PROBLEM SET 1.

PROBLEM 1: Let X, Y be Banach spaces, $A \in \mathcal{L}(X,Y)$ surjective. Show that there exists $M>0$ such that given $y \in Y$ we can find $x \in X$ with $\|x\|_X \leq M\|y\|_Y$ for which $A(x)=y$

PROBLEM 2: Let X be a normed space, Y a Banach space and $A \in \mathcal{L}(X,Y)$ a surjective isometry. Show that X is a Banach space.

PROBLEM 3: Let X, Y be normed spaces with $\dim X < \infty$ and $A: X \to Y$ a linear operator. Show that $A \in \mathcal{L}(X,Y)$

PROBLEM 4: Let X be an infinite dimensional Banach space and $C \subseteq X$ nonempty, compact. Show that $\operatorname{int} C = \emptyset$

PROBLEM 5: Let X be a normed space, $V \subseteq X$ a vector subspace. Assume that $\operatorname{int} V \neq \emptyset$. Show that $V = X$.