18.0

Or in pseudovector-matrix form, we can write

$$\begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{8}$$

Noting that the differential operator matrix in the brackets is just the transform of the one that appeared in Eqn. 7 of Module 8, we can write this as:

$$\mathbf{L}^{\mathrm{T}}\boldsymbol{\sigma} = \mathbf{0} \tag{9}$$

Example 2

It isn't hard to come up with functions of stress that satisfy the equilibrium equations; any constant will do, since the stress gradients will then be identically zero. The catch is that they must satisfy the boundary conditions as well, and this complicates things considerably. Later modules will outline several approaches to solving the equations directly, but in some simple cases a solution can be seen by inspection.

Figure 6: A tensile specimen.

Consider a tensile specimen subjected to a load P as shown in Fig. 6. A trial solution that certainly satisfies the equilibrium equations is

$$[\sigma] = \begin{bmatrix} c & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where c is a constant we must choose so as to satisfy the boundary conditions. To maintain horizontal equilibrium in the free-body diagram of Fig. 6(b), it is immediately obvious that $cA = P$, or $\sigma_x = c = P/A$. This familiar relation was used in Module 1 to define the stress, but we see here that it can be viewed as a consequence of equilibrium considerations rather than a basic definition.

Problems

1. Determine whether the following stress state satisfies equilibrium: (χωρίς μαζικές δυνάμεις)

$$[\sigma] = \begin{bmatrix} 2x^3y^2 & -2x^2y^3 \\ -2x^2y^3 & xy^4 \end{bmatrix}$$

2. Develop the two-dimensional form of the Cartesian equilibrium equations by drawing a free-body diagram of an infinitesimal section:

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Από ~~eq dvi~~ eq.pdf, problem 1, page 5.

$$\sigma_x = 2x^3y^2 \qquad \tau_{xy} = \tau_{yx} = -2x^2y^3$$

$$\sigma_y = xy^4$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} = 6x^2y^2 + (-6x^2y^2) = 0 \quad \checkmark$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = -4xy^3 + 4xy^3 = 0 \quad \checkmark$$

Ικανοποιούνται