PROBLEM SET 5

PROBLEM 1: Let X be a separable metric space and $f: X \to \mathbb{R}$ a function. Let L be the set of all strict local minimizers of $f(\cdot)$. Show that L is at most countable.

PROBLEM 2: Let X be a topological and $f: X \to \mathbb{R}$ is a function. Show that $f(\cdot)$ is continuous <u>iff</u> $\forall \lambda \in \mathbb{R}$ the sets $\{f \geq \lambda\}$ and $\{f > \lambda\}$ are closed and open respectively.

PROBLEM 3: Let (X,d) be a metric space and $f: X \to \overline{\mathbb{R}} = \mathbb{R} \cup \{+\infty\}$ is a nontrivial function.

Show that $f(\cdot)$ is lower semicontinuous and bounded below <u>iff</u> $\exists$ a sequence $\{f_n\}_{n \in \mathbb{N}} \subseteq C_b(X)$ s.t $f_n \uparrow f$.

(<u>Hint</u>: Consider the functions $\hat{f}_n(x) = \inf_{y \in X} [f(y) + n d(x,y)]$)

PROBLEM 4: Show that the uniform limit of upper semicontinuous functions is upper semicontinuous.

PROBLEM 5: Let (X,d) be a complete metric space and $f: X \to X$ such that $f^{(k)}$ is a contraction for some $k \in \mathbb{N}$ ($f^{(k)} = \overbrace{f \circ f \circ \ldots \circ f}^{k\text{-times}}$). Show that $f(\cdot)$ has a unique fixed point.