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E. Hoek, C. Carranza-Torres and B. Corkum

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The program RocLab mentioned in this paper will be available for downloading (free) after the meeting from www.rocscience.com.

HOEK-BROWN FAILURE CRITERION – 2002 EDITION

Evert Hoek
Consulting Engineer, Vancouver, Canada

Carlos Carranza-Torres
Itasca Consulting Group Inc., Minneapolis, USA

Brent Corkum
Rocscience Inc., Toronto, Canada

ABSTRACT: The Hoek-Brown failure criterion for rock masses is widely accepted and has been applied in a large number of projects around the world. While, in general, it has been found to be satisfactory, there are some uncertainties and inaccuracies that have made the criterion inconvenient to apply and to incorporate into numerical models and limit equilibrium programs. In particular, the difficulty of finding an acceptable equivalent friction angle and cohesive strength for a given rock mass has been a problem since the publication of the criterion in 1980. This paper resolves all these issues and sets out a recommended sequence of calculations for applying the criterion. An associated Windows program called "RocLab" has been developed to provide a convenient means of solving and plotting the equations presented in this paper.

1. INTRODUCTION

Hoek and Brown [1, 2] introduced their failure criterion in an attempt to provide input data for the analyses required for the design of underground excavations in hard rock. The criterion was derived from the results of research into the brittle failure of intact rock by Hoek [3] and on model studies of jointed rock mass behaviour by Brown [4]. The criterion started from the properties of intact rock and then introduced factors to reduce these properties on the basis of the characteristics of joints in a rock mass. The authors sought to link the empirical criterion to geological observations by means of one of the available rock mass classification schemes and, for this purpose, they chose the Rock Mass Rating proposed by Bieniawski [5].

Because of the lack of suitable alternatives, the criterion was soon adopted by the rock mechanics community and its use quickly spread beyond the original limits used in deriving the strength reduction relationships. Consequently, it became necessary to re-examine these relationships and to introduce new elements from time to time to account for the wide range of practical problems to which the criterion was being applied. Typical of these enhancements were the introduction of the idea of "undisturbed" and "disturbed" rock masses Hoek and Brown [6], and the introduction of a modified criterion to force the rock mass tensile strength to zero for very poor quality rock masses (Hoek, Wood and Shah, [7]).

One of the early difficulties arose because many geotechnical problems, particularly slope stability issues, are more conveniently dealt with in terms of shear and normal stresses rather than the principal stress relationships of the original Hoek-Brown criterion, defined by the equation:

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m\frac{\sigma_3'}{\sigma_{ci}} + s\right)^{0.5} \tag{1}$$

where σ_1' and σ_3' are the major and minor effective principal stresses at failure
σ_{ci} is the uniaxial compressive strength of the intact rock material and
m and s are material constants, where s = 1 for intact rock.

An exact relationship between equation 1 and the normal and shear stresses at failure was derived by J. W. Bray (reported by Hoek [8]) and later by Ucar [9] and Londe[1] [10].

Hoek [12] discussed the derivation of equivalent friction angles and cohesive strengths for various practical situations. These derivations were based upon tangents to the Mohr envelope derived by

[1] Londe's equations were later found to contain errors although the concepts introduced by Londe were extremely important in the application of the Hoek-Brown criterion to tunnelling problems (Carranza-Torres and Fairhurst, [11])

Bray. Hoek [13] suggested that the cohesive strength determined by fitting a tangent to the curvilinear Mohr envelope is an upper bound value and may give optimistic results in stability calculations. Consequently, an average value, determined by fitting a linear Mohr-Coulomb relationship by least squares methods, may be more appropriate. In this paper Hoek also introduced the concept of the Generalized Hoek-Brown criterion in which the shape of the principal stress plot or the Mohr envelope could be adjusted by means of a variable coefficient a in place of the square root term in equation 1.

Hoek and Brown [14] attempted to consolidate all the previous enhancements into a comprehensive presentation of the failure criterion and they gave a number of worked examples to illustrate its practical application.

In addition to the changes in the equations, it was also recognised that the Rock Mass Rating of Bieniawski was no longer adequate as a vehicle for relating the failure criterion to geological observations in the field, particularly for very weak rock masses. This resulted in the introduction of the Geological Strength Index (GSI) by Hoek, Wood and Shah [7], Hoek [13] and Hoek, Kaiser and Bawden [15]. This index was subsequently extended for weak rock masses in a series of papers by Hoek, Marinos and Benissi [16], Hoek and Marinos [17, 18] and Marinos and Hoek [19].

The Geological Strength Index will not be discussed in the following text, which will concentrate on the sequence of calculations now proposed for the application of the Generalized Hoek Brown criterion to jointed rock masses.

2. *GENERALIZED HOEK-BROWN CRITERION*

This is expressed as

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s\right)^a \qquad (2)$$

where m_b is a reduced value of the material constant m_i and is given by

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right) \qquad (3)$$

s and a are constants for the rock mass given by the following relationships:

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \qquad (4)$$

$$a = \frac{1}{2} + \frac{1}{6}\left(e^{-GSI/15} - e^{-20/3}\right) \qquad (5)$$

D is a factor which depens upon the degree of disturbance to which the rock mass has been subjected by blast damage and stress relaxation. It varies from 0 for undisturbed in situ rock masses to 1 for very disturbed rock masses. Guidelines for the selection of D are discussed in a later section.

The uniaxial compressive strength is obtained by setting $\sigma_3' = 0$ in equation 2, giving:

$$\sigma_c = \sigma_{ci} . s^a \qquad (6)$$

and, the tensile strength is:

$$\sigma_t = -\frac{s\sigma_{ci}}{m_b} \qquad (7)$$

Equation 7 is obtained by setting $\sigma_1' = \sigma_3' = \sigma_t$ in equation 2. This represents a condition of biaxial tension. Hoek [8] showed that, for brittle materials, the uniaxial tensile strength is equal to the biaxial tensile strength.

Note that the "switch" at GSI = 25 for the coefficients s and a (Hoek and Brown, [14]) has been eliminated in equations 4 and 5 which give smooth continuous transitions for the entire range of GSI values. The numerical values of a and s, given by these equations, are very close to those given by the previous equations and it is not necessary for readers to revisit and make corrections to old calculations.

Normal and shear stresses are related to principal stresses by the equations published by Balmer[2] [20].

$$\sigma_n' = \frac{\sigma_1' + \sigma_3'}{2} - \frac{\sigma_1' - \sigma_3'}{2} \cdot \frac{d\sigma_1'/d\sigma_3' - 1}{d\sigma_1'/d\sigma_3' + 1} \qquad (8)$$

$$\tau = (\sigma_1' - \sigma_3')\frac{\sqrt{d\sigma_1'/d\sigma_3'}}{d\sigma_1'/d\sigma_3' + 1} \qquad (9)$$

where

$$d\sigma_1'/d\sigma_3' = 1 + am_b\left(m_b\sigma_3'/\sigma_{ci} + s\right)^{a-1} \qquad (10)$$

3. *MODULUS OF DEFORMATION*

The rock mass modulus of deformation is given by:

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2 The original equations derived by Balmer contained errors that have been corrected in equations 8 and 9.

$$E_m(GPa) = \left(1 - \frac{D}{2}\right)\sqrt{\frac{\sigma_{ci}}{100}} \cdot 10^{((GSI-10)/40)} \qquad (11)$$

Note that the original equation proposed by Hoek and Brown [14] has been modified, by the inclusion of the factor D, to allow for the effects of blast damage and stress relaxation.

4. MOHR-COULOMB CRITERION

Since most geotechnical software is still written in terms of the Mohr-Coulomb failure criterion, it is necessary to determine equivalent angles of friction and cohesive strengths for each rock mass and stress range. This is done by fitting an average linear relationship to the curve generated by solving equation 2 for a range of minor principal stress values defined by $\sigma_t < \sigma_3 < \sigma'_{3\max}$, as illustrated in Figure 1. The fitting process involves balancing the areas above and below the Mohr-Coulomb plot. This results in the following equations for the angle of friction ϕ' and cohesive strength c':

$$\phi' = \sin^{-1}\left[\frac{6am_b(s + m_b\sigma'_{3n})^{a-1}}{2(1+a)(2+a) + 6am_b(s + m_b\sigma'_{3n})^{a-1}}\right] \qquad (12)$$

$$c' = \frac{\sigma_{ci}\left[(1+2a)s + (1-a)m_b\sigma'_{3n}\right](s + m_b\sigma'_{3n})^{a-1}}{(1+a)(2+a)\sqrt{1 + \left(6am_b(s + m_b\sigma'_{3n})^{a-1}\right)/\left((1+a)(2+a)\right)}} \qquad (13)$$

where $\sigma_{3n} = \sigma'_{3\max}/\sigma_{ci}$

Note that the value of $\sigma'_{3\max}$, the upper limit of confining stress over which the relationship between the Hoek-Brown and the Mohr-Coulomb criteria is considered, has to be determined for each individual case. Guidelines for selecting these values for slopes as well as shallow and deep tunnels are presented later.

The Mohr-Coulomb shear strength τ, for a given normal stress σ, is found by substitution of these values of c' and ϕ' in to the equation:

$$\tau = c' + \sigma \tan\phi' \qquad (14)$$

The equivalent plot, in terms of the major and minor principal stresses, is defined by:

$$\sigma'_1 = \frac{2c'\cos\phi'}{1-\sin\phi'} + \frac{1+\sin\phi'}{1-\sin\phi'}\sigma'_3 \qquad (15)$$

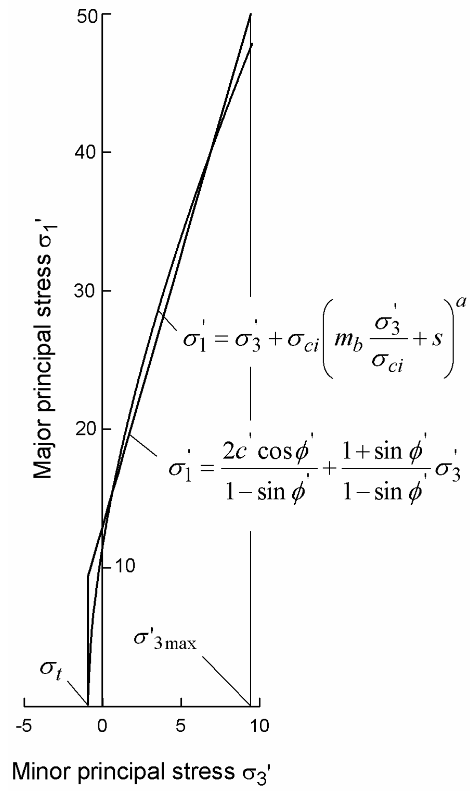

Figure 1: Relationships between major and minor principal stresses for Hoek-Brown and equivalent Mohr-Coulomb criteria.

5. ROCK MASS STRENGTH

The uniaxial compressive strength of the rock mass σ_c is given by equation 6. Failure initiates at the boundary of an excavation when σ_c is exceeded by the stress induced on that boundary. The failure propagates from this initiation point into a biaxial stress field and it eventually stabilizes when the local strength, defined by equation 2, is higher than the induced stresses σ'_1 and σ'_3. Most numerical models can follow this process of fracture propagation and this level of detailed analysis is very important when considering the stability of excavations in rock and when designing support systems.

However, there are times when it is useful to consider the overall behaviour of a rock mass rather than the detailed failure propagation process described above. For example, when considering the strength of a pillar, it is useful to have an estimate of the overall strength of the pillar rather than a detailed knowledge of the extent of fracture propagation in the pillar. This leads to the concept of a global "rock mass strength" and Hoek and Brown [14] proposed that this could be estimated from the Mohr-Coulomb relationship:

$$\sigma'_{cm} = \frac{2c'\cos\phi'}{1-\sin\phi'} \quad (16)$$

with c' and ϕ' determined for the stress range $\sigma_t < \sigma'_3 < \sigma_{ci}/4$ giving

$$\sigma'_{cm} = \sigma_{ci} \cdot \frac{(m_b + 4s - a(m_b - 8s))(m_b/4 + s)^{a-1}}{2(1+a)(2+a)} \quad (17)$$

6. *DETERMINATION OF σ'_{3MAX}*

The issue of determining the appropriate value of $\sigma'_{3\max}$ for use in equations 12 and 13 depends upon the specific application. Two cases will be investigated:

1. Tunnels – where the value of $\sigma'_{3\max}$ is that which gives equivalent characteristic curves for the two failure criteria for deep tunnels or equivalent subsidence profiles for shallow tunnels.
2. Slopes – here the calculated factor of safety and the shape and location of the failure surface have to be equivalent.

For the case of deep tunnels, closed form solutions for both the Generalized Hoek-Brown and the Mohr-Coulomb criteria have been used to generate hundreds of solutions and to find the value of $\sigma'_{3\max}$ that gives equivalent characteristic curves.

For shallow tunnels, where the depth below surface is less than 3 tunnel diameters, comparative numerical studies of the extent of failure and the magnitude of surface subsidence gave an identical relationship to that obtained for deep tunnels, provided that caving to surface is avoided.

The results of the studies for deep tunnels are plotted in Figure 2 and the fitted equation for both cases is:

$$\frac{\sigma'_{3\max}}{\sigma'_{cm}} = 0.47\left(\frac{\sigma'_{cm}}{\gamma H}\right)^{-0.94} \quad (18)$$

where σ'_{cm} is the rock mass strength, defined by equation 17, γ is the unit weight of the rock mass and H is the depth of the tunnel below surface. In cases where the horizontal stress is higher than the vertical stress, the horizontal stress value should be used in place of γH.

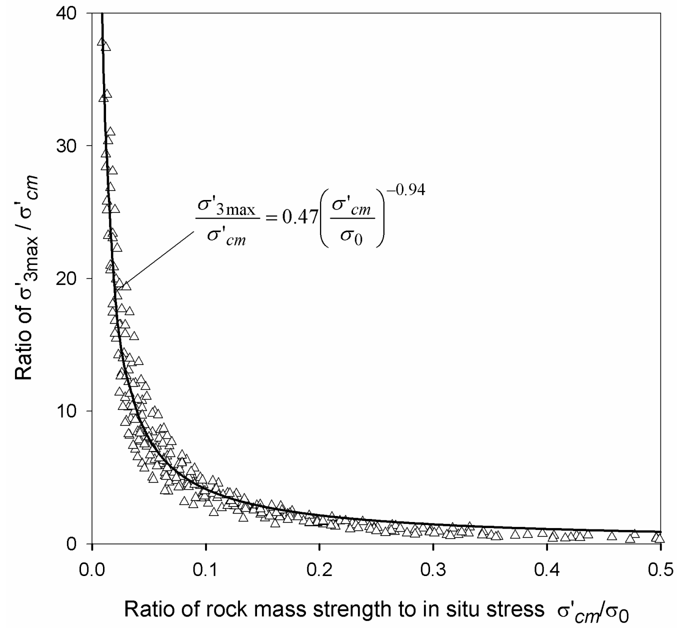

Figure 2: Relationship for the calculation of $\sigma'_{3\max}$ for equivalent Mohr-Coulomb and Hoek-Brown parameters for tunnels.

Equation 18 applies to all underground excavations, which are surrounded by a zone of failure that does not extend to surface. For studies of problems such as block caving in mines it is recommended that no attempt should be made to relate the Hoek-Brown and Mohr-Coulomb parameters and that the determination of material properties and subsequent analysis should be based on only one of these criteria.

Similar studies for slopes, using Bishop's circular failure analysis for a wide range of slope geometries and rock mass properties, gave:

$$\frac{\sigma'_{3\max}}{\sigma'_{cm}} = 0.72\left(\frac{\sigma'_{cm}}{\gamma H}\right)^{-0.91} \quad (19)$$

where H is the height of the slope.

7. *ESTIMATION OF DISTURBANCE FACTOR D*

Experience in the design of slopes in very large open pit mines has shown that the Hoek-Brown criterion for undisturbed in situ rock masses ($D = 0$) results in rock mass properties that are too optimistic [21, 22]. The effects of heavy blast damage as well as stress relief due to removal of the overburden result in disturbance of the rock mass. It is considered that the "disturbed" rock mass properties [6], $D = 1$ in equations 3 and 4, are more appropriate for these rock masses.

Lorig and Varona [23] showed that factors such as the lateral confinement produced by different radii of curvature of slopes (in plan) as compared with their height also have an influence on the degree of disturbance.

Sonmez and Ulusay [24] back-analysed five slope failures in open pit coal mines in Turkey and attempted to assign disturbance factors to each rock mass based upon their assessment of the rock mass properties predicted by the Hoek-Brown criterion. Unfortunately, one of the slope failures appears to be structurally controlled while another consists of a transported waste pile. The authors consider that the Hoek-Brown criterion is not applicable to these two cases.

Cheng and Liu [25] report the results of very careful back analysis of deformation measurements, from extensometers placed before the commencement of excavation, in the Mingtan power cavern in Taiwan. It was found that a zone of blast damage extended for a distance of approximately 2 m around all large excavations. The back-calculated strength and deformation properties of the damaged rock mass give an equivalent disturbance factor $D = 0.7$.

From these references it is clear that a large number of factors can influence the degree of disturbance in the rock mass surrounding an excavation and that it may never be possible to quantify these factors precisely. However, based on their experience and on an analysis of all the details contained in these papers, the authors have attempted to draw up a set of guidelines for estimating the factor D and these are summarised in Table 1.

The influence of this disturbance factor can be large. This is illustrated by a typical example in which σ_{ci} = 50 MPa, m_i = 10 and GSI = 45. For an undisturbed in situ rock mass surrounding a tunnel at a depth of 100 m, with a disturbance factor $D = 0$, the equivalent friction angle is $\phi' = 47.16°$ while the cohesive strength is $c' =$ 0.58 MPa. A rock mass with the same basic parameters but in highly disturbed slope of 100 m height, with a disturbance factor of $D = 1$, has an equivalent friction angle of $\phi' = 27.61°$ and a cohesive strength of $c' =$ 0.35 MPa.

Note that these are guidelines only and the reader would be well advised to apply the values given with caution. However, they can be used to provide a realistic starting point for any design and, if the observed or measured performance of the excavation turns out to be better than predicted, the disturbance factors can be adjusted downwards.

8. CONCLUSION

A number of uncertainties and practical problems in using the Hoek-Brown failure criterion have been addressed in this paper. Wherever possible, an attempt has been made to provide a rigorous and unambiguous method for calculating or estimating the input parameters required for the analysis. These methods have all been implemented in a Windows program called “RocLab” that can be downloaded (free) from www.rocscience.com. This program includes tables and charts for estimating the uniaxial compressive strength of the intact rock elements (σ_{ci}), the material constant m_i and the Geological Strength Index (*GSI*).

9. ACKNOWLEDGEMENTS

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Table 1: Guidelines for estimating disturbance factor D

Appearance of rock mass	Description of rock mass	Suggested value of D
	Excellent quality controlled blasting or excavation by Tunnel Boring Machine results in minimal disturbance to the confined rock mass surrounding a tunnel.	$D = 0$
	Mechanical or hand excavation in poor quality rock masses (no blasting) results in minimal disturbance to the surrounding rock mass. Where squeezing problems result in significant floor heave, disturbance can be severe unless a temporary invert, as shown in the photograph, is placed.	$D = 0$ $D = 0.5$ No invert
	Very poor quality blasting in a hard rock tunnel results in severe local damage, extending 2 or 3 m, in the surrounding rock mass.	$D = 0.8$
	Small scale blasting in civil engineering slopes results in modest rock mass damage, particularly if controlled blasting is used as shown on the left hand side of the photograph. However, stress relief results in some disturbance.	$D = 0.7$ Good blasting $D = 1.0$ Poor blasting
	Very large open pit mine slopes suffer significant disturbance due to heavy production blasting and also due to stress relief from overburden removal. In some softer rocks excavation can be carried out by ripping and dozing and the degree of damage to the slopes is less.	$D = 1.0$ Production blasting $D = 0.7$ Mechanical excavation

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