Problem Set 3

Problem 1: X = B-space and $A \in \mathcal{L}_c(X)$.
Show that id $-$ A is injective and has a continuous inverse on $(\mathrm{id} - A)(X)$.

Problem 2: X, Y B-spaces and $A \in \mathcal{L}_c(X, Y)$.
Show that $R(A)$ is separable.

Problem 3: X = B-space, $\{A_n\}_{n \in \mathbb{N}} \subseteq \mathcal{L}(X)$ isomorphisms such that $\|A_n^{-1}\|_{\mathcal{L}} < 1$ $\forall n \in \mathbb{N}$, $A_n \to A$ in $\mathcal{L}(X)$. Show that A is an isomorphism too.

Problem 4: X = B-space, $A, B \in \mathcal{L}(X)$ and $AB = BA$.
Show that AB is invertible if and only if A and B are invertible.

Problem 5: X B-space and $A : X \to X^*$ linear operator such that $\langle A(u), u \rangle \geq 0$ $\forall u \in X$.
Show $A \in \mathcal{L}(X, X^*)$.