PROBLEM SET 2

PROBLEM 1: X = Banach space, $A : X \to X^*$ linear operator and $\langle A(u), x \rangle = \langle A(x), u \rangle$ $\forall\, x, u \in X$. Show that $A \in \mathcal{L}(X, Y)$.

PROBLEM 2: X = Banach space, $V \subseteq X$ vector subspace, $u \in X \setminus V$. Show that $\exists\, u^* \in X^*$, $\|u^*\|_{X^*} = 1$ and

$$\langle u^*, u \rangle = d(u, V), \quad u^*|_V = 0.$$

PROBLEM 3: X = Banach space. Show that the following are equivalent

(a) X = Reflexive;

(b) every decreasing sequence $\{C_n\}_{n \in \mathbb{N}}$ of nonempty, closed, convex and bounded sets has a nonempty intersection

PROBLEM 4: X = infinite dimensional Banach space and it is reflexive. Show that $\exists\, \{u_n\}_{n \in \mathbb{N}} \subseteq \partial B_1$ such that $u_n \xrightarrow{w} 0$.

PROBLEM 5: Show that every normed space is a dense vector subspace of a Banach space.