gamma^* y - \gamma_w \Delta h^* y/L - \gamma_w^* L^* y/L) = (\gamma' - \gamma_w \Delta h/L) y$

$= 0 \rightarrow i_{cr} = \gamma'/\gamma_w$



$h = u / \gamma_w + z$

Total head controls the flow of underground water

$u = -\kappa * i$

Permeability in velocity units

$i = \Delta h/L$

Hydraulic gradient

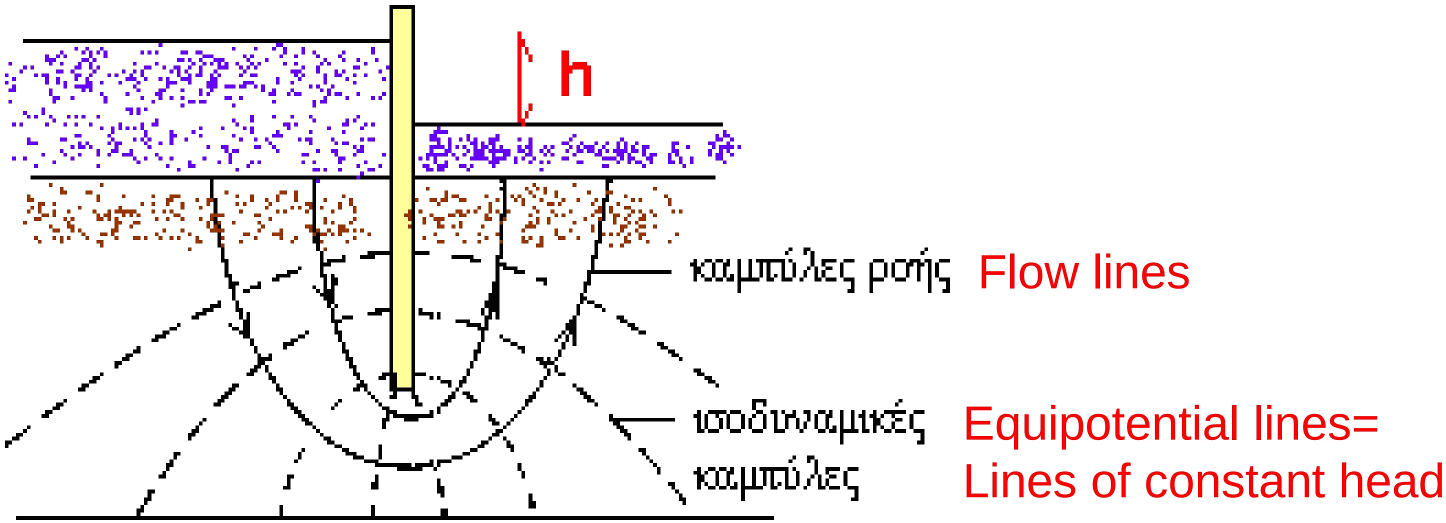
$\sigma' = (\gamma' - \gamma_w \Delta h/L)y$

$= 0 \rightarrow i_{cr} = \gamma'/\gamma_w$

Effective stresses under seepage conditio

Critical hydraulic gradient

Two dimensional seepage



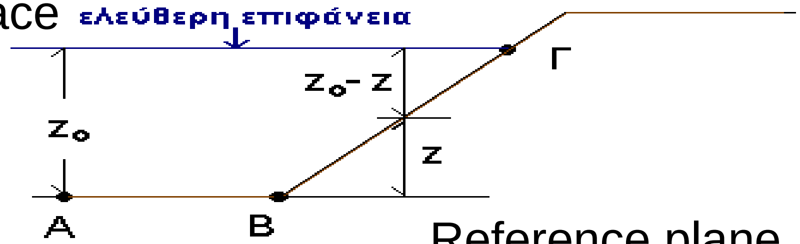
a graphical representation of two-dimensional steady-state groundwater flow equation

$$\frac{\partial^2 h}{\partial z^2} + \frac{\partial^2 h}{\partial x^2} = 0$$

- Under two dimensional seepage the sum of the change in hydraulic gradient in the two directions is zero

Boundary conditions

Free surface

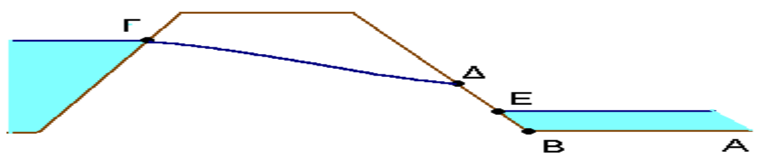


Reference plane

$$h = \frac{p}{\gamma_w} + z$$

$$p = \gamma_w \cdot (z_o - z)$$

$$h = z_o - z + z = z_o$$

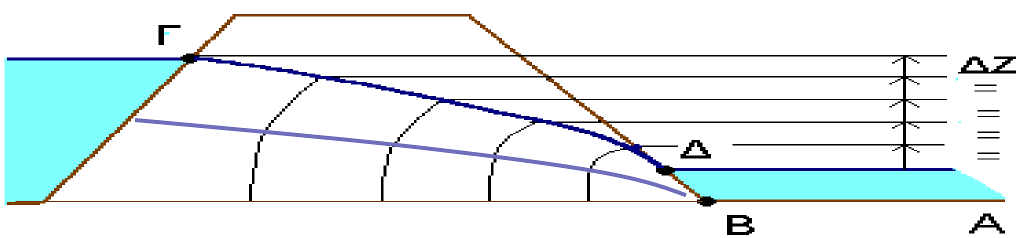


$$h = \frac{p}{\gamma_w} + z$$

$$p = 0$$

$$h = z$$

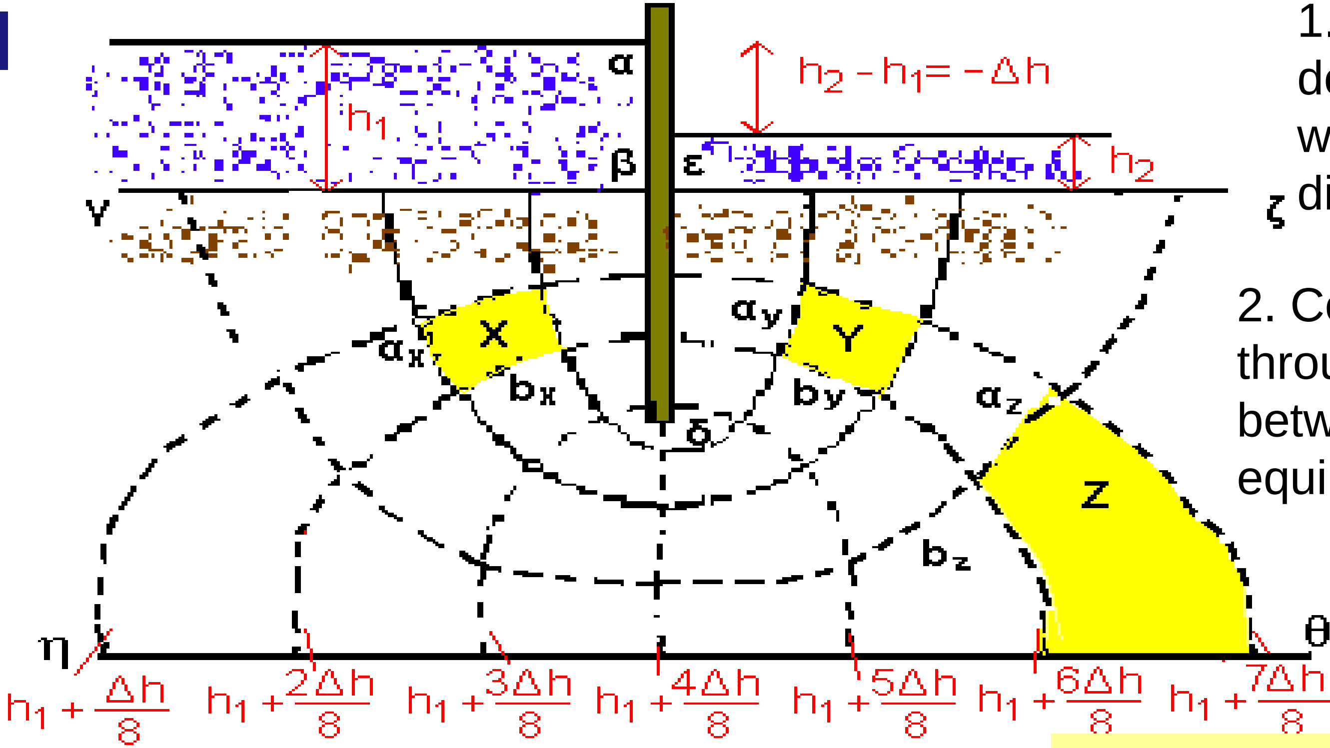
Freatic line



Each interval between 2 equipotentials corres-ponds to a head loss Δh equal to 1/N of the total head loss h through the soil: Δh=h/N

Equipotentials intersect the freatic line at constant drops in height

Ισοδυναμικές τέμνουν τη φρεατική γραμμή ανά ίσες μεταβολές στάθμης



1. A pair of flow lines define a tube (channel where flow is one dimensional

2. Consider the flow through an area delimited between 2 flow and 2 equipotential lines

$$q_x = -k \cdot \frac{-\frac{\Delta h}{8}}{a_x} \cdot b_x \cdot 1 \qquad q_y = -k \cdot \frac{-\frac{\Delta h}{8}}{a_y} \cdot b_y \cdot 1 \qquad q_z = -k \cdot \frac{-\frac{\Delta h}{8}}{a_z} \cdot b_z \cdot 1$$

$$q_x = q_y \longrightarrow \frac{b_x}{a_x} = \frac{b_y}{a_y} \text{ εάν } \frac{b_z}{a_z} = \frac{b_y}{a_y} \longrightarrow q_z = q_y$$

$$q_x = q_y = q_z$$

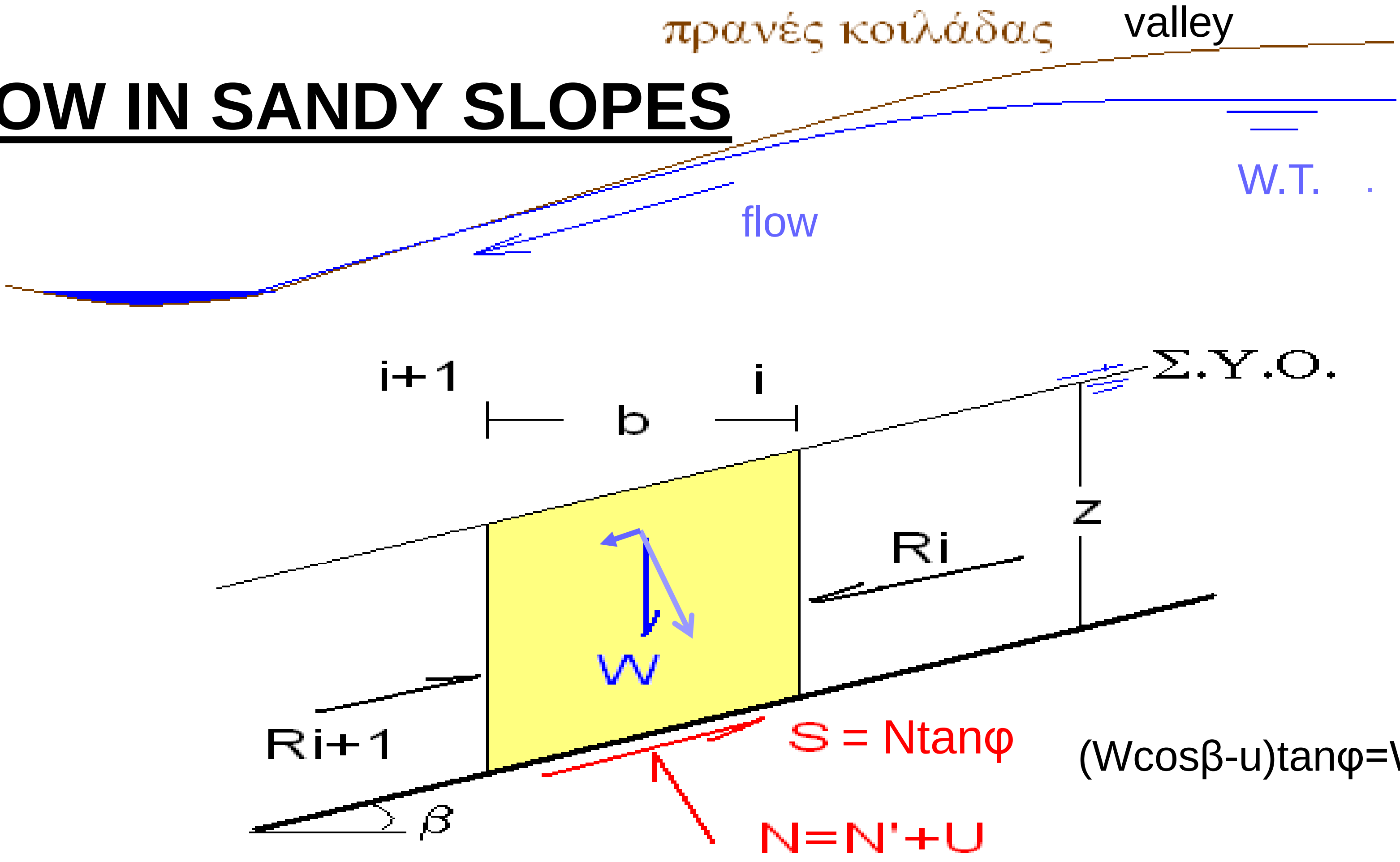
$N_\rho =$ Number of flow channels (4)

Εάν $N_\delta =$ Number of drops in head (8)

$$Q = N_\rho \cdot q_x = -N_\rho \cdot k \cdot \frac{-H}{N_\delta} \cdot \frac{b_x}{a_x} = k \cdot (h_1 - h_2) \cdot \frac{N_\rho}{N_\delta}$$

ΑΠΕΙΡΟΜΗΚΕΣ ΠΡΑΝΕΣ ΣΕ *ΑΜΜΟ*

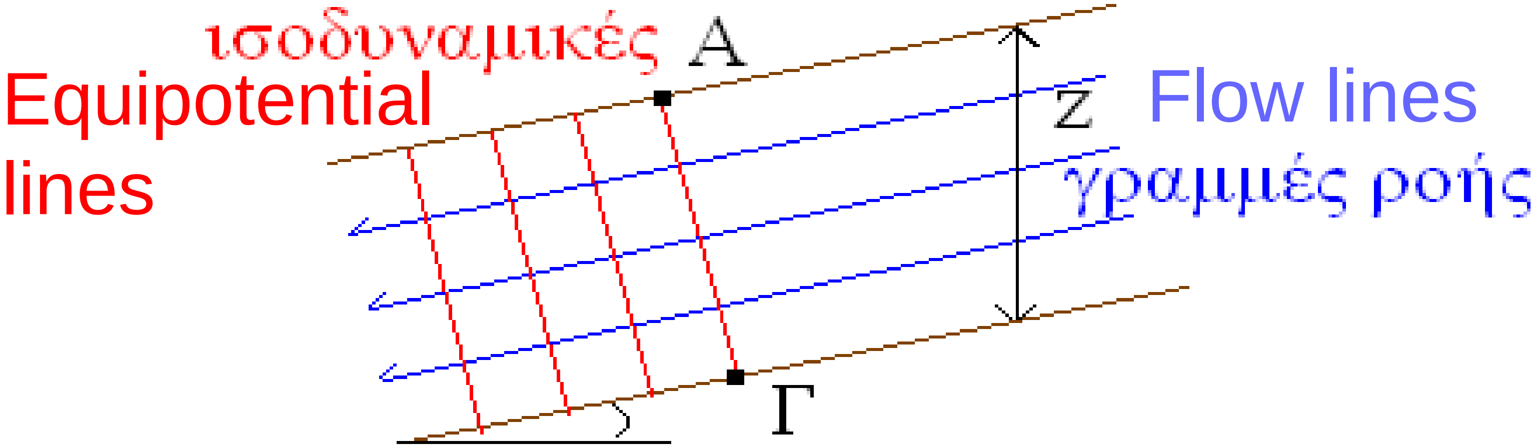
FLOW IN SANDY SLOPES



(Wcosβ-u)tanφ=Wsinβ

Flow reduces slope steepness by 50 percent

FLOW IN SLOPES



$$h_A = h_\Gamma = u_A / \gamma_w + z_A = u_\Gamma / \gamma_w + z_\Gamma \rightarrow$$

$$u_\Gamma = (z_A - z_\Gamma) * \gamma_w = A\Gamma * \cos\beta * \gamma_w =$$

$$\gamma_w * \cos^2\beta * z$$

c'=0 & seepage: phreatic line at soil surface

$u = \gamma_w * z * \cos^2\beta$

$(W\cos\beta - \gamma_w \cos^2\beta z)\tan\varphi = W\sin\beta \rightarrow [(1 - \gamma_w \cos^2\beta z / (\gamma z \cos\beta)\cos\beta]\tan\varphi = \tan\beta$

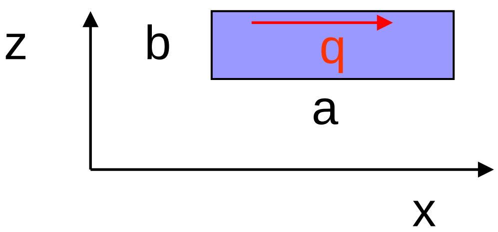
$\mathbf{(1-\gamma_w/\gamma)\tan\varphi' = \tan\beta}$ (for FS=1)$\rightarrow$

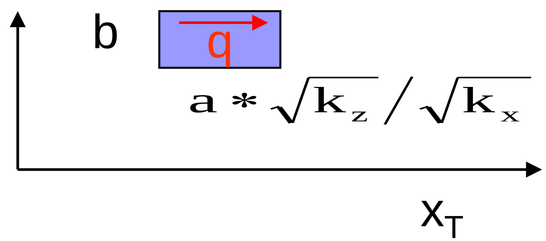
$\tan\beta = (\gamma'/\gamma) * \tan\varphi' \sim \tan\varphi'/2$

Anisotropic flow conditions

$$k_z \cdot \frac{\partial^2 h}{\partial z^2} + k_x \cdot \frac{\partial^2 h}{\partial x^2} = 0 \longrightarrow \frac{\partial^2 h}{\partial z^2} + \frac{\partial^2 h}{(\frac{k_z}{k_x}) \cdot \partial x^2} = 0$$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial x_T^2} = 0 \xrightarrow{\text{οπου}} x_T = (\frac{k_z}{k_x})^{\frac{1}{2}} \cdot x$$





$q = -k_x * (\delta h/a) * b * 1 \; = \; q = -k' * (\delta h/a) * (\sqrt{k_x} / \sqrt{k_z}) * b * 1 \rightarrow k' = \sqrt{k_x} * \sqrt{k_z}$

$$Q = \sqrt{k_x \cdot k_z} \cdot (h_1 - h_2) \cdot \frac{N_\rho}{N_\delta}$$