Some aspects of the finite element formulation

Field equations: 1) Balance of momentum (essentially equilibrium)
2) Balance of mass for the whole porous medium (global)
3) Balance of mass of ExtraFibrillar ions. (basically concerns 3 ions)
4) Balance of mass for the transfer between EF and IF.

Unknowns to be found via the FE solution
1) Displacement field of the solid phase $\vec{u}$ (vectorial field regarding dimensional space and nodal locations)
2) The electrochemical potentials μ^{ec}_{kE}
3) The intrafibrillar independet mass contents m^{iT}

All other unknowns (pressures, concentrations etc) can be found from these unknown fields.

Weak forms are needed for the 4 field equations.

E.g. for the momentum equation, the weak form is

$$\int_V \vec{\nabla}(\delta\vec{w}) : \underset{\sim}{\sigma}\, dV = \int_{\partial V} \delta\vec{w} \cdot \underset{\sim}{\sigma} \cdot \vec{n}\, dS$$

$\delta\vec{w}$: virtual displacement vector

Unknown's vector: (field unknowns)

$$\begin{bmatrix} \vec{u} \\ \vec{\mu}^{ec}_{wE} \\ \vec{\mu}^{ec}_{NaE} \\ \vec{\mu}^{ec}_{CaE} \\ \vec{\mu}^{ec}_{ceE} \\ \vec{m}^{wI} \\ \vec{m}^{NaI} \\ \vec{m}^{CaI} \\ \vdots \end{bmatrix} = X$$

$n_{sd} + 7$
↓
(order of space dimensions)

Residual to be minimised

$$R = F_{surf}(S, X) + F_{int}\left(X, \frac{dX}{dt}\right)$$

R: residual

F_{surf}: change to the residual due to interaction of the medium with the exterior space

$\mathbb{S}$: gereralised forces

$\mathbb{F}_{int}$: the part of $\mathbb{R}$ concerned with the volume inside the medium

$\frac{d\mathbb{X}}{dt}$: rate of change of $\mathbb{X}$ time.

Discritisation in space and in time

Existence of non-linearities. Newton-Raphson based methods are used for the solution.